New Synthetic Seismogram Technique with Application to Coal Measures of the Central City Basin

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Abstract
The effect of shallow coal seams on recording seismic reflections from deeper beds is examined by means of synthetic seismograms. An expression based on acoustic impedance permits the calculation of reflection coefficient, as a function of frequency, and synthetic seismograms, with or without surface multiples. Results indicate that using lower frequencies (5-25 Hertz) would permit recording below the coal measures encountered in a particular hole in the Central Sydney Basin.

Introduction
Because of its low density and low velocity, coal provides a strong seismic contrast, compared to any normal sedimentary rock. Coal seams within a clastic sequence have been found to create strong seismic reflections and to effectively prevent the recording of reflections from deeper beds. For example, examination of field data and synthetic seismograms lead to the conclusion that Upper Permian coal measures of the Central Sydney Basin (Packham and Emerson, 1975) cause this kind of interference. The thought occurred that such a sequence might behave somewhat like a low-pass mechanical filter, where each coal seam would provide compliance and each sedimentary bed would move as a mass element. On this line of thought, reflections from deeper beds could be obtained by using lower frequencies, within the pass-band of the filter.

The lumped-element filter seemed an over-simplification, so an extract expression was derived which yields the reflection from and transmission through any sequence of beds. As an intermediate step, acoustic impedance looking into the sequence was derived, and it was recognized that the Fourier transform of the reciprocal of this impedance yields the usual synthetic seismogram, with all surface and inter-bed multiples included. Fourier transform of the reflection coefficient yields a synthetic seismogram without surface multiples. Comparison of the two types of seismograms for the coal measures encountered in the M.I.H. St. Albans No. 1 permits the distinction between loss of deep reflections due to low transmission and loss of deep reflections due to surface multiples.

Expressions for Synthetic Seismograms
Expressions have been derived for computing the reflection and transmission of plane waves within a layered medium, using matrices which describe the continuity of stress and displacement at the boundaries (Wueneschel, 1960). The same layered geometry is treated here, using an expression for acoustic impedance which is particularly simple (Morse, 1948, p. 238). For a wave in a medium with density \( \rho \), attenuation \( a \), and velocity \( c \), the acoustic impedance at a distance \( x \) above a boundary offering an impedance \( Z_f \) is:

\[
Z(x) = Z_0 \tanh(\phi + \gamma x)
\]

where \( \phi = \tanh^{-1}(Z_r / Z_0) \)

\[
\gamma = (a + j\omega/c) \quad Z_0 = (j\omega\rho/c)
\]

Applied to a sequence of layers with no attenuation, as sketched in Figure 1, the impedance at the top of the \( n \)th layer is:

\[
Z_n = \rho_n C_n \tanh(\tanh^{-1}(Z_{n-1}/\rho_n C_n) + j(\omega L_n/C_n))
\]

\[
Z_n = \rho_n C_n \left[ \frac{(Z_{n-1}/\rho_n C_n) + i\tan(\omega L_n/C_n)}{1 + (Z_{n-1}/\rho_n C_n)i\tan(\omega L_n/C_n)} \right]
\]

\[
Z_n = \rho_n C_n \left[ \frac{Z_{n-1} + i\rho_n C_n \tan(\omega L_n/C_n)}{\rho_n C_n + i Z_{n-1} \tan(\omega L_n/C_n)} \right]
\]

We note that

\[
P_n = Z_n V_n \quad V_n = (1/Z_n) P_n
\]

Where \( P_n e^{i\omega t} \) is the pressure (negative of normal stress) at the top of the \( n \)th layer and \( V_n e^{i\omega t} \) is the particle velocity. If we specified \( P_n \) at the top layer, then \( V_n e^{i\omega t} \) would be the velocity at the top. This corresponds to a pressure source on a free surface, and \( V_n \) includes all multiples with this free surface.

\[ p_n \quad c_n \quad L_n \quad Z_n \]

\[ p_2 \quad c_2 \quad L_2 \quad Z_n \]

\[ p_1 \quad c_1 \quad L_1 \quad Z_n \]

\[ p_0 \quad c_0 \quad L_0 \]

**FIGURE 1**  
Layered elastic solid.

If we instead considered a medium of density \( \rho_n \) and velocity \( C_n \) to be present above the \( \text{Nth} \) layer, then the reflection coefficient \( R \) at this interface is

\[
R = \frac{Z_n - \rho_n C_n}{Z_n + \rho_n C_n} \\
R = \frac{Z_n - \rho_n C_n}{Z_n + \rho_n C_n}
\]

or \( P_r = R P_i \).

In this case, a down-going incident wave is assumed, described by \( P_i e^{\text{j}2 \pi f t} \) at the boundary. This gives rise to an up-going reflected wave which at the boundary is \( P_r e^{\text{j}2 \pi f t} \).

The customary synthetic seismogram, that is, velocity waveform on the free surface, is obtained by inverse Fourier transform of \( V_n \). Inverse transform of \( P_r \) yields the reflection from a sequence of beds, without surface multiples, assuming that the layered sequence is overlain by a thick uniform medium.

**Computed Example**

Table I lists the properties of a layered sequence which corresponds to the coal measures encountered in one of the holes in the Sydney Basin reported by Packham and Emerson. The "deep reflector" at about 0.48 sec was put in to illustrate transmission through the coal measures and was not taken from the reference.

Figures 2 and 3 show the reflection record, 390 meters above the top coal seam, with the presence of a thick bed over the conglomerate which matches the conglomerate acoustically. For Fig. 2, the spectrum extends from 20-80 Hz; for Fig. 3 from 5-25 Hz. The incident waveform is shown in the upper, part of each figure, scaled so as to have a maximum peak height of unity. The contrast in acoustic impedance at the deep reflector gives a reflection coefficient of 0.093, so the deep reflection should have the same waveform as the incident pressure and a maximum value of 0.093. Similarly, reflection from the boundary between a thick conglomerate and a thick coal bed would be inverted, with a peak of \(-0.47\). Reflection from a single thin coal seam would be much weaker, but it is seen in Figure 2 that the composite reflection from the coal sequence does peaks in this range, with a waveform quite different from the incident signal. On the other hand, the waveform of the deep reflection is quite similar to the incident pulse, with a peak value of 0.054. In Figure 3, the coal seams again produce a distorted waveform, with maximum excursion less than 0.2. The deep reflector duplicates the low-frequency pulse, with a peak of 0.089.

**FIGURE 2**  
Waveform of reflected pressure due to a pressure pulse as shown arriving from above in a thick overburden. Frequency range 20-80 Hz.

**FIGURE 3**  
Waveform of reflected pressure due to a pressure pulse as shown arriving from above in a thick overburden. Frequency range 5-25 Hz.

Figure 4 shows that the magnitude of the reflection coefficient varies substantially with frequency. Below 35 Hertz, the average is about 0.2, whereas the average above 40 Hz is about 0.7. Multiple reflections with a free surface are drastically different in the two cases. For the higher frequencies, amplitudes of successive reflections would be 0.7, 0.49, 0.34, 0.24, etc.; for low frequencies, 0.2, 0.04, 0.008, etc. The potential for masking a deep reflector of amplitude 0.093 is obviously quite different. In Fig. 5, the reflection at a time of 0.48 is masked; in Fig. 6, the deep reflection is clear, as a relatively undistorted (inverted) version of the incident pulse.
Conclusions

Even a modest number (8) of coal seams in a conglomerate can give rise to very large reflection coefficients (above 0.5) at frequencies normally used in seismic prospecting, even though the coal seams are thin (two-way time of one to four milliseconds). High reactivity means poor transmission, and it might be anticipated that deep reflections would be lost because of lack of penetration. In the example treated this is not the case. In the band 20-80 Hertz, the deep reflection amplitude is 0.5 as great as it would have been without the coal sequence, not a drastic reduction. Multiples with the free surface simply masked the deep reflection. For the frequency band 5-25 Hertz, transmission through the coal sequence left the deep reflection at 0.96 of its full value and multiples with the free surface caused no serious masking. Going to the lower frequencies would provide a solution to the problem of mapping reflectors below the coal measures.

Acknowledgements

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References


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