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A model of impulse-driven coupled hydromagnetic waves in the magnetosphere

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Hydromagnetic oscillations in the magnetosphere may be separated into two modes; toroidal oscillations (azimuthal field motions) and poloidal oscillations (radial motions). These modes are usually coupled, but the nature of the coupling has only been studied analytically in restricted geometries (e.g. one-dimensional uniform field models) or restricted regions (e.g. near the equatorial plane). Most workers have considered steady-state solutions. In this paper, preliminary numerical solutions of the coupled hydromagnetic wave equations in a cylindrical model magnetosphere, under the action of an impulse on the outer (magnetopause) boundary, are presented.

The magnetosphere is represented by an infinitely long half cylinder, with circular magnetic field lines and magnetopause at $r = r_0$. The magnetic field strength decreases as r^{-1}, and s represents azimuth. The electric field components of the hydromagnetic wave E_r, E_s ($E_\theta = 0$) are defined to be

$$E_r = T(r,t)\, f(\theta) \exp(i\lambda s)$$

$$E_s = iP\,(r,t)\, f(\theta) \exp(i\lambda s)$$

where

$$f(\theta) = \cosh K\,(\pi/2-\theta)\sin(n\theta) - i\sinh K\,(\pi/2-\theta)\cos(n\theta)$$

represents a standing wave structure of harmonic n along the field line, and damping decrement K to simulate ionospheric dissipation.

The coupled hydromagnetic wave equations are:

$$\left[A_N^{-2}(x)\frac{\partial^2}{\partial\tau^2} + x^{-2}(n+iK)^2 + m^2\right]T = m\frac{\partial P}{\partial x}$$

$$\left[A_N^{-2}(x)\frac{\partial^2}{\partial\tau^2} + x^{-2}(n+iK)^2 - x^{-1}\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}\right)\right]P = -mx^{-1}\frac{\partial}{\partial x}(xT)$$

where $x = r/r_0$, $m = \lambda r_0$ is the azimuthal wavenumber, and $\tau = t/t_0$. $A_N(x)$ is the Alfven speed, normalized to the magnetopause value.

Note the wave mode coupling via m, and involving $\partial/\partial x$ of the perpendicular component. These equations are solved by analytical Laplace transformation, finite difference methods and subsequent numerical inverse Laplace transformation to give the time dependent solutions. The radial variation of the Alfven speed is chosen such that the resulting uncoupled eigen-periods of the axi-symmetric toroidal mode are the same as those in a dipole geomagnetic field where equatorial ion mass density is proportional to R^{-3} in the equatorial plane. An impulse is applied at $r = r_0$ in the azimuthal electric field component, corresponding to a radial motion of the magnetopause boundary.

The results presented consider $n = 1$ (fundamental resonance along the magnetic field), $m = 3$, and $K = 0.02$. Figure 1 shows the solutions for the 'azimuthal' and radial components of the electric field at selected radial positions x.

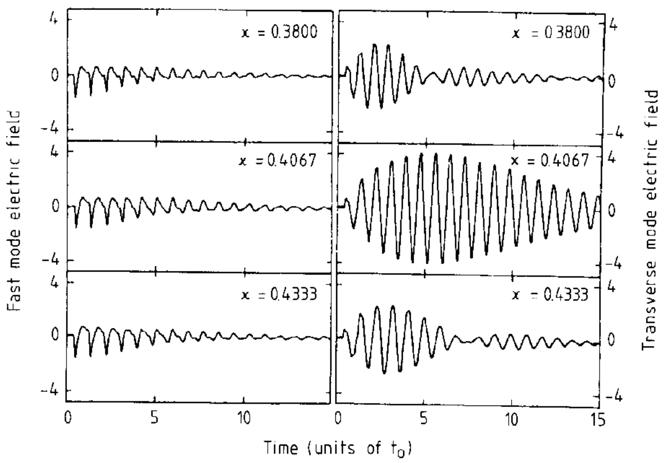

Fig 1 Waveforms of the fast mode and transverse mode electric field components at selected radial positions.

The azimuthal components show a decaying complex waveform after the arrival of the initial impulse. Fourier analysis shows the same set of discrete frequencies at all radial positions. The corresponding radial electric field components exhibit a period increase with x and a decaying wave packet structure.

The discrete azimuthal electric field components correspond to resonant fast mode (radial motion) oscillations within the magnetospheric cavity. Figure 2 displays the radial variation of the amplitude of these standing resonances up to order 4. Where the cavity resonance period matches the local toroidal mode (radial electric field) period, energy is coupled into a monochromatic toroidal mode resonance (Fig. 2). The combined effects of the monochromatic resonance and a transient oscillation at the local toroidal mode eigen-period result in the wave packet structure in Fig. 1. Note that the cavity resonances have the form of a quarter-wave and its higher harmonics, and that they appear to be evanescent inside the radial position of the appropriate toroidal mode resonance.

Monochromatic resonant ULF pulsations are usually attributed to steady-state driving forces such as the Kelvin-Helmholtz instability on the surface of the magnetopause. The above model shows that impulsive sources may also drive monochromatic resonances via the intermediary of magnetospheric cavity resonances.

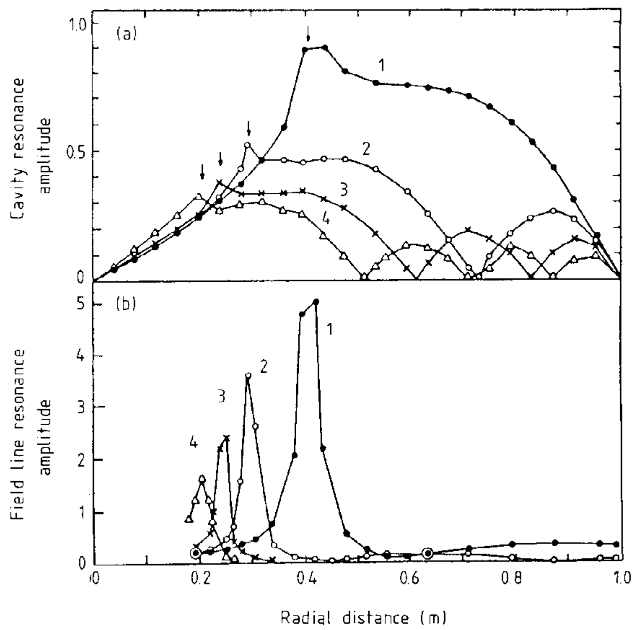

Fig 2 (a) Radial variation of the amplitude of the cavity resonance harmonics up to order 4 and (b) their associated toroidal mode monochromatic resonances.

The use of geomagnetic pulsations in determining magnetospheric plasma properties

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The existence of cold or cool heavy ions in the magnetospheric plasma of the earth has been known since early GEOS-I ion composition experiment results in 1977 (Geiss *et al.* 1978). Helium (He^+) and oxygen (O^+) relative concentrations of up to 50% of the total ion concentration were found. The reliability of these particle concentrations are difficult to estimate. Unresolved wave modulations, giving rise to plasma flows and other problems including spacecraft charging, make concentration measurements difficult. Mauk (1984) has recently shown, by computer simulation, that particle concentrations measured in association with linear interaction between a wave and He^+ ions may be artificially inflated by an order of magnitude. An extremely simple and sensitive measure of heavy ion (He^+, O^+) relative ion concentrations can be made using the bounding surfaces associated with the propagation of ion cyclotron waves in a multicomponent cold plasma. Obviously, measurements can be made only in the presence of ion cyclotron wave energy which occurs in the Pc1-2 geomagnetic pulsation frequency range (0.1-5 Hz).

The purpose of this paper is to present the properties of ion cyclotron waves seen by magnetometers on board the ATS-6 geostationary spacecraft at $L=6.6$ and interpret them in terms of simple multicomponent cold plasma propagation theory. This procedure leads to the application of wave diagnostic techniques to determine relative He^+ and O^+ ion concentrations.

The typical dynamic spectra recorded by ATS-6 show that the wave spectrum is organized by the He^+ cyclotron frequency (f_{He}). Two 4 month data sets of similar events are available, one from 1974 when ATS-6 was situated 12°N of the geomagnetic equator and the other when the spacecraft was on the geomagnetic equator. Figure 1 shows polarization ellipticity plotted as a function of wave frequency normalized to the equatorial proton cyclotron frequency (f_H). The $\lambda=12°$ data are mapped back to the equator using the Olsen-Pfitzer 1977 geomagnetic field model. The He^+ slot is apparent in both plots by the absence of data points around f_{He}. For the $\lambda=0°$ data most waves are LH-polarized with some almost linear cases observed. Off the equator the wave regime above