Generalised ray parameters for vertically inhomogeneous and anisotropic media

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SUMMARY

In this presentation we derive a concise equation for a generalised ray parameter in inhomogeneous and anisotropic media. We illustrate the direct application of this conserved quantity by several examples, involving anisotropic parameters and linear velocity fields.

The ray parameter, or raypath parameter, *p*, is a conserved quantity which plays an important role in both exploration and global seismology. The constancy of the ray parameter contributes to convenient methods of raytracing and imaging. The standard form of *p* in a homogeneous isotropic medium is the horizontal component of the slowless, expressed in terms of the angle measured from the normal to the interface between media, θ, and the velocity, *v*. This ray parameter, *p*, is a conserved quantity, ie the *first integral* of the *Euler-Lagrange* equation.

Increased interest in anisotropic characteristics of sedimentary rocks motivated this work, which provides an exact form of the ray parameter for vertically inhomogeneous and anisotropic media. We use the mathematical tools provided by *calculus of variations*, in particular the Euler-Lagrange equation with its first integrals, to arrive at this new form of the ray parameter. There always exists a conserved quantity, such as the ray parameter for arbitrarily complex velocity fields. For exploration seismology in sedimentary basins, a relevant form would account for vertical inhomogeneity *and* anisotropy. In such a case, the ray parameter resembles the standard form mentioned above with an additional term due to the anisotropy.

The results allow for convenient modelling and raytracing. They permit certain investigations of the influence of anisotropy on ray trajectories and traveltimes, which play an important role in AVO analysis. Furthermore, presented results could be incorporated in data-processing applications.

Key words: ray, parameter, anisotropy, inhomogeneity.

INTRODUCTION

The ray parameter, p, is a conserved quantity and as such is useful for raytracing and imaging purposes. Its standard form for a homogeneous and isotropic medium is

$$p = \frac{\sin\theta}{v}. \qquad (1)$$

Here we develop a form for a medium that is both vertically inhomogeneous and anisotropic, and in turn has the potential to increase the scope of applications for the ray parameter.

We use the Calculus of Variations to derive a first integral, which gives us a new conserved quantity in the form of a ray parameter that is valid for media with vertical inhomogeneity and anisotropy. This technique is powerful and has many applications (eg Epstein and Slawinski, 1998) and in this case is approached by solving the Euler-Lagrange equation, for the first integral.

In such a case, the ray parameter resembles equation (1) with an additional term due to the anisotropy. The results allow for convenient adaptation of existing algorithms for modelling and raytracing. They permit certain investigations of the influence of anisotropy on ray trajectories and traveltimes, which play an important role in AVO analysis. Furthermore, presented results could be incorporated in data-processing applications.

METHOD AND RESULTS

The traveltime, *t*, between two points, A and B, which might represent a source and a receiver, or any two points on the ray trajectory, can be expressed as

$$t = \int_A^B \frac{ds}{v(s)}, \qquad (2)$$

where *ds* denotes the element of distance and *v(s)* is the magnitude of speed over this infinitesimal distance. The integrand is called the Lagrange density, *L*. Fermats principle of stationary time implies a stationary value for the above integral, which can be determined using the calculus of variations (eg Fox 1987).

In two dimensions, for vertically inhomogeneous and anisotropic media equation (2) can be written as

$$t = \int_{B_x}^{A_x} \frac{\sqrt{1+z'}}{v(z,z')}dx, \qquad (3)$$

where *x* denotes the horizontal axis and $z' = dz/dx$ represents anisotropic dependence since

$$z' = \cot\theta, \qquad (4)$$

with θ being the group angle. The expression of anisotropy as a function of *z'* yields a convenient form to be used later in equation (8). In order to obtain the raypath trajectory such that *t* is minimised one needs to solve the Euler-Lagrange equation whose second form we use in this work, namely,

$$\frac{\partial L}{\partial x} - \frac{d}{dx}\left[z'\frac{\partial L}{\partial z'} - L\right] = 0. \qquad (5)$$

From equation (3) we can see that the Lagrange density, L, is explicitly independent of x, ie,

$$L = L(z, z') = \frac{\sqrt{1+z'}}{v(z, z')}, \qquad (6)$$

which in turn implies a simpler form for equation (5),

$$\frac{d}{dx}\left[z'\frac{\partial L}{\partial z'} - L\right] = 0. \qquad (7)$$

Consequently, from properties of differential calculus,

$$z'\frac{\partial L}{\partial z'} - L = -p, \qquad (8)$$

where p is a constant. This is precisely the conserved quantity of fundamental importance in physical considerations and in a seismological context, p, represents the ray parameter. We now apply equation (8) to equation (6) and arrive at an exact form of the ray parameter in terms of $v(z,\theta)$ and θ with the use of equation (4).

Inserting equation (6) into equation (8) yields

$$\frac{1}{v\sqrt{1+(z')^2}} - \frac{\partial}{\partial z'}\left(\frac{1}{v}\right) z'\sqrt{1+(z')^2} = p. \qquad (9)$$

Expression (9) is the conserved quantity. To state it in a more familiar fashion in terms of the propagation angle, θ, we use equation (4) and arrive at

$$p = \frac{\sin\theta}{v} + \cos\theta \frac{\partial}{\partial\theta}\left(\frac{1}{v}\right) \qquad (10)$$

Equation (10) is the ray parameter for vertically inhomogeneous and anisotropic media. The second term on the right hand side corresponds to the contribution of the anisotropic properties of the medium. If the medium is isotropic, ie $v(z,\theta) = v(z)$, then equation (10) reduces to the standard expression (1). The conserved quantity, p, stated in expression (10) results from the explicit absence of the argument x in the integral (3). No restrictions, however, are imposed on anisotropy. Hence, all types of anisotropy can be considered.

CONCLUSIONS

The work to be presented in this talk has followed the mathematical-physics approach of calculus of variations invoking the key seismological principle of Fermat, and the resultant notion of conserved quantities known as first integrals. The present lithological interest in anisotropy makes it a timely and relevant exploration tool. The notion of vertically inhomogeneous media appears to describe a large variety of real velocity fields. Moreover the linear velocity function, leading to convenient expressions, has been established as a good seismological description of many sedimentary basins (eg Slotnick, 1959).

The presented method shows the power of calculus of variations as a generalised tool for raypath analysis. Also, rayparameters for significantly more complex velocity fields (eg both horizontal and vertical inhomogeniety) exist, and while not necessarily exhibiting elegant expressions, can be incorporated in computer codes. The use of a conserved quantity provides a computationally viable option for both exactness and efficiency of processing algorithms.

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