Quantifying the errors in gravity reduction

Philip Heath *
Geological Survey of South Australia
4/101 Grenfell St Adelaide SA 5000
Philip.heath@sa.gov.au

**presenting author asterisked*

SUMMARY

Mistakes in processing gravity data lead to errors in the final product. This can mean that overlapping gravity surveys are often incompatible, and can lead to incorrect geological interpretations. In this paper I demonstrate the magnitude of the errors introduced at various stages of the gravity reduction process. I have focussed on errors relating to calibration factors, time zones and time changes, height, geodetic datums, gravity datums, and the equations involved therein. The errors range from below the level of detection, to many milligals.

The results highlight the need to not only be diligent and thorough in processing gravity data, but also how it is necessity to document the steps taken when processing data. Without properly documented gravity surveys they cannot be reprocessed should an error be identified.

Key words: Gravity, Reduction, Geodesy, Processing, Error

INTRODUCTION

A common product of state and territory geological surveys is a regional Bouguer Gravity map. These images are compilations of multiple gravity surveys and it is an ongoing problem that overlapping surveys rarely match. Most commonly there appears to be an offset between the surveys leading to anomalies in the final image corresponding to mismatched points. Numerous attempts have been undertaken to create a smooth image of the gravity in South Australia using merging tools and different gridding algorithms (e.g., Heath et. al 2012) but to date a perfect image hasn't been created.

There are multiple reasons that surveys might mismatch. Different resolutions from different gravity meters as well as different precision and accuracy in elevation techniques are two common issues. Another issue is in how the data are processed. The process of gravity reduction is straightforward, however at each step of the process decisions need to be made regarding which formulae to use, which geographic datum is required and so on. This paper attempts to quantify some of the errors involved in gravity reduction.

Multiple software packages exist that have the ability to process raw gravity data. A simple spreadsheet has been constructed to undertake this process in the field with minimal effort and to avoid software licensing issues. (The spreadsheet has been scrutinised by undertaking gravity reduction with similar software and producing equivalent results. The approach has allowed me to analyse the errors involved at different stages of the process should erroneous information be inputted. It is freely available to anyone who requests it.)

A series of demonstrations have been undertaken to illustrate quantitatively how much error is introduced at various stages of the gravity reduction process. Each demonstration takes an element of the process, and inserts realistic, incorrect values and presents the results in a series of tables.

The first two demonstrations relate to the calibration factor. The calibration factor is a multiplication factor that must be applied to all raw gravity readings. A typical Scintrex CG5 gravity meter measures a relative gravity value in mGals, however there isn't 1:1 relation with reality. A calibration factor must be calculated prior to all survey work by recording values at the bottom and top of a calibration range with a significant (over 50mGal) change in the gravity values. The calibration factor is a ratio of the actual difference in gravity to the measured difference in gravity. One potential source of error is using the inverse of the calibration factor (i.e., using it to multiply instead of divide). The first demonstration is the effect of using the inverse of the calibration factor.

The second demonstration illustrates how the calibration factor itself can be calculated in different ways. At the most basic it can be calculated from two measurements, it can also be calculated from an ABABA-type loop between the two points. In both cases the processor must decide to assume a linear drift in the measurements, or to undertake a reduction to determine the calibration factor. The processor may have multiple readings to choose from. Does the processor choose the readings with the lowest Standard Deviations (SDs) or should they average them all? The second demonstration shows the effects of computing different calibration factors from the same set of observations.

Table 1 shows the raw data collected from a gravity calibration run, between Adelaide and Norton Summit, South Australia.

A			B			A			B			A		
reading	SD	time	reading	SD	time	reading	SD	time	reading	SD	time	reading	SD	time
4939.358	0.056	9:08 AM	4863.987	0.035	9:31 AM	4939.35	0.057	9:52 AM	4863.979	0.044	10:15 AM	4939.338	0.06	10:37 AM
4939.368	0.035	9:09 AM	4863.999	0.049	9:32 AM	4939.364	0.056	9:53 AM	4863.993	0.049	10:16 AM	4939.351	0.046	10:38 AM
4939.373	0.077	9:10 AM	4864.005	0.066	9:33 AM	4939.369	0.05	9:54 AM	4864	0.047	10:17 AM	4939.354	0.037	10:39 AM
4939.376	0.034	9:11 AM	4864.035	0.035	9:34 AM	4939.373	0.073	9:55 AM	4864.002	0.047	10:18 AM	4939.357	0.042	10:40 AM
4939.379	0.053	9:12 AM	4864.011	0.044	9:35 AM	4939.374	0.037	9:56 AM	4864.006	0.034	10:19 AM	4939.359	0.052	10:41 AM

Table 1. Raw readings taken from a calibration run between Adelaide (A) and Norton Summit (B).

Eight scenarios are considered, each a different way to calculate the calibration factor. The scenarios are:

1. Calibration factor using two ‘best’ values only (from first A & B):
2. Calibration factor using two average values (from first A & B):
3. Calibration factor using average values, assuming linear drift (from first ABA):
4. Calibration factor using average values, assuming linear drift (from second ABA):
5. Calibration factor using average values, assuming linear drift (ABABA):
6. Calibration factor using best values only, treated as a gravity loop (from first ABA):
7. Calibration factor using best values only, treated as a gravity loop (from second ABA):
8. Calibration factor using best values only, treated as a gravity loop (ABABA):

The third demonstration relates to the time that is recorded at a gravity station. This is typically recorded to the nearest minute. A reading on a CG5 gravity meter is effectively an average of a series of measurements, so the time recorded on the internal computer is the average time of the measurements, to a precision of the nearest second. Textbooks (e.g., Telford, Geldart & Sheriff (1990)) claim that the difference in up to 30 seconds won’t create noticeable change. This can be demonstrated through taking a simple gravity loop (shown in table 2) and modifying the time component. Changes of an hour due to incorrectly setting daylight savings time on a gravity meter can also be tested by simply modifying the time.

A			B			A		
Reading (mGals)	Standard Deviation (mGals)	time	Reading (mGals)	SD (mGals)	time	Reading (mGals)	SD (mGals)	time
4939.376	0.034	9:11 AM	4863.987	0.035	9:31 AM	4939.374	0.037	9:56 AM

Table 2. A simple gravity loop can be used to demonstrate what happens when a time is modified.

The fourth item considered is the effect of height. Each time a gravity meter is placed on a tripod and levelled, the sensor will be at a different height above the ground. This height should be measured, recorded and added to the elevation as measured by Differential GPS. For regional surveys this might be added as a constant throughout the survey (approximately 27cm when using a CG5), but for microgravity surveys (measuring to tens of microgals (Sheriff (1991)) the elevation must be carefully taken into account.

Another potential error in height comes when using Ellipsoidal heights instead of Orthometric heights in the Bouguer calculation. Orthometric heights should be used in gravity calculations as they represent a height from an equi-gravity datum. I haven’t included any further demonstrations here, as the difference between Elliposidal and Orthometric heights are generally in the order of meters, so the differences in gravity value will be greater than the error involved in incorrectly taking into account the height of the meter.

The fifth demonstration relates do gravity datums. From a state geological survey perspective there are three gravity datums in common use: Isogal65 (Potsdam), Isogal84 (ISGN1971) and AAGD07. To add further confusion these are often presented in different units: either mgals or micro ms-2.

There are two commonly used equations to convert between Isogal65 and Isogal84 in Australia (Wellman, Barlow and Murray (1985)):

$$obsg_{1984} = 979671.88 + 1.00053(obsg_{1965} - 979685.74) \quad \text{(mGal)} \quad (1)$$

$$obsg_{1984} = obsg_{1965} + V \quad \text{(mGal)} \quad (2)$$

where:

$$V = 14.166 - 0.001838X + 0.0405366Y - 0.000220256X^2 + 0.000476101XY + 0.00070915Y^2 + 0.0000016635X^3 - 0.0000964709X^2Y - 0.0000373075XY^2 - 0.00014788Y^3 + 0.000000915962X^4 + 0.000000764998X^3Y - 0.0000000772339X^2Y^2 - 0.00000673367XY^3 - 0.00000982392Y^4 + 0.00000000910128X^5 + 0.000000147126X^4Y + 0.0000000666914X^3Y^2 + 0.000000568814X^2Y^3 + 0.00000045662XY^4 + 0.000000438121Y^5 \quad (3)$$

In equation (3), X is the longitude subtract 135 degrees east, and Y is the latitude subtract 25 degrees south (i.e., if using the equations use a positive value of latitude in the southern hemisphere).

Another issue between datums occurs when published values don't match. The gravity values for De Rose Hill in South Australia are: 978975.48mGal (Isogal65), 9789612.42ums-2 (AAGD07) & therefore 978961.32mGal (Isogal84). Using equations (1) and (2) to convert between the values results in differences of either 0.661 or 0.764mGal, approximately 7μms^{-2}. A comprehensive analysis of all Isogal stations in Australia is beyond the scope of this paper, but should be examined.

The sixth demonstration relates to theoretical gravity. Geodetic datums can also be a source of error in Bouguer Anomaly calculation (they won't cause error in the calculation of observed gravity). The latitude of the gravity measurement is part of the theoretical gravity correction component of gravity reduction. Telford, Geldart and Sheriff (1990) give the theoretical gravity equation for GRS1967, however different sources provide slightly different equations (see list below). There are newer equations for GRS80 and WGS84, as well as the older 1930 equation. Some of these are listed here:

$$g_{t_{1930}} = 978031.8(1 + 0.0053024 sin^2\theta - 0.0000059 sin^2 2\theta) \text{ (mGal)} \quad (4)$$
(Sheriff (1991), Blakely (1995) and Reynolds (1997))

$$g_{t_{1967}} = 978031.8(1 + 0.0053024 sin^2\theta - 0.0000059 sin^2 2\theta) \text{ (mGal)} \quad (5)$$
(Kearey, Brookes & Hill (1984))
$$g_{t_{1967}} = 978031.8(1 + 0.0053024 sin^2\theta - 0.0000058 sin^2 2\theta) \text{ (mGal)} \quad (6)$$
(Sheriff (1991))
$$g_{t_{1967}} = 978031.846(1 + 0.0053024 sin^2\theta - 0.0000058 sin^2 2\theta) \text{ (mGal)} \quad (7)$$
(Blakely (1995))
$$g_{t_{1967}} = 978031.846(1 + 0.005278895 sin^2\theta + 0.000023462 sin^4\theta) \text{ (mGal)} \quad (8)$$
(Telford, Geldart & Sheriff (1990) and Reynolds (1997))
$$g_{t_{1967}} = 978031.85(1 + 0.005278895 sin^2\theta + 0.000023462 sin^4\theta) \text{ (mGal)} \quad (9)$$
(Kearey, Brookes & Hill (1984))

$$g_{t_{1980}} = 978032.67715 \frac{1+0.001931851353 sin^2\theta}{\sqrt{1-0.0066943800229 sin^2\theta}} \text{ (mGal)} \quad (10)$$
(Wikipedia)

$$g_{t_{1984}} = 978032.67714 \frac{1+0.00193185138639 sin^2\theta}{\sqrt{1-0.00669437999013 sin^2\theta}} \text{ (mGal)} \quad (11)$$
(Blakely (1995))
$$g_{t_{1984}} = 978032.53359 \frac{1+0.00193185265241 sin^2\theta}{\sqrt{1-0.00669437999013 sin^2\theta}} \text{ (mGal)} \quad (12)$$
(Wikipedia)

$$g_{t_{1987?}} = 978032.68(1 + 0.00193185138639 sin^2\theta)(1 - 0.00669437999013 sin^2\theta) \text{ (mGal)} \quad (13)$$
(Sheriff (1991))
$$g_{t_{1987?}} = 978032.68 \frac{1+0.00193185138639 sin^2\theta}{\sqrt{1-0.00669437999013 sin^2\theta}} \text{ (mGal)} \quad (14)$$
(modified from Sheriff (1991))

Equation 13 is as presented in Sheriff (1991). It appears to be an obvious misprint and equation 14 here is a modified version of the equation.

The seventh demonstration illustrates what happens when latitudes in a different geographic datum are used in the theoretical gravity calcualtion. Using AGD66 instead of GDA94 values (or vice versa) in the theoretical gravity equation will yield different values.

The eighth demonstration illustrates how all these errors feed into the final Bouguer anomaly. These arise as combinations of all the previously described errors.

There are many other potential sources of error in gravity surveying (not least of all potential confusion arising from the terms Isogal65, AGD66, GRS67). Some of the operation errors include (when using a CG5 instrument) setting the internal clock incorrectly and incorrectly setting the reference point on the meter. Other sources of error involve terrain corrections and atmospheric corrections. There are far too many sources of error to include in this paper, so I've restricted myself to those commonly involved in the processing of data.

METHOD AND RESULTS

First demonstration

One potential source of error is using the inverse of the calibration factor, or using it to multiply instead of divide. For a typical calibration factor of 1.000271973, a typical raw reading of 3785.98mGals becomes 3783.93mGals if the inverse is used, creating a difference of 2.06mGals. The dynamic range of a Scintrex CG5 is 1000 to 9000 mGals, meaning this error could range from anywhere between 0.5mGals to 5.0mGals.

Second demonstration

For each calculation technique (listed 1 to 8 in the introduction) I've calculated the calibration factor and the minimum and maximum corrected value of a CG5 gravity meter assuming a dynamic range of 1000 to 9000 mGals. I've then calculated the difference between the values calculated from each scenario with the final scenario (which is arguably the most accurate). The differences range from 0.01 to (almost) 1.00mGal. Table 3 shows these results.

Scenario	Calibration factor	Min reading (mGals)	Max reading (mGals)	Difference from scenario 8 (min) (mGals)	Difference from scenario 8 (max) (mGals)
1	1.000337147	999.6629664	8996.966697	-0.021357902	-0.192221115
2	1.000424752	999.5754279	8996.178851	-0.108896386	-0.980067476
3	1.000305291	999.6948024	8997.253221	0.010478076	0.094302685
4	1.00033051	999.6695987	8997.026388	-0.014725574	-0.132530162
5	1.000332723	999.6673879	8997.006491	-0.016936359	-0.152427234
6	1.000303843	999.6962495	8997.266245	0.011925214	0.107326928
7	1.000328416	999.6716916	8997.045224	-0.012632687	-0.113694185
8	1.000315775	999.6843243	8997.158918	0	0

Table 3. The effect of an incorrect calibration factor could lead to errors up to 1mGal.

Third demonstration

The third demonstration is the effect of incorrect time. This can happen if a time is written into a notebook incorrectly, or if daylight savings time is not taken into account on the internal clock of an instrument. Consider the loop in table 1. Given the value at A is 979706.660mGal, the reduced value of B is 979631.272mGal. The results of altering the time of B by adding and subtracting half-minutes, whole minutes and whole hours are shown in table 4.

	Gravity reading (mGal)	Difference (mGal)
Time of B subtract one hour:	979630.7633508	-0.5085381
Time of B subtract one minute:	979631.2718444	-0.0000444
Time of B subtract half minute:	979631.2718667	-0.0000222
Time of B:	979631.2718889	0
Time of B add one half minute:	979631.2719111	0.0000222
Time of B add one minute:	979631.2719333	0.0000444
Time of B add one hour:	979630.7686825	0.5032064

Table 4. The effect of getting the time wrong by a minute is less than below the level of detection, but an hour difference is noticeable.

Fourth demonstration

The effect of using an incorrect height in processing is demonstrated by taking an existing loop and modifying the height and looking at the differences. Considering again the loop shown in table 2, table 5 shows the same reading repeated 14 times but with modified heights. The final two columns are simple Bouguer Anomalies and the difference between the actual values.

Fifth Demonstration

The difference between Isogal65 and Isogal84 values is approximately 14mGal, however the difference varies depending on which equation is used. For a point in Adelaide the Isogal65 value is 979706.660mGal, giving Isogal84 values of 979692.811mGal (using equation 1) and 979692.237mGal using equation 2. The difference between the values is 0.574mGals. Generally this isn't a problem unless converting values back and forth between datums in which case a single formula should be used only. These issues occur in databases (notably between SA Geodata and GADDS) which use different equations to automatically populate whichever values are not present in the database.

Sixth Demonstration

To demonstrate the difference between equations (4) to (14) I've taken a series of Australian latitudes (-10 to -45 degrees) and calculated the theoretical gravity for each, using equations (4) to (14). I've then subtracted each value from selected 'benchmark' equations (those with most references), to demonstrate the difference in values between 1967 and 1984 values. Table 9 at the end of the paper contains all this information. Typically the 1930 equation (equation 4) is different by around 16 to 17mGals, the 1967 equations differ up to nearly 1mGal, the 1984 equations by up to 0.14mGal, and the differences between 1967 and 1984 values are in the order of 0.8mGal.

Station	Reading	time	AHD	Height change	BA (mGal)	Difference (mGal)
A	4939.376	9:11 AM	85.000	N/A	-16.76068	N/A
B	4863.987	9:31 AM	482.500	+ 50cm	-14.47805	0.09835
B	4863.987	9:31 AM	482.200	+ 20cm	-14.53706	0.03934
B	4863.987	9:31 AM	482.100	+ 10cm	-14.55673	0.01967
B	4863.987	9:31 AM	482.050	+ 5cm	-14.56657	0.00984
B	4863.987	9:31 AM	482.010	+ 1cm	-14.57444	0.00197
B	4863.987	9:31 AM	482.005	+ 5mm	-14.57542	0.00098
B	4863.987	9:31 AM	482.001	+ 1mm	-14.57621	0.00020
B	4863.987	9:31 AM	482.000	0	-14.57640	0.00000
B	4863.987	9:31 AM	481.999	- 1mm	-14.57660	-0.00020
B	4863.987	9:31 AM	481.995	- 5mm	-14.57739	-0.00098
B	4863.987	9:31 AM	481.990	- 1cm	-14.57837	-0.00197
B	4863.987	9:31 AM	481.950	- 5cm	-14.58624	-0.00984
B	4863.987	9:31 AM	481.900	- 10cm	-14.59607	-0.01967
B	4863.987	9:31 AM	481.800	- 20cm	-14.61574	-0.03934
B	4863.987	9:31 AM	481.500	- 50cm	-14.67475	-0.09835
A	4939.374	9:56 AM	85.000	N/A	-16.76068	N/A

Table 5. The effect of an incorrect height can be up to 0.1mGal at 50cm difference.

Seventh Demonstration

To demonstrate the difference in theoretical gravity between AGD66 and GDA94 values, I've taken a range of Australian AGD66 latitudes (-10 to -45), calculated the equivalent GDA94 values, and calculated various theoretical gravity values at these points. Table 6 shows the gravity values and table 7 shows the differences: up to around 0.83mGal.

AGD66 latitude (degrees)	GDA94 latitude (degrees)	gt1967 AGD66 (mGals)	gt1967 GDA94 (mGals)	gt1984 AGD66 (mGals)	gt1984 GDA94 (mGals)
-10	-9.998537539	978047.7648	978047.7601	978048.5964	978048.5917
-15	-14.99852332	978067.6178	978067.6108	978068.45	978068.443
-20	-19.9985195	978095.3274	978095.3181	978096.1604	978096.151
-25	-24.99852609	978130.8092	978130.7976	978131.6431	978131.6315
-30	-29.99854302	978173.955	978173.9413	978174.7901	978174.7764
-35	-34.99857012	978224.6335	978224.6179	978225.4699	978225.4543
-40	-39.99860714	978282.6902	978282.673	978283.5281	978283.5109
-45	-44.99865374	978347.9481	978347.9296	978348.7876	978348.7691

Table 6. Theoretical gravity calculated for various latitudes in AGD66 and GDA94.

AGD66 latitude (degrees)	Difference in 67s (mGals)	Difference in 84s (mGals)	Difference in 66s (mGals)	Difference in 94s (mGals)
-10	0.004651056	-0.004651188	0.831593179	0.831593047
-15	0.00702672	-0.007026917	0.832153419	0.832153221
-20	0.009359912	-0.009360172	0.832926274	0.832926014
-25	0.011594556	-0.011594872	0.833900607	0.833900291
-30	0.013676634	-0.013676999	0.835062579	0.835062215
-35	0.015555423	-0.015555828	0.836395992	0.836395588
-40	0.017184499	-0.017184932	0.837882688	0.837882255
-45	0.018522732	-0.018523185	0.839502992	0.839502539

Table 7. The difference in theoretical gravity is up to 0.84mGal.

Eighth demonstration

To demonstrate a combination of errors in calculating the Bouguer anomaly, if an incorrect datum is used in a calculation (error 0.83mGals), and the elevation is out by 20cm (error 0.4mGal), and incorrect calibration number used (error 1mGal), the total isn't simply the sum of these errors. To demonstrate consider again the simple loop in table 4. The difference in Observed gravity is 0.5297mGals and the difference in Bouguer Anomaly is -0.3667mGal.

station	raw	Calibration factor	Observed gravity (mGals)	latitude	elevation	Bouguer (mGals)	(difference, mGals)
A	4939.376	1.000315775	979706.6600	-34.92309965	85.0	-16.76068	
B	4863.987	1.000315775	979631.2957	-34.92901048	482.0	-14.57641	
B - wrong	4863.987	1.000424752	979630.7660	-34.92755000	482.2	-14.94310	-0.36670
A	4939.374	1.000315775	979706.6600	-34.92309965	85.0	-16.76068	

Table 8. This simple loop with a combination of errors yields a difference in the final Bouguer Anomaly of -0.3667mGal.

CONCLUSIONS

Mistakes in processing gravity data can lead to errors in the final product that are geologically wrong. I've demonstrated that errors due to calibration factors, time zones, height, geodetic datum, gravity datum and the equations used all lead to an incorrect value for gravity at a point. These errors – combined with other potential sources of error not discussed here – mean that adjacent or overlapping gravity surveys will often not correlate.

As new geodetic models will undoubtedly be created from time to time it would be naive to set some sort of standard, rather I suggest that all gravity surveys should be documented to a level where the observed and Bouguer gravity anomalies can be recreated from raw readings. This transparency in gravity processing will allow gravity surveys to be reprocessed should any issues with the survey be found, and should ultimately allow better gravity products.

REFERENCES

Blakely, R.J., 1995, Potential Theory in Gravity and Magnetic Applications: Cambridge University Press.

Heath, P., Reed, G., Dhu, T., Keeping, T. & Katona, L., 2012, Regional Terrain Corrected Gravity Grid production, examples from South Australia: Preview, 156, 120

Kearey, P., Brooks, M., and Hill, I., 1984, An Introduction to Geophysical Exploration (third edition): Blackwell Publishing.

Reynolds, J.M., 1997, An Introduction to Applied and Environmental Geophysics: John Wiley & Sons.

Sheriff, R.E., 1991, Encyclopedic Dictionary of Applied Geophysics (fourth edition): The Society of Exploration Geophysicists.

Telford, W.M., Geldart, L.P., and Sheriff, R.E., 1990, Applied Geophysics (second edition): Cambridge University Press.

Wellman, P., Barlow, B.C., and Murray, A.S., 1985, Gravity base-station network values, Australia (report 261): Bereau of Mineral Resources, Geology & Geophysics.

Latitude	Equation 4	Equation 5	Equation 6	Equation 7	Equation 8	Equation 9	Equation 10	Equation 11	Equation 12	Equation 13	Equation 14
-10	978064.929724	978047.771661	978047.771963	978047.817964	978047.764792	978047.768792	978048.596395	978048.596385	978048.452837	978018.318403	978048.599245
-15	978084.795801	978067.690038	978067.690715	978067.736717	978067.617846	978067.621847	978068.450010	978068.450000	978068.306453	978000.407612	978068.452860
-20	978112.522436	978095.489667	978095.490869	978095.536872	978095.327441	978095.331441	978096.160377	978096.160367	978096.016823	977975.409300	978096.163227
-25	978148.024076	978131.084769	978131.086644	978131.132648	978130.809191	978130.813192	978131.643102	978131.643092	978131.499551	977943.399987	978131.645952
-30	978191.191175	978174.365511	978174.368203	978174.414210	978173.955026	978173.959027	978174.790098	978174.790089	978174.646552	977904.477663	978174.792949
-35	978241.890537	978225.198346	978225.201997	978225.248006	978224.633504	978224.637505	978225.469909	978225.469900	978225.326368	977858.761495	978225.472760
-40	978299.965721	978283.426421	978283.431171	978283.477183	978282.690197	978282.694198	978283.528088	978283.528079	978283.384553	977806.391474	978283.530940
-45	978365.237531	978348.870066	978348.876053	978348.922068	978347.948144	978347.952145	978348.787656	978348.787647	978348.644126	977747.527993	978348.790508

Difference between Equation 11 and the rest (mGal)

Latitude	Equation 4	Equation 5	Equation 6	Equation 7	Equation 8	Equation 9	Equation 10	Equation 11	Equation 12	Equation 13	Equation 14
-10	16.333339	-0.824724	-0.824422	-0.778422	-0.831593	-0.827593	0.000010	0.000000	-0.143549	-30.277982	0.002860
-15	16.345801	-0.759962	-0.759284	-0.713283	-0.832153	-0.828153	0.000010	0.000000	-0.143547	-68.042387	0.002860
-20	16.362069	-0.670700	-0.669498	-0.623495	-0.832926	-0.828926	0.000010	0.000000	-0.143544	-120.751067	0.002860
-25	16.380984	-0.558323	-0.556448	-0.510444	-0.833901	-0.829900	0.000010	0.000000	-0.143541	-188.243105	0.002860
-30	16.401086	-0.424577	-0.421886	-0.375879	-0.835063	-0.831062	0.000010	0.000000	-0.143537	-270.312426	0.002860
-35	16.420637	-0.271554	-0.267903	-0.221894	-0.836396	-0.832395	0.000009	0.000000	-0.143532	-366.708405	0.002861
-40	16.437642	-0.101659	-0.096908	-0.050896	-0.837883	-0.833882	0.000009	0.000000	-0.143527	-477.136605	0.002861
-45	16.449884	0.082420	0.088406	0.134421	-0.839503	-0.835502	0.000009	0.000000	-0.143521	-601.259654	0.002861

Difference between Equation 8 and rest (mGals)

Latitude	Equation 4	Equation 5	Equation 6	Equation 7	Equation 8	Equation 9	Equation 10	Equation 11	Equation 12	Equation 13	Equation 14
-10	17.164932	0.006869	0.007171	0.053172	0.000000	0.004000	0.831603	0.831593	0.688045	-29.446389	0.834453
-15	17.177954	0.072191	0.072869	0.118871	0.000000	0.004000	0.832163	0.832153	0.688607	-67.210234	0.835014
-20	17.194995	0.162226	0.163428	0.209431	0.000000	0.004000	0.832936	0.832926	0.689382	-119.918141	0.835786
-25	17.214884	0.275578	0.277452	0.323457	0.000000	0.004000	0.833910	0.833901	0.690360	-187.409204	0.836761
-30	17.236149	0.410485	0.413177	0.459184	0.000000	0.004001	0.835072	0.835063	0.691526	-269.477363	0.837923
-35	17.257033	0.564842	0.568493	0.614502	0.000000	0.004001	0.836405	0.836396	0.692864	-365.872009	0.839257
-40	17.275525	0.736224	0.740975	0.786987	0.000000	0.004001	0.837892	0.837883	0.694356	-476.298722	0.840743
-45	17.289387	0.921923	0.927909	0.973924	0.000000	0.004001	0.839512	0.839503	0.695982	-600.420151	0.842364

Table 9. Variations in theoretical gravity equations can yeild differences greater than 0.01mGal.