Gravity gradient data filtering using translation invariant wavelet

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SUMMARY

Full tensor gradient (FTG) data is highly useful in hydrocarbon exploration and the detection of some geological targets with small size as its higher detailed information abundance and finer resolution. At the same time, there are some high-frequency Gaussian white noise mixed in the target signal and which has closer frequency range than the conventional gravity data. Thus, one key step before inversion is to remove as much Gaussian white noise as possible and reserve the subtle details. For this pre-processing step, several effective methods have been used, including low-pass filters, least square fitting methods based on Laplace equation and wavelet filtering methods. In this paper, we would utilize the translation invariant wavelet for the reason that it can suppress Gaussian white noise through multi-resolution analysis and at the same time can avoid pseudo-Gibbs phenomenon. The other point different from wavelet method used before is that we applied a mixed threshold constructed according to the curve of both soft threshold and hard threshold. Compared to soft and hard threshold, mixed threshold can keep more details and remove more noise respectively in terms of the energy distribution of signal and noise. Then we process wavelet coefficients with mixed threshold and do inverse transform to recover the data. The results demonstrate that translation invariant wavelet can not only remove the major part of Gaussian white noise, but also reserves high-frequency detailed information of FTG data. Obviously, translation invariant wavelet with mixed thresholding has preferable application effect in filtering FTG data.

Key words: full tensor gradient; filter; translation invariant wavelet; mixed thresholding.

INTRODUCTION

Full tensor gradient (FTG) data is playing a more and more important role in hydrocarbon exploration, under both airborne and seaborne conditions. The components of FTG data can be effectively used to geometric inversion and physical properties inversion of exploration target. With more localized features, the power spectrum of FTG data is more focused on high-frequency section (Julio et al., 2004). In order to extract the detailed signal caused by the target accurately, it is necessary to suppress Gaussian white noise which has close frequency range.

Apart from low-pass filtering in Fourier domain, two other methods were utilized to suppress Gaussian white noise while retain minor details in the signal, that is, Laplace equation fitting method and wavelet filtering method. The former one can remove the parts of signal that are not subject to the mutual relations among the components of FTG data, and which are addressed with Laplace equation. That is to say, if a signal in one tensor is not supported in the other tensors, it will be removed from the data. This method combines independently measured gradient components to reduce noise and it requires a method of calculating one gradient component from measurements of a different gradient component. Fourier method, equivalent source method, direct fitting of harmonic functions and space-domain integrals have been used to do the calculation (Nabighian and Misac, 1984; Sanchez et al., 2005; Barnes and John, 2011; Yuan et al., 2013). The latter one removes noise through vanishing or diminishing the wavelet coefficients which are regarded to be ascribed to noise, and threshold is used in this procedure. After that, the remaining wavelet coefficients are used to recover the target signal.

Translation invariant wavelet transform got prevailing since the application of Coifman and Donoho (1995). It is based on a cycle-spinning algorithm which consists of a collection of signal shift, filtering each shifted signal and align-average the estimates. With the utility of translation invariant wavelet transform, the pseudo-Gibbs effects, which are commonly caused by singularities or discontinuities in the signal, will counteract each other and the final results will show less aliasing. Besides, for the pursuit of recovering the majority part of the target signal, we created a mixed thresholding method instead of using the conventional soft threshold or hard threshold. Three steps exist in the conduction of this method: transforming the FTG data into wavelet coefficients with translation invariant wavelet, thresholding the coefficients with mixed threshold and doing inverse transform of the thresholded coefficients. We can obtain better results with better recovery accuracy, not only on removing high-frequency noise, but also on reserving subtle response of the exploration target.

METHOD AND RESULTS

Translation invariant wavelet

Wavelet transform is a kind of time-frequency transform and on its basis, one function can be decomposed into a weighted sum of wavelets obtained from dyadic translation and dilation of a specified mother wavelet. For discrete wavelet transform (DWT) utilized in this paper, the parameters are discretised and we obtain the family wavelets of a mother wavelet function $\psi(t)$:

$$\psi_{m,k}(t) = \frac{1}{\sqrt{2^m}}\psi(\frac{t-k\cdot 2^m}{2^m}) \tag{1}$$

where m is the scale parameter (wavelet decomposition level) and k is the shift parameter. The wavelet coefficients $\tilde{W}$ of a one dimensional signal $f(t)$ is the projection of $f(t)$ onto a wavelet, and let $f(t)$ be a signal of length 2^N . The coefficients of $f(t)$ are as follows:

$$\tilde{W}_{m,k} = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{2^m}}\psi(\frac{t-k\cdot 2^m}{2^m})dt \tag{2}$$

Based on equation (2) and the representation of a one-dimensional signal with wavelet transform, we can get the discrete wavelet transforming expression of a two-dimensional matrix $M(x,y)$:

$$M(x,y) = \sum_{k=l=0}^{2^j-1} c_{j_0;k,l}\phi_{j_0;k,l}(x,\mathrm{y}) + \sum_i \sum_{j=0}^{J-1} \sum_{k=l=0}^{2^j-1} d^i_{j;k,l}\psi^i_{j;k,l}(x,\mathrm{y}) \tag{3}$$

where $\phi(t)$ denotes scaling function, $c_{0,0}$ is scaling coefficient and $d_{j,k}$ are corresponding wavelet coefficients, k, l are translations in X and Y directions, $i = h,\ v,\ d$ are sub-band wavelet coefficients in horizontal, vertical and diagonal direction, respectively. In practice, we use 2D-DWT to process FTG data, which is based on the algorithm of Mallat with complexity of $O(n)$ rather than the slow $O(n^3)$ matrix multiplication. Then we can obtain the matrix contains approximation coefficients and detail coefficients by decomposition of several levels. We created a diagram to demonstrate the procedure on level $j+1$, as shown in Figure 1.

Before the decomposition, we proposed the translation invariant wavelet with circular shifts to remove the undesirable artefacts caused by misalignments between features in the signal and those in the wavelet basis. This would help to supress pseudo-Gibbs effects by averaging the signals of all circular shifts. The main operation is to alter the sequence positions of singularities so that the features of the signal could have a better alignment with the basis. Actually, a certain phase shift of the signal can exhibit evidently less amplitude oscillation compared to the original signal. In this paper, we use h to denote different shifts and H to define the range of all shifts. According to simulation test, I chose $H = 13$ to make sure that a satisfied result will be obtained with low computation consumption.

We would use two different wavelet families, Daubechies and Symlets to filter the synthetic FTG data. Daubechies wavelets and Symlets wavelets have similar properties with orthogonality, compact support and are near symmetric and these properties make the wavelets a good way to suppress the noise and keep the original nature of the useful part of data. Let us use N denote the number of vanishing moments, then Daubechies wavelets can be expressed as dbN and symN for Symlets wavelets. We will use Daubechies and Symlets with different vanishing moments and different decomposition level to filter the data and pick a better result.

Mixed thresholding

According to theories of Donoho and Johnstone (1994) and Donoho (1995), wavelet transform can make signal energy focused on large wavelet coefficients and noise energy distributed throughout the entire wavelet domain. Therefore, wavelets coefficients with smaller amplitude are likely to be attributed to Gaussian white noise in signal and by setting these coefficients to zero can we obtain the filtered signal. The selection of the threshold determines the filtering effect and an appropriate threshold will help to remove noisy part of signal and keep the useful part at the same time. Two thresholds are commonly used at the moment, universal threshold and adjusting threshold. Universal threshold is calculated on the basis of the integral statistic characteristics of the signal, including the size of the data and noise level, which can be estimated with the energy of coefficients of signal and noise, respectively. The expression of universal threshold is: $T = \sigma\sqrt{2\ln N}$, in which N is the dimension of data and $\sigma = MAD(w_1)/0.6745$, estimated with the median absolute deviation of diagonal details coefficients on scale one w_1 , denotes noise level.

With this universal threshold, there are two main thresholding methods: hard thresholding and soft thresholding. Expressions of hard thresholding and soft thresholding are as follows:

$$\hat{w}^h_{j,k} = \begin{cases} w_{j,k} & \left|w_{j,k}\right| \geq T \\ 0 & \left|w_{j,k}\right| < T \end{cases} \tag{4}$$

and

$$\hat{w}_{j,k}^{s} = \begin{cases} \mathrm{sgn}(w_{j,k})(|w_{j,k}| - T) & |w_{j,k}| \ge T \\ 0 & |w_{j,k}| < T \end{cases} \tag{5}$$

Hard thresholding might leave out some Gaussian white noise in data as it made no change to larger coefficients, while soft thresholding may cause over-smoothing because of its shrinkage to all the coefficients. Based on these two kinds of thresholding methods, we created a mixed thresholding which takes the energetic distribution of signal and high-frequency noise. Mixed thresholding has a minor shrinkage over wavelet coefficients when they are large, and the amplitude of this shrinkage increases as coefficients become smaller. It can be expressed as:

$$\hat{w}_{j,k} = \begin{cases} w_{j,k} - T \cdot \cos(\frac{\pi}{2} - (\frac{T}{w_{j,k}})^{\alpha}) & w_{j,k} \ge T \\ 0 & |w_{j,k}| < T \\ w_{j,k} + T \cdot \cos(\frac{\pi}{2} - (\frac{T}{|w_{j,k}|})^{\alpha}) & w_{j,k} \le -T \end{cases} \tag{6}$$

The curves of the three thresholding methods are shown in Figure 3. In equation(6), $\alpha \in R$ is a shrink factor and can be chosen from the range of $[2,\ 5]$. In Figure 3, we chose $\alpha = 3$. With mixed thresholding, we can get a better result which contains more useful contents and less high-frequency noise.

FTG filtering

In this section, we built an underground model with two connected prisms and calculated its gravity gradient response. The dimension of the first prism is $240m \times 240m \times 300m$ and that of the second one is $400m \times 760m \times 100m$, and their relative density is $0.4 \times 10^3\ kg \cdot m^{-3}$ and $0.6 \times 10^3\ kg \cdot m^{-3}$, respectively. The depth of the prisms' top surface is $50m$, $250m$ and the survey altitude is $100m$ from the ground. We set the survey grid to $25m \times 25m$ and the length on northern and eastern direction is $2000m$. For this model, there will be some subtle details which can reflect the relative position and response magnitude of the two prisms. With these details, we can demonstrate the filtering effect of the method in this paper. Then we calculated the forward gravity gradient components of the model, and the original data is shown in Figure 4. Before adding Gaussian white noise, the power or the variance of each component was estimated. In this paper we added the same magnitude of Gaussian white noise as the raw data. Noise level (variance) of Txx, Txy, Txz, Tyy, Tyz, Tzz is 13.6, 4.7, 20, 13.5, 20.5 and 38, respectively.

Wavelet bases we used are db2~db6 and sym4~sym8, the decomposition level is 1~5. Root mean square error (RMS) was utilized to demonstrate the filtering effect of each combination and the average value of RMS of each component was calculated. The results are shown in Table 1. We can see that the best result among all is achieved with sym4 on level 2. In Figure 4, we show the filtering results of the six components with sym4 on level 2. For Txx, Txz, Tyy, this filtering method keeps most useful information and suppresses noise effectively, for Txy, Tyz, it recovers the raw data with a few errors and for Tzz, it leaves a bit of noise. It is obvious that we can obtain satisfied filtering results with translation invariant wavelet.

Figures and Tables

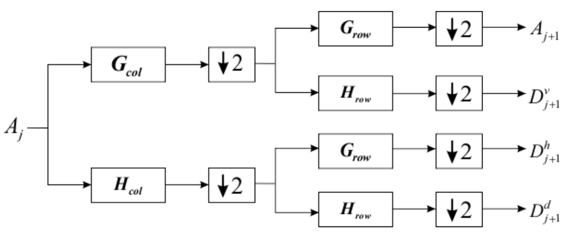

Figure 1: Decomposition structure of 2D-DWT on level j+1.

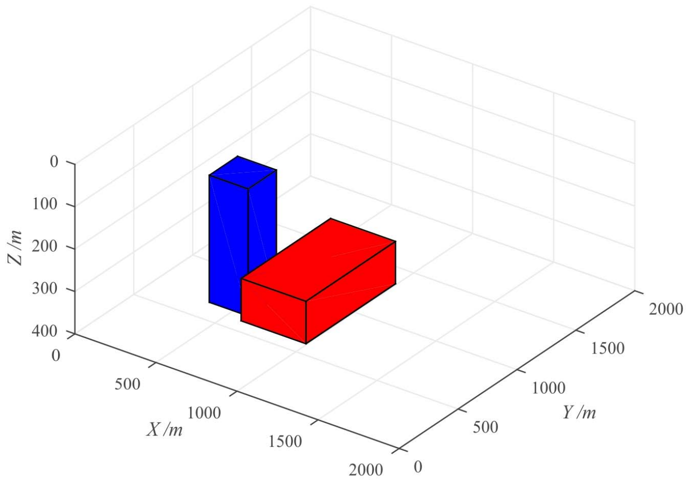

Figure 2: Model with two prisms.

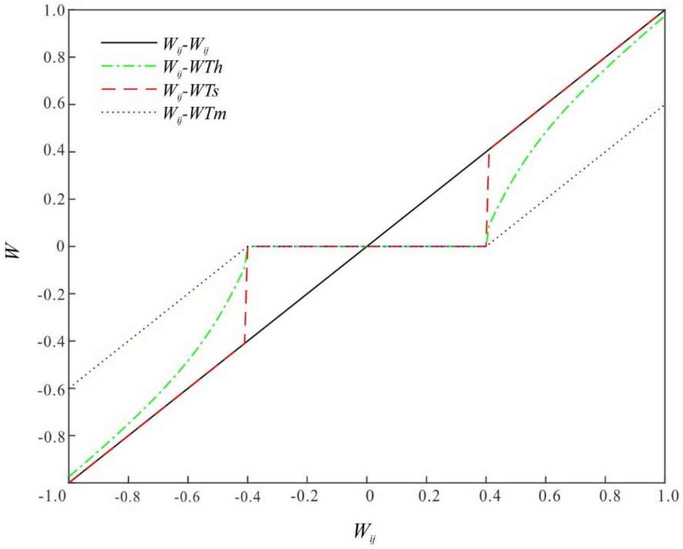

Figure 3: Curve of original signal, hard thresholding, soft thresholding and mixed thresholding.

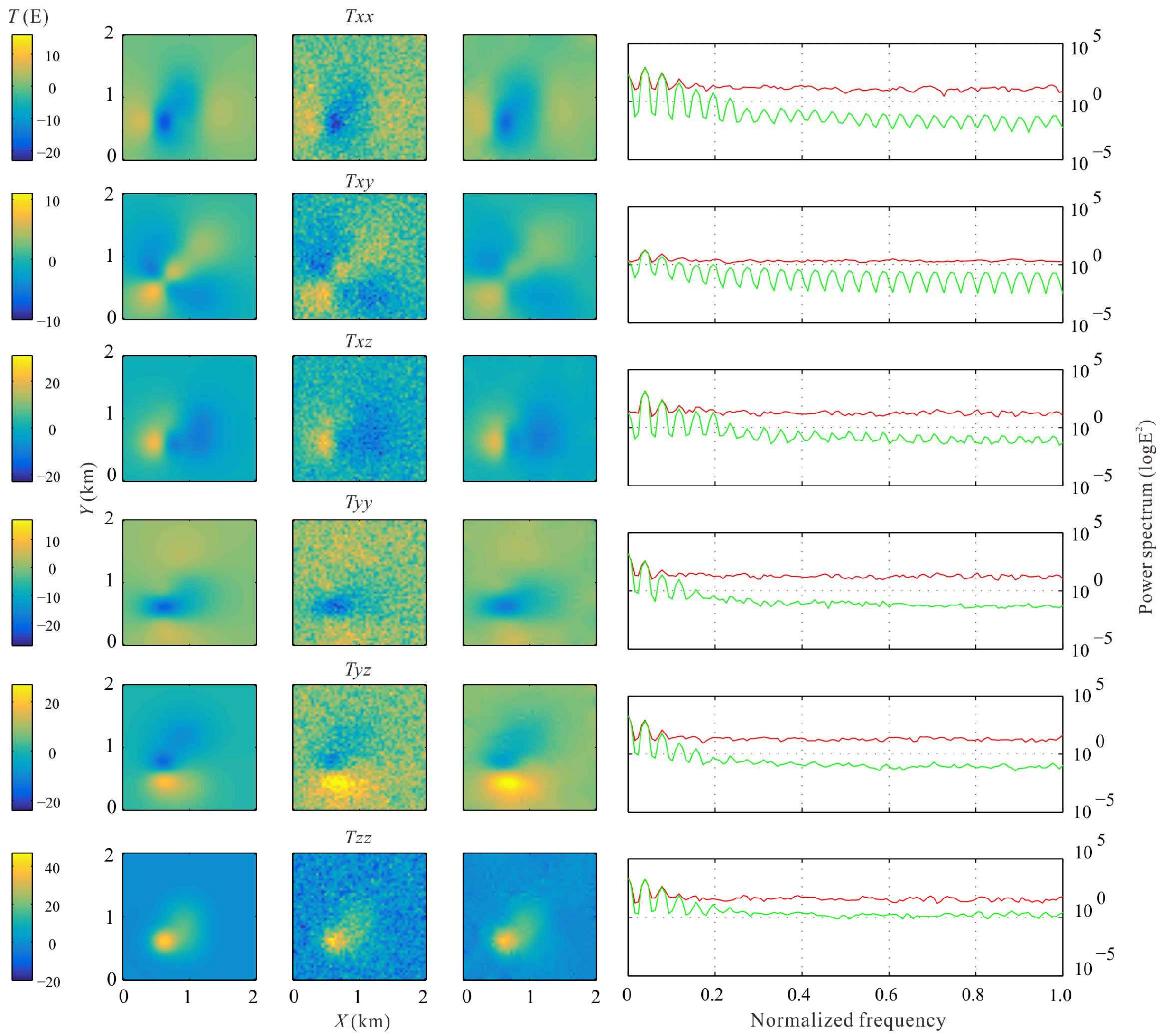

Figure 4: Filtering result of the six FTG components. Column one is the raw model data, column two is noisy data and column three is the filtered data. The forth column is the power spectrum, red line is noisy data and green line denotes filter data.

Wavelet basis	Decomposition level				
	1	2	3	4	5
db2	0.6074	0.5994	0.6064	0.7002	1.2178
db3	0.6134	0.6068	0.6069	0.6781	1.2369
db4	0.6359	0.6307	0.6287	0.6767	1.2482
db5	0.6464	0.6442	0.6256	0.6648	1.2558
db6	0.6795	0.6772	0.6504	0.6657	1.2645
sym4	0.5932	0.5849	0.5904	0.6676	1.2260
sym5	0.6231	0.6178	0.6221	0.6765	1.2464
sym6	0.6111	0.6029	0.6052	0.6664	1.2285
sym7	0.6351	0.6305	0.6306	0.6781	1.2424
sym8	0.6245	0.6187	0.6252	0.6761	1.2508

Table 1: RMS between raw data and filtered data with different wavelet bases and decomposition level.

CONCLUSIONS

Translation invariant wavelet helps to suppress pseudo-Gibbs phenomenon effectively and at the same time, the application of mixed threshold makes a preferable separation between raw data and Gaussian white noise. Using translation invariant wavelet combined with mixed threshold, we achieve a satisfied filtering result which keeps most subtle details and removes high frequency noise effectively. In the model test, we applied Daubechies and Symlets wavelets and chose sym4 as the wavelet basis used in filtering. Symlets wavelets with vanishing moments smaller than 4 are not appropriate and the best combination of different vanishing moments and decomposition level can be achieved only by trial-and-error. In this paper, we used the universal threshold which was determined with the general statistical information of the noisy data and this is a point that can be promoted in the future work. An auto-adjusting threshold on different decomposition level might get a better result on the filtering of FTG data.

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REFERENCES

Barnes, G., and John L., 2011, Processing gravity gradient data: Geophysics, 76, I33-I47.

Coifman R. R., and Donoho, D. L., 1995, Wavelets and Statistics, chapter, Translation-invariant de-noising: New York: Springer-Verlag, 103, 125–150.

Donoho D. L., 1995, De-Noising by Soft-Thresholding: IEEE transactions on information theory, 41, 613-627.

Donoho D. L., and Johnstone I. M., 1994, Ideal spatial adaptation by wavelet shrinkage: Biometrika, 81, 425–455.

Julio C. S., Luis T., and Yaoguo L., 2003, Efficient automatic denoising of gravity gradiometry data: Geophysics, 69, 772-782.

Misac N., 1984, Toward a three-dimensional automatic interpretation of potential field data via generalized Hilbert transforms: Fundamental relations: Geophysics, 49, 780-786.

Sanchez, V., David S., Yaoguo L., Misac N., David W., and David S., 2005, Processing and inversion of magnetic gradient tensor data for UXO applications: 18th EEGS Symposium on the Application of Geophysics to Engineering and Environmental Problems, 1193-1202.

Yuan, Y., Da-Nian H., Qing-Lu Y., and Mei-Xia G., 2013, Noise filtering of full-gravity gradient tensor data: Applied Geophysics, 10, 241-250.