Processing of Airborne Gamma-Ray Spectrometry using Inversions

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SUMMARY

Standard processing of Airborne Gamma-Ray Spectrometry data generally gives good results when the geological situation is uniform and the conditions of measurements are quite constant within footprint area with possible exception of flight height variations in a small range. Any violation of these conditions leads to certain problems. In reality, violations such as large changes of flight height and/or rugged terrain are not that rare as well as sharp changes in composition of surface rocks. This article proposes an approach where the solutions of inverse problems are used for data processing. The approach is quite natural in the processing of field data measured along the flight lines: it explicitly takes into account one-dimensional models of survey and flight parameters – from topography to sources distribution on the surface. Also, it clearly demonstrates that the inverse problem of Airborne Gamma-Ray Spectrometry data does not have a unique solution. This feature can be used in accordance with the geological problem in hands because various formulations of inverse problems can lead to various geological solutions. The use of the approach is illustrated by several examples given for both flight lines and survey areas.

Key words: Inverse Problem, Flight Line Processing, Airborne Spectrometry, Gamma-Ray spectra.

INTRODUCTION

Data processing in Airborne Gamma-Ray Spectrometry (AGRS) is the system of procedures to solve a geophysical inverse problem, i.e., to estimate the distribution of radioactive isotopes in the ground by measuring gamma radiation in the air. In these problems, some conventional in geophysics simplifications and approximations are natural and inevitable. Here are some of them related mainly to flight lines. First, the discretization of measured and sought functions on time and coordinates according to measurement techniques. Next, in the airborne geophysics usually some unknown functions of coordinates are replaced by constant values, e.g., depth distribution of sources, while some other functions are replaced by one-dimensional (1D) functions. In AGRS, it is convenient to accept that the point of data record is the point of data measurement, although, unlike many other airborne methods, in AGRS it is a significant exaggeration. With such simplifications, the inverse problem for any particular flight line is a 1D problem - all the quantities (known and unknown) vary along one direction. All the record point coordinates are assumed to be known (from GPS), and for every record point geometrical and physical parameters of measurements are known (flight altitude, pressure, temperature and topography). The only unknown function is the concentration of radioactive isotopes in the ground (or the radiation intensity on the surface). There are also unknown noises associated both with errors of measurements, model approximations and natural sources like radioactive decay. Such inverse problems are called the discrete linear inverse problems, or the parameter estimation problems, and they are well studied in mathematics. These problems do not have simple and unique solutions.

The well-known standard processing of AGRS data is based, essentially, on an even simpler model of geological and geometrical parameters of the gamma-ray radiation propagation (Grasty and Minty, 1995; Guidelines, 2003; Airborne, 1991), where only the flight height can vary slightly along the survey line within footprint area. The methodology of the standard processing is carefully developed, and it successfully avoids the need for the solutions, and the very formulation of the inverse problem. Thus, the standard method, instead of solving ill-posed problem for the unknown vector, reduces the problem to finding the number – a concentration, and therefore it allows the point-by-point processing. In many cases, this leads to quite satisfactory results and at the same time it creates the illusion of simple and unambiguous solutions of the AGRS data processing, though, as it is known in geophysics, such solutions may exist only in a few carefully selected cases. In some situations, e.g., relating to the topography (Schwarz et al., 1992), the results can be improved by the introduction of proper corrections, while in other situations the distortions may remain unnoticed.

Sometimes, the correctness and accuracy of standard calculations are not reliable enough. One reason is that the process of radiation is stochastic and it results to very noisy measurement data. Therefore, in AGRS data processing, well statistically justifiable rules should be used for highly noisy empirical input data. This implies the relevant smoothing to obtain better spectra statistics. Another reason is an understandable desire to fly with the lightest available equipment and this also leads to the necessity of smoothing. Any natural deviations from homogenous model of data collection also increase the complexity of the situation. Therefore the results of the processing might (and should) be improved by the use of various methods of smoothing or even averaging the data. Such smoothing is usually done in flight lines (in time) and in grids (in space). It leads to loss of spatial resolution, and the results are actually of noticeably smaller scale than the survey scale. To suppress the noises even more, there are powerful successful approaches like NASVD method (Hovgaard and Grasty, 1997). On the first stage, the method ignores many details (coordinates, heights, etc.), then it uses easily interpretable principal components to get very smooth results. So that the method essentially uses averaging and this explains the recommendation of preliminary data zoning that should correct this flaw (Guidelines, 2003). It is interesting that such a recommendation itself might mean that the method is not sufficiently robust to the natural changes of geological and geophysical conditions and/or to the (geometric) survey conditions.

The proposed approach appears to overcome some of the problems associated with the standard processing. In this approach one of the processing stages is replaced with the solution of inverse problems. That is, having obtained the intensities in the air from the measured spectra with standard (Airborne, 1991) or another (Coetzee, 2009) technique, it is possible to find the unknown parameters of certain geophysical model of measurements. In AGRS, the only important unknown parameter is the concentration of the radioactive elements. One convenient and natural model of the problem is to represent the topography by flat strips stretched across the line, to place breaks or jumps to the points of records, and to consider the density of sources in each strip as a constant. The heights, temperature, pressure, etc. at each record point are taken directly from measurements (after elimination of outliers). The flight data is sufficient to build the strips and solve such 1D problem.

The strips will have variable widths (sizes along the flight line) equal to one-second distance of flight and they should preferably be of finite length (size across the flight line). For example, under typical helicopter survey conditions, the length to each side of the flight line may be equal to double nominal flight height if strips of this length give the main contribution to the measured values. When the strips are of finite length, the interpretation of the data becomes easier (it's hard to imagine strips of infinite length), and in the mountains, where such strips are extrapolating the data of the flight line, the errors for the finite strips are smaller (it is even more difficult to imagine the endless strip in the rugged terrain).

The proposed technology differs from the available one in the following major aspects:
- it explicitly uses a 1D geophysical model of the AGRS problem;
- it clearly and more correctly uses the measurement model, in particular it does not use nominal flight height for inversion;
- it definitely shows the lack of unique solution; this is hardly a disadvantage: obtained solutions can be adjusted according to the geological or geophysical problem at hand, for example, it is quite feasible to get smooth, stepwise or spiked solutions;
- it does not use explicit data smoothing, does not lose arbitrary details, and distinctly shows the spatial resolution of the survey data.

Some pros and cons of the approach will be shown below on the examples. Please note that some real examples are not typical and were intentionally chosen to emphasize some features.

THEORY

First, the formula for model of the direct radiation from a horizontal strip will be shown for the case when both half-spaces are homogeneous. Then, the method to solve the linear inverse problem for a set of such strips will be described.

Forward problem

The model of radioactive sources consists of rectangular strips extended across the flight line. The strip is horizontal segment [a, b] along the line (X-axis), and has length 2w, across the line (Y-axis), see Figure 1. Starting from the field of elementary rod of constant radioactivity (Kogan et al., 1969; Minty, 1997; Schwarz et al., 1992), the integration over the horizontal rectangle [a, b] × [-w, w] gives the unscattered field

$$J = J_0 \frac{h}{2\pi} \int_a^b \int_{-w}^{w} \frac{\exp(-\mu_a r)\,dx\,dy}{r^3} \qquad (1)$$

where $J_0 = q/2\mu_e$ is intensity on the surface, and q is source activity (Kogan et al., 1969),
μ_e, μ_a are linear attenuation coefficients in the earth and air,
h is the height of the detector above the topography,
r is the distance between the detector and topography point, $r^2 = h^2 + x^2 + y^2$
dx dy is the area of surface element (ds).

Equation (1) does not require a reduction to special functions (as in Schwarz et al., 1992) because with today's much faster computing, it is enough to use a few Gaussian nodes in each direction to obtain accuracy about 0.01% even for a central strip. A more correct model will also include (inside or outside of the integrals) some calibration parameters, e.g., efficiency or directional characteristics of the detector, but here and below any real detector parameters are ignored. Note the model of homogeneous half-space does not allow finding the activities (concentrations) independently of the attenuation coefficients.

In Table 1, the leftmost column is for the number of strips starting from the central point. To the right, every left column indicates the relative contribution, in percent, of the pair of strips, and every right one shows the cumulative contribution of all the strips from the central to the current one, in percent. To estimate the field within 10% error, at least three strips to each side should be used, even for low flight height and low energy. For typical survey parameters, the doubled height as half-length of the strips may be enough for processing, even for the highest energies, as one can see from columns for 200 and 800 meters of strip half-length.

The number of strips that are required to compute the field at a record point depends on energy and current measurement conditions - height, strip sizes, etc. Often it is an even, equally on each side of the record point.

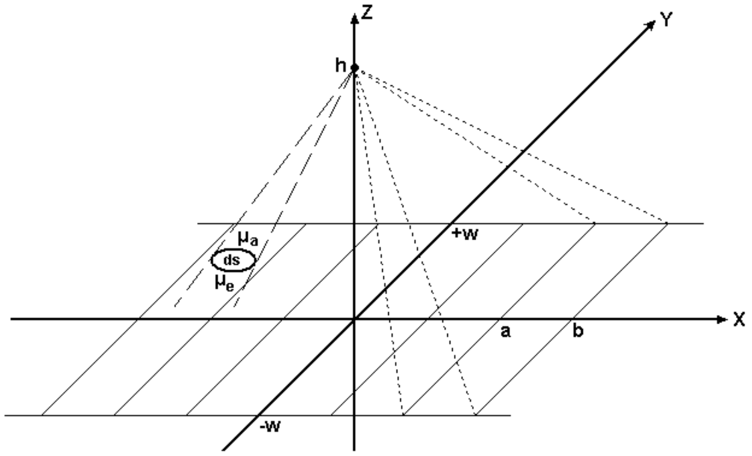

Figure 1. Geometry of the forward problem. Left: intersection of elementary rod with the surface. Right: current strip. The record point is (0, 0, h).

Number of strips	E = 0.66 MeV h = 60 m; w = 200 m		E = 2.62 MeV h = 100 m; w = 200 m		E = 2.62 MeV h = 100 m; w = 800 m	
	Each pair	Cumulative	Each pair	Cumulative	Each pair	Cumulative
2	46	46	29	29	28	28
6	13	87	16	69	16	67
10	3	96	7	87	7	85
14			3	94	3	93
18					1	96

Table 1. Contribution (in percent), of the strip number (for 30 m width strips).

Extending one strip to the entire plane, the well-known expression for the homogeneous half space (Grasty, 1979; Grasty et al., 1979; Guidelines, 2003; Kogan et al., 1969; Schwarz et al., 1992) is:

$$J = J_0 E_2(\mu_a h) \tag{2}$$

where $E_2(x)$ is the exponential integral of the second kind.

Inverse problem

Let the intensities (i.e., photopeak areas) at the data record points on the flight line for all the energies are already available. Now they have to be computed at the earth's surface, in accordance with known geometrical and physical parameters: coordinates, topography, altitude, temperature, pressure. Radon in the air, in the proposed approach has to be ignored for the time being. Thus, the discrete linear inverse problem to solve is

$$Af = b \tag{3}$$

where A – the system matrix with elements built on base of equation (1),
b – the measured vector of intensities in the record points,
f – the sought vector of intensities on the ground.

The problem (3) is considered as ill-posed if (a) the singular values of the matrix smoothly, without jumps, decrease to zero, and (b) the ratio of the largest eigenvalue of the matrix to the smallest is very large (Hansen, 2008). In AGRS, the problem (3) for geometry model in Figure 1 has these features, and it should be addressed by appropriate methods considering the equation (3) as an approximate equation. There exist myriad (though countable) of approaches to solve such inverse problems and many ways to write their formulations. Here one of the oldest and most studied approaches will be applied, remarkably described by Press et al. (2007). The notations and terms are taken mainly from Hansen (2008), Miller and Karl (2003), Tan et al. (2002). More theoretical and historical description is given by Vasin (2006). This approach is known as the least squares method with Tikhonov regularization, and in essence it comes down to minimization of the functional

$$f_\lambda = \arg\min\{\|Af - b\|^2 + \lambda\Omega(f - f_0)\} \tag{4}$$

where $\lambda > 0$ is a regularization parameter,
$\Omega(f)$ is a stabilizer, often in Sobolev norm of derivative operators:

$$\Omega(f) = \sum_{i=n}^{N} \alpha_i \|L_i(f)\|^2, \qquad \alpha_i \geq 0; \quad \sum_i \alpha_i > 0 \tag{5}$$

f_0 is an initial guess or default solution; often $f_0 = 0$.

Here n and N are small integers; quite often $n = N = 1$ are used. The combination with $n = 0$, $N = 2$ usually gives better results with relevant α_i. Components L_i are difference operators:

$$L_0 = \begin{vmatrix} 1 & & & 0 \\ & 1 & & \\ & & \dots & \\ 0 & & & 1 \end{vmatrix}, \; L_1 = \begin{vmatrix} 1 & -1 & & 0 \\ & 1 & -1 & \\ & & \dots & \\ 0 & & 1 & -1 \end{vmatrix}, \; L_2 = \begin{vmatrix} 1 & -2 & 1 & & & 0 \\ & 1 & -2 & 1 & & \\ & & & \dots & & \\ 0 & & & 1 & -2 & 1 \end{vmatrix}. \tag{6}$$

All the norms $\|\cdot\|$ are Euclidian. First term in (4) is the residual norm, arising from the impossibility and needlessness of exact match of measurements and calculations. The second term in (4), the stabilizer, specifies the desired shape of solution and the norm of regularization. The impact of the regularization is determined by the parameter, while the structure of the stabilizer determines the form of the sought solutions. E.g., for a smooth stabilizer (5) the solution is also smooth, and the smoothness can be controlled. The stabilizer in form (5) is quite acceptable (Vasin, 2006), and it can be written in the form (Hansen, 2008; Tan et al., 2002):

$$\Omega(f) = \|L f\|^2 , \tag{7}$$

where matrix L is built from L_i matrices. In these notations the problem (4) has the explicit solution (Miller and Karl, 2003)

$$f_\lambda = (\lambda L^T L + A^T A)^{-1}(\lambda L^T L f_0 + A^T b) \tag{8}$$

for every given value of $\lambda > 0$. To get a “good” solution of (4) it is important to find an “optimal” value of parameter λ. There are several methods to choose the optimal parameter value and many modifications (Bauer and Lukas, 2011; Hansen, 2008). In case of AGRS, the L-curve method (Hansen, 2000) seems to be an appropriate method (note: this L has nothing to do with any previous).

Advantages of L-curve method is that it does not depend on unknown error norms and can be easily automated (Hansen, 2000). With L-curve method the solutions f_λ are computed for a number of values of λ. The optimal value of λ is the point of maximal curvature (Hansen, 2008) on the curve of log ($\|Af_\lambda - b\|$) versus log ($\|L(f_\lambda - f_0)\|$), where L is from (7). The vast majority of the optimal points on the L-curve can be found without excessive computing, although the actual curve is not as magnificent as its educational form (Tan et al., 2002). In rare cases when a solution is not found (usually this means a very smooth solution), the initial value (Press et al., 2007) is quite acceptable. Additional advantage of the L-curve method is that both choosing the optimal parameter and finding the solutions themselves may be calculated with the same formula (8).

Applications and Examples

Altitude and Spatial Resolution

This case study of helicopter AGRS survey in mountain region is devoted to spatial resolution and its influence on the results. Therefore, the flight heights are of large range intentionally. The topography, DEM, is shown in Figure 3. The lines are in West-East direction, with nominal altitude 120 m and line distance 200 m. Actual radar altitudes used for processing are approximately from 50 m to 450 m. The map of altitudes is shown in Figure 4. It is seen that in the middle of the Eastern part the actual altitudes are mostly much higher then nominal altitude. There are also many lines parts with increased but acceptable altitudes.

The results of the standard processing for Thorium are shown in Figure 5. In the Eastern part, there is no apparent connection between the heights and fields, and there are large field gradients in the direction of flights, in fact, maximum gradients are at the greatest heights. Of course, these details are incorrect, because even for Thorium energy reliable data hardly can be measured at altitudes greater than 350 m. Data for Potassium and Uranium are similar to these but somewhat worse.

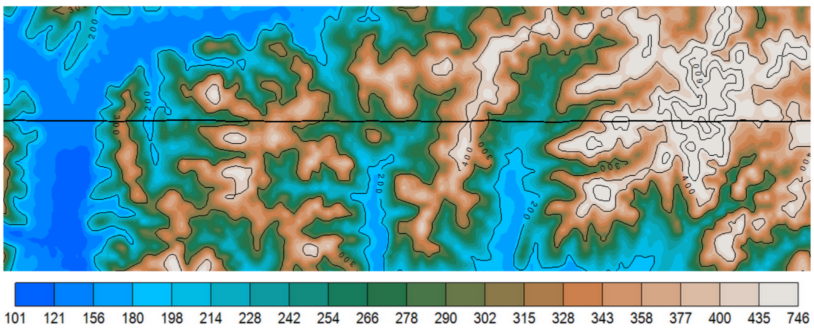

Figure 3. Survey area topography (DEM), meters.

Solutions of the inverse problem for Thorium are shown in Figure 6. In this case, relation of gamma fields with flight altitude has several forms: (a) in the regions of greatest heights there are no large gradients along the line of flight, while there are high gradients across these lines (remind that the solutions on adjacent lines are independent); (b) large field values are caused, obviously, by the lack of information from the ground, since even a very weak field (noise) at higher altitudes requires a very strong sources on the ground – use (2) to estimate, also this is a cause of apparent correlation between inverse solution and large flight height; and (c) the flight direction is best seen in the areas of greater flight heights, and one reason for this is in variable spatial resolutions on the adjacent lines. Thus, one drawback of the proposed approach is that the flight lines, being independently processed, might have inconsistent spatial resolution, and this may require some kind of levelling. Generally, such inconsistency should not be considered solely as a result of different heights of measurements; it can also be associated with other violations of 1D assumptions in the problem formulation, and for the greater heights it can be related to tiny changes in the source data, because of low Signal-to-Noise Ratio. Yet, by experience, in most cases the inversion data require less strong levelling than the standardly processed data.

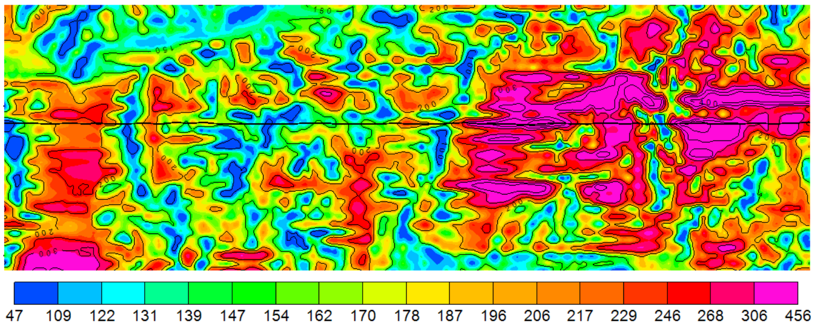

Figure 4. Radar altitudes, meters.

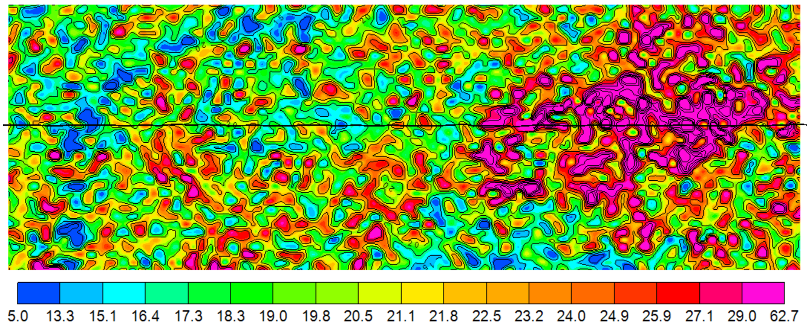

Figure 5. Thorium map, standard processing (smoothed and levelled), cps.

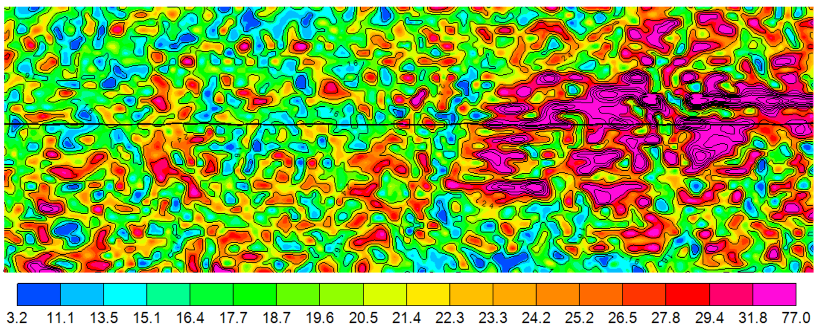

Figure 6. Thorium map, inverse processing (no smoothing, no levelling), cps.

The loss of spatial resolution with increasing flight altitudes is a well known phenomenon (Billings et al., 2003). The spatial resolution can be roughly estimated as number of extremes per unit length. In this example, it can be demonstrated on the flight line shown on all the maps. The field data along the line are shown in Figure 7. Figure 7e shows that radar altitude changes from about 60 to 450 m and that the acceptable data can be measured mostly in the Western part of the line. Figure 7b shows output of standard processing. It is clear that the spatial resolution is associated with the smoothing filter size rather then with the height. Figure 7a shows the result of inversion process where the spatial resolution has good correlation with radar altitude. Particularly, note decreased resolution (inversion vs standard) near the points 38000, 42000, and increased resolution near the points 25000, 32000, 36000 and even 41500.

DISCUSSION

Various Formulations of Inverse Problems

The regularization method (4), (5) is not the only one among the studied and successful methods. Generally, the inverse problem can be formulated and solved in many ways, each having its own disadvantages and some having advantages. These mathematically various ways can give geologically various types of solutions - smooth, stepwise, spiked. This feature could be used in various geological situations. Below are some examples.

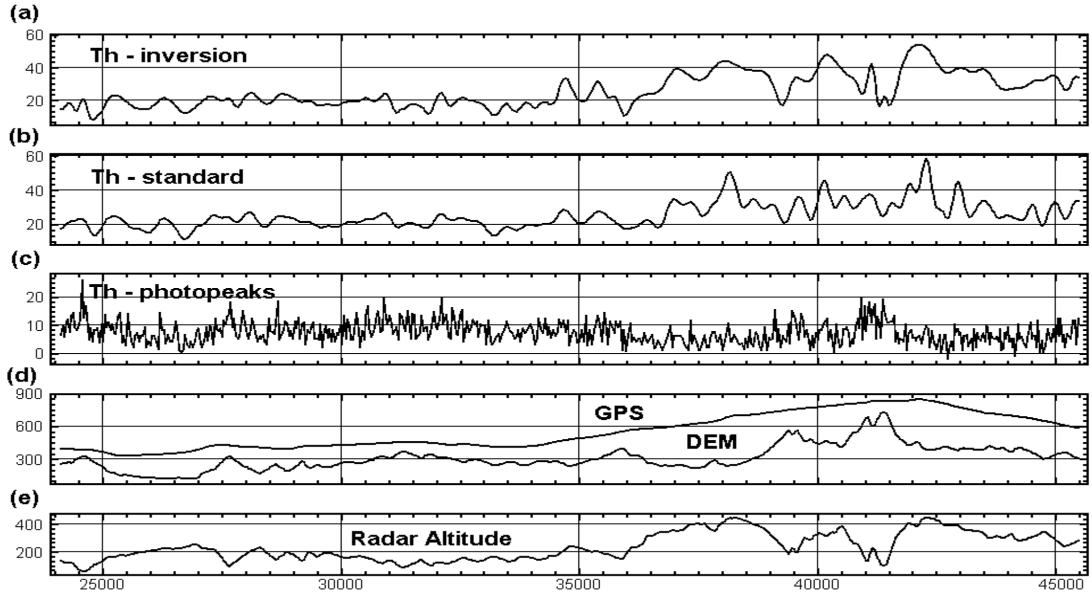

Figure 7. Field plots along the line shown on the maps: (a) inversion processing; (b) standard processing; (c) input data for processing; (d) topography and GPS altitude; (e) radar altitude.

Mathematically, the inverse problem can be formulated as Total Least Squares problem (Beck and Ben-Tal, 2006; Markovsky and Van Huffel, 2007), which includes the inaccurate calculation of the problem matrix, as well as errors in the topography and other parameters in the problem formulation. Thus, the accuracy and correctness of the results might be improved.

Geologically, more interesting is the formulation of the inverse problem with the stabilizer of the total variation, when the solution is stepwise, i.e., piecewise constant, and the locations of jumps are not set in advance (Vogel, 2002). Shown on the maps, such fields are not much different from usual smooth fields. However, consideration of profiles can give qualitatively characterized information.

Figure 8 shows a semi-synthetic example of inverse processing with total variation stabilizer. This example was created as follows. All the non-gamma parameters (coordinates, temperatures, etc) from a survey data were retained, and intensity values on the surface were assigned to every grid node of the DEM. The intensity on the surface was set as piecewise constant with the values close to 20, 40 and 60; see Figure 8b (thick). These values have been converted to the data record points (it is a 1D forward problem) and supplied with appropriate Poisson noise, Figure 8b (thin). These "raw" data were processed with the standard method and as inverse problem. Figures 8c and 8d show the geometry along the line. Figure 8a shows the standard solution (thin, slightly smoothed), and inverted solution (thick). It is clearly seen that the inverse solution is more consistent with "true" solution than the standard one. Generally, for real data, where sources are not really step-wised, the results are not so obviously step-wised, but still are pretty different from the smooth solutions.

Another interesting approach is to model the radiation sources with point emitters on a smooth background (Courbin et al., 2000). These points are points only in the given task, in fact they are the same strips, but with a much more powerful source than nearby. In this problem, the sought sources are subjected to quite different constraints and the solution is obtained by iterative methods where the number of iterations is the regularization parameter. Figure 9 shows the line processed using this method where the number of iterations was deliberately chosen to better show the structure of the solution. In this case also the spatial resolution (as number of extremes per unit length) is connected to altitude, but point sources are not related in any simple manner to topography or the measured radiation. Therefore, there is no simple relation between the smooth solution described above, and spiked solution. In this approach, using its multiplicative solutions, to make a non-negative solution it is sufficient to make the positive initial guess. One practically useful example of this class of solutions has been associated with the search for emitting remnants in the moraine areas.

Note about 2D inversion. The 1D inversion uses at each point the data obtained for a short time (less than a minute) and for a short line segment (a few mean free paths of photons in air). Thus, in 1D case the data are local and simultaneous. The same data in 2D inversion still are local data, but generally they are no longer simultaneous. This makes it easier to get into the contradictory situation where the field sources (earth) and the intermediate medium (air), and even (scary to think) the detector are changed during the survey measurements. Therefore, the 2D inversion requires a different formulation that includes a time in some form.

Stabilizer Weights

Optimal value of regularization parameter depends on the stabilizer weights (5), since they largely determine the form and quality of the solutions. By experience, a good set of weights is $\alpha = (1,7,1)$. This combination can be explained as follows. When searching for smooth solutions, the first differences (derived from the desired solution) have most important role, their weight α_1 should be large. To keep the solution away from unreasonably large range of values, it is useful to limit its norm, and positive weight of the solution α_0 is used for this purpose. Finally, in order not to miss the essential details of the sought solution the higher derivative of the solution should be included, with lower weight α_2. Such solutions have at least a couple of attractive properties. Namely, they are close enough to the standard results (when they can be compared) and they generally have fewer negative values than the standard processing results.

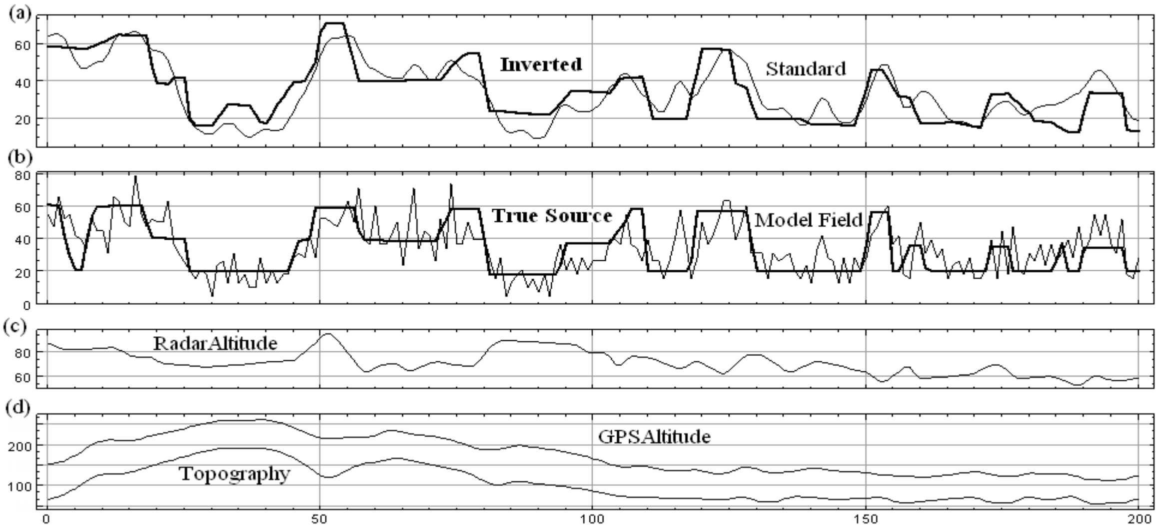

Figure 8. Semi-synthetic example of inversion processing: (a) standard (thin) and inverse (thick) solutions; (b) predetermined "true" source intensities (thick) and the model field (thin); (c) and (d) actual survey data along the line: radar altitude, GPS altitude and topography.

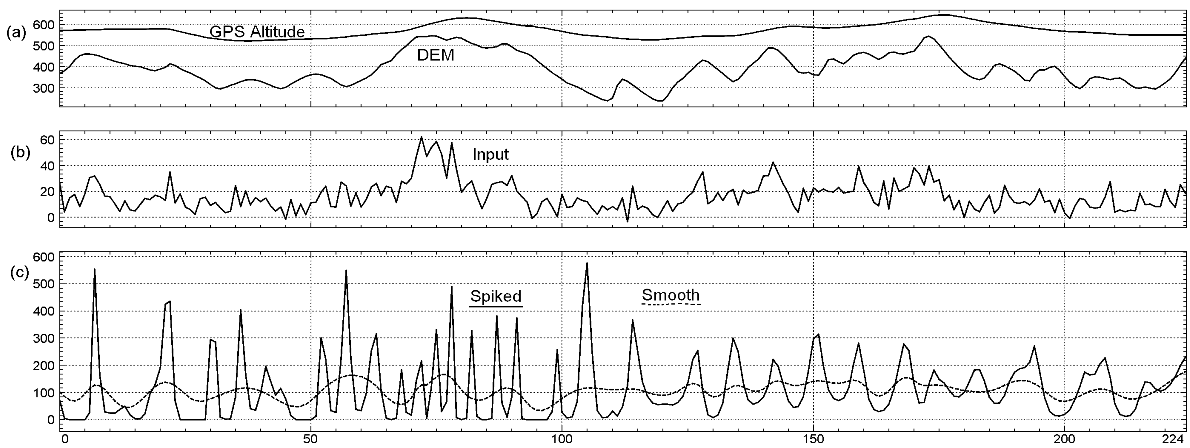

Figure 9. Example of spiked solutions: (a) flight GPS altitude and DEM; (b) measured intensity; (c) spiked solution (solid line) and smooth solution (dotted line).

CONCLUSIONS

Proposed new approach to processing of AGRS data assigns the essential role to the solution of the linear 1D inverse problem. Fully automated processing is quite possible (so far except for radon). The approach has several advantages. All the raw data can be taken directly from the measurements and many deviations from the standard uniform conditions are automatically resolved (topography effects, altitude and airspeed changes, uneven distribution of resources, and other noises). In the inversion process the smoothing is consistent with input data and their errors. Additional smoothing usually may be needed mostly in a levelling procedure required because of mismatched spatial resolution. Various mathematical formulations are possible, resulting in geologically diverse solutions in the form of smooth, stepwise, spiked (peakwise) functions, which might depend on the geological problem to solve, the survey conditions and the interests of the interpreter.

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