Effects of rock porosity on acoustic wave velocities estimation from sonic logs

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SUMMARY

Estimation of S-wave velocity from P-wave velocity has been widely used in geomechanical applications as both velocity measurements are not always available. In present study, some of previously suggested equations for estimation of shear velocity have been presented. These equations have local validity and new calibration of constants is needed if used in new region. On the other hand, theoretical background in geomechanics shows linear relationship between the squares of these velocities. Measured data points were tested in this linear equation and it was found that good matching is observed only in clean sandstone samples. The idea that alteration in linearity is mainly due to porosity effect, motivated the authors to make a new regression considering porosity as one of input parameters. The results highly confirmed this method and new correlations with previously published data showed correlation coefficient nearly equal to unity.
Key words: compressional velocity, shear velocity, wave propagation, porosity.

INTRODUCTION

The physics of wave propagation in rocks is defined according to elastic disturbance from point to point through a medium. The P-wave (also known as dilatational, irrotational, longitudinal, compressional) velocity is in the direction of propagation while S-wave (also shear, transverse, rotational) velocity is perpendicular to the direction of propagation. Many parameters can affect wave velocity in rocks including lithology, porosity and cementation, degree of consolidation, fracturing, percentage and type of pore fluid, depth, age and in situ stresses. It should be reminded that fluids do not support shear strength; therefore, S-waves do not propagate in fluids. Compressional and shear velocities (Vp and Vs, respectively) are important in seismic inversion and petrophysical evaluation of formations, especially for analysis of reservoir geomechanical properties. For estimating the geomechanical parameters such as Young's modulus, Poisson's ratio, and Lame parameters, both Vp and Vs plus density are needed. Accordingly, knowing Vp, Vs, and density, other elastic parameters of a rock formation can be calculated in terms of the acoustic wave velocities (Liu et al. 2012). The main elastic moduli and the relationship between them are as follows:

$$\mu = \rho V_s^2 \tag{1}$$

$$E = \frac{\rho V_s^2 \left(3V_p^2 - 4V_s^2\right)}{\left(V_p^2 - V_s^2\right)} \tag{2}$$

$$\lambda = \rho V_p^2 - 2\rho V_s^2 \tag{3}$$

$$K = \rho V_p^2 - \frac{4}{3}\rho V_s^2 \tag{4}$$

$$\nu = \frac{\left(V_p^2 - 2V_s^2\right)}{2\left(V_p^2 - V_s^2\right)} \tag{5}$$

In Equations (1) to (5), the parameters μ, ρ, E, λ, K and ν are shear modulus, density, Young's modulus, Lame coefficient, bulk modulus and Poisson's ratio, respectively. Knowing rock density and compressional and shear velocities will be lead to calculation of all other rock moduli. On the other hand, acoustic wave propagation theory states that the P-wave (Vp) and S-wave (Vs) velocities can be expressed as:

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \tag{6}$$

$$V_s = \sqrt{\frac{\mu}{\rho}} \tag{7}$$

The above-mentioned geomechanical parameters are useful in estimating maximum and minimum horizontal stresses, mud weight design, etc. Therefore, rock mechanical properties can be estimated using some sonic log providing P- and S-wave velocity

information such as dipole sonic log and so on. However, very often S-wave velocity is not recorded in the field all the time due to the cost constrains and lack of technology. Therefore, prediction of the S-wave velocity is an interesting objective for researchers (Farrokhrouz and Asef 2010). Alternatively, if sonic tools to measure VS are not available, we may use a prediction equation for estimating shear wave velocity based on compressional wave velocity obtained from monopole sonic log (Liu et al. 2012). Almost all such equations are empirical (Asef and Farrokhrouz 2010) and a brief list of them is shown in Table 1.

Predicting Parameter	Formulation	R^2	Rock Types	Reference
ϕ	V_s (m/s) = 2998.3 – 18.086ϕ	0.141	Sandstone	Miller and Stewart (1990)
V_p	V_s (km/s) = 0.79 V_p – 0.787	---	Shaly Sandstone	HAN (1986)
ϕ	V_s (km/s) = 4.06 – 6.28ϕ	0.99	Sandstone	HAN (1986)
V_p	Vs (m/s) = 0.456 V_p +264.3	0.90	Different Rock Types	Fahimifar and Soroush (2003)
V_p	Vs (m/s) = 0.862 V_p +1172	---	Different Rock Types	Castagna et al (1985)

Table 1: Prediction of S-wave velocity using P-wave velocity or porosity as estimator

Brocher (2005) developed a relation between elastic wave velocity in the Earth's crust and developed Equation (10) indicating a nonlinear relationship between Vp and Vs (velocities in km/s).

$$V_s = 0.7858 - 1.2344V_p + 0.7949V_p^2 - 0.1238V_p^3 + 0.0064V_p^4 \qquad 1.5 < V_p < 8 \tag{8}$$

It is noticed that each of the above empirical equations was developed for a specific field and a specific lithology. Nevertheless, if they are used at any other fields with different lithology, they may result in erroneous predictions.

Although a number of correlations have been suggested to predict V_s from V_p, none of them has a universal applicability. Therefore, the focus of current study is to promote new equation form with some theoretical backgrounds. General form of such regression will be presented and will be validated with new series of data sets to check whether it has a wider applicability.

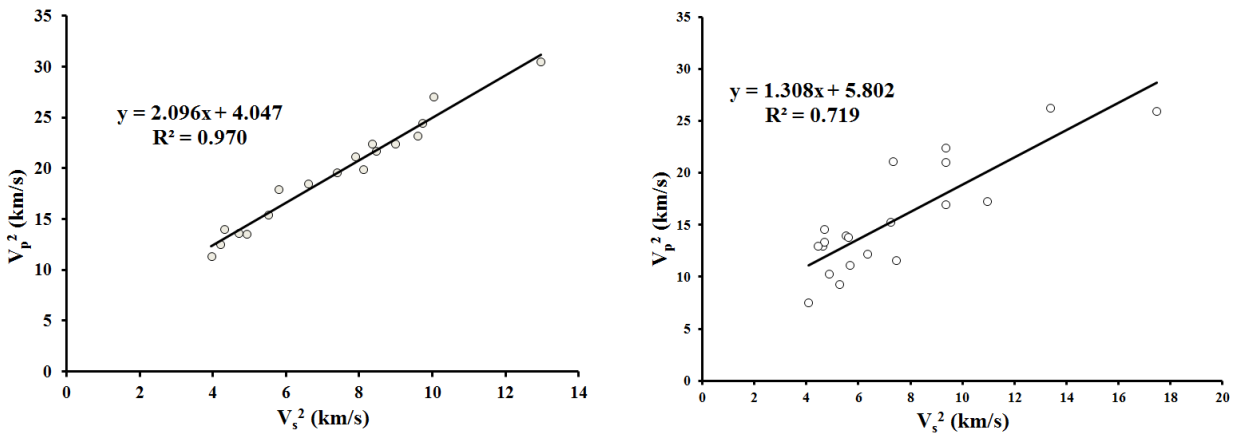
METHOD AND RESULTS

Estimation of Vs from Vp (or vice versa) has been a common approach for determination of shear velocity in formations. Although all presented correlations are empirical, there is also a theoretical rock physics relationship between them. Equations (3) and (4) can also be written as follows:

$$V_p^2 = \frac{\lambda}{\rho} + 2V_s^2 \tag{9}$$

$$V_p^2 = \frac{K}{\rho} + \frac{4}{3}V_s^2 \tag{10}$$

Equations (9) and (10) reveal that there is a linear relationship between squares of P-wave and S-wave velocities. In other words, plotting square of S-wave versus P-wave will result in a linear equation with constant value equal to 4/3 or 2. This fact is seen in Figure 1.



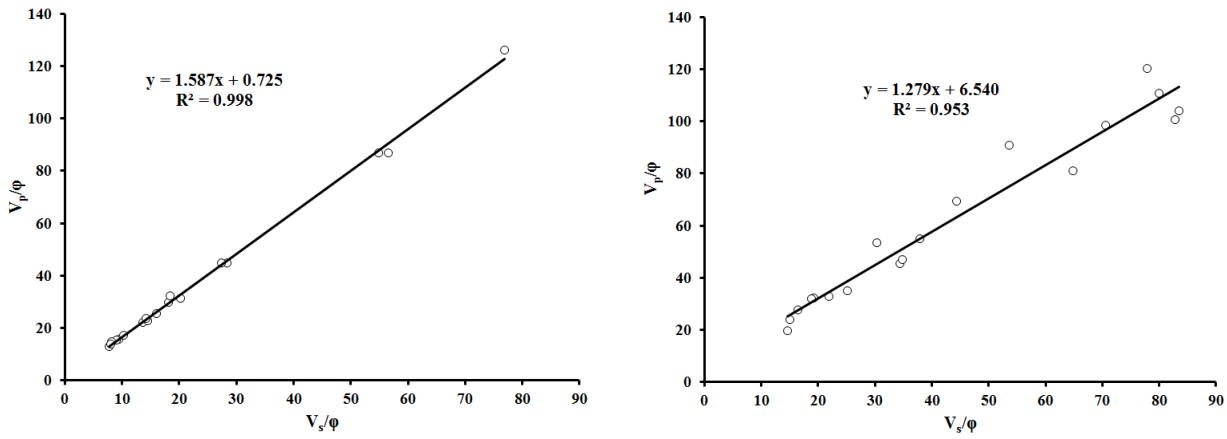
(a) (b)

Figure 1: Plot of S-wave square versus P-wave square; data obtained from a) Han (1986) b) Jizba

Eq. No.	Correlation Form	R-Square	Rock Type	Reference Data
(11)	$V_p^2 = 0.82 + 2.69V_s^2$	0.784	Shale	Lashkaripour (1996)
(12)	$V_p^2 = -0.35 + 2.33V_s^2$	0.762		Yale and Jameison
(13)	$V_p^2 = 0.17 + 3.17V_s^2$	0.644	Basalt	Planke et al (1999)
(14)	$V_p^2 = 3.37 + 2.44V_s^2$	0.861	Limestone	Author's Data
(15)	$V_p^2 = 43.85 + 3.38V_s^2$	0.728	Mix Carbonates	Rafavich et al (1984)

Table 2: Correlations between velocity squares for measured data points

The slope of the linear line in Figure 1(a) and (b) is nearly equal to 2 and 4/3, respectively. These real data points with related R-square confirm the validity of Equations (9) and (10). The point is that these relations show good matching with clean sandstone samples. But the samples of other rock types do not show similar trends. A list of these correlations is shown in Table 2. As it is observed, the slope of the linear line is not equal to 2 or 4/3 and/or the correlation coefficient is low in other rock types. For sedimentary rocks, good correlations with considerable R-square are obtained, but the constant values are neither 2 nor 4/3. On the other hand, for igneous or metamorphic rocks neither constant values nor R-squares show good matching. The objective of current study is to determine whether an appropriate correlation exists or not.



(a) (b)

Figure 2: Plot of S-wave versus P-wave with consideration of porosity; data obtained from a) Han b) Jizba

Initially, the only difference between completely elastic materials and rocks is porosity. Porosity in rocks reduces the strength of the rocks (Farrokhrouz et al. 2014) and causes a new material behavior called poroelasticity. Such property can divert rock behavior from completely elastic material into a poroelastic material. Applying porosity into Equations (9) and (10) for the same data points in Figure 1 will improve correlation coefficient and data points scatter show some convergence and isotropy behavior (Figure 2). As it is seen, correlation coefficient is close to unity in both cases and slope of the linear line is 1.587 and 1.279, respectively. Table 3 shows similar modification for the same data points in Table 2.

Eq. No.	Correlation Form	R-Square	Rock Type	Reference
(16)	$\frac{V_p}{\varphi} = 1.74\frac{V_s}{\varphi} + 0.18$	0.978	Shale	Lashkaripour (1996)
(17)	$\frac{V_p}{\varphi} = 1.47\frac{V_s}{\varphi} + 0.71$	0.992		Yale and Jameison
(18)	$\frac{V_p}{\varphi} = 1.81\frac{V_s}{\varphi} + 0.48$	0.989	Basalt	Planke et al (1999)
(19)	$\frac{V_p}{\varphi} = 1.72\frac{V_s}{\varphi} + 2.02$	0.977	Limestone	Author's Data
(20)	$\frac{V_p}{\varphi} = 1.75\frac{V_s}{\varphi} + 31.00$	0.9999	Mix Carbonates	Rafavich et al (1984)

Table 3: Correlations between velocities for measured data points considering porosity effect

CONCLUSIONS

According to the statistical studies performed here, the following statements could be concluded:

1. Theoretical relations between squares of P-wave and S-wave velocity showed good correlation in clean sandstone applying real data points. But in other rock types, correlation coefficient showed some discrepancies.
2. The most effective parameter in rock velocities is porosity. Other parameters may also influence wave velocities like degree of cementation, consolidation history and so on.
3. Applying porosity in wave velocity estimation in new equation form lead to a very good equation with very high correlation coefficient.
4. The results showed that if porosity is considered in wave velocity estimations, very precise results will be obtained and in some cases extra costs of measurements can be canceled.

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