Field-dependent susceptibility of rocks and ores – implications for magnetic petrophysics and magnetic modelling

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SUMMARY

It is usually assumed that the initial magnetisation curve for a rock, soil or ore sample is linear in the applied field, for fields much less than the coercivity of the magnetic minerals in the sample. This implies that the measured susceptibility, defined as the induced magnetisation divided by the inducing applied field, is independent of the field H that is used in the measurement and that the induced magnetisation of the rock unit *in situ* can be calculated, irrespective of the field used by the measuring instrument, by multiplying the measured susceptibility by the Earth's field at the location of the rock unit. A better approximation for many materials that contain ferromagnetic (*sensu lato*) minerals is a quadratic dependence of the weak-field magnetisation on the applied field, given by Rayleigh's Law, which yields a linear dependence of susceptibility on applied field. This field-dependent susceptibility is associated with hysteresis and a phase lag of magnetisation behind the applied field for AC measurements, which can masquerade as a phase lag produced by magnetic viscosity. Field-dependence of susceptibility is strongly affected by self-demagnetisation, so measurements of the Rayleigh coefficient η of strongly magnetic samples, as well as the initial susceptibility χ, must be corrected for self-demagnetisation in order to calculate intrinsic properties of the rock unit. Self-demagnetisation also largely explains why rocks containing low-Ti magnetite grains, which have high intrinsic susceptibility, exhibit only weak field-dependence of susceptibility, whereas rocks bearing titaniferous magnetite, monoclinic pyrrhotite or multidomain hematite exhibit relatively pronounced field-dependence of susceptibility. Under the conditions of the Néel approximation ($\eta H << \chi$), the Rayleigh laws are still obeyed even when self-demagnetisation is considered. However, considerable departures from the Rayleigh relations occur when $\eta H > \chi$. This paper examines implications of field-dependent susceptibility for measurements of susceptibility and its anisotropy, and methods for correcting calculations of induced magnetisation.

Key words: magnetic susceptibility, field-dependence, Rayleigh Law, self-demagnetisation

INTRODUCTION

Magnetic susceptibility is an important rock property for many geophysical applications, including magnetic and EM modelling, lithological characterisation and correlation, petrofabric studies using magnetic anisotropy, magnetic petrology, and environmental magnetism. To a first approximation, the induced magnetisation of rocks and ores is proportional to the applied field, for fields of the same order of magnitude as the relatively weak geomagnetic field, which ranges from about H = 20 to 50 A/m ($B \approx$ 25-65 μT) depending on geographic location. The constant of proportionality is defined as the magnetic susceptibility. However when the magnetisation is not perfectly proportional to the applied field, the nominal susceptibility obtained after dividing the measured magnetisation by the applied field is field-dependent.

The assumption of linearity is strictly obeyed for weakly magnetic rocks and ores that contain only paramagnetic and/or diamagnetic minerals. For many moderately to strongly magnetic rocks and ores that contain ferromagnetic *sensu lato* minerals, however, the induced magnetisation is not perfectly proportional to the applied field (Néel, 1955; Clark, 1983, 1984; Bloemendal et al., 1985; Smith and Banerjee, 1987; Worm, 1991; Worm et al., 1993; Markert and Lehmann, 1996; Jackson et al., 1998; de Wall, 2000; Hrouda , 2002; de Wall and Nano, 2004; Vahle and Kontny, 2005; Hrouda et al., 2006; Martin-Hernandez et al., 2008; Guerrero-Suarez and Martin-Hernandez, 2012) and the concept of susceptibility becomes less clear. Ideally, susceptibilities of rocks and ores should be measured in a field that is similar to that of the geomagnetic field at their location. However, many susceptibility instruments use fields that are up to an order of magnitude stronger than the geomagnetic field, in order to improve sensitivity (see Table 1). Furthermore the apparent susceptibility can be somewhat different, depending on the physical principle used in the measurement. This means that, in some circumstances, field-dependence of susceptibility may need to be taken into account when using susceptibility measurements. Measured anisotropy of susceptibility values can be significantly affected by field-dependence of susceptibility (Markert and Lehmann, 1996; de Wall, 2000; Hrouda, 2002, 2007; Vahle and Kontny, 2005; Hrouda et al., 2006; Guerrero-Suarez and Martin-Hernandez, 2012), with implications for petrofabric studies and for magnetic modelling of highly anisotropic rock units, although orientations of magnetic foliations and lineations are usually less affected. Although field-dependence of susceptibility complicates magnetic petrophysics, magnetic fabric studies and magnetic modelling, it may have merit as a tool for characterising magnetic mineralogy and magnetic granulometry (Bloemendal et al., 1985; Jackson et al., 1998; de Wall, 2000; Hrouda , 2002; de Wall and Nano, 2004; Vahle and Kontny, 2005). Except at very low fields (generally much less than the geomagnetic field), field-dependent susceptibility reflects irreversible magnetisation processes and hence it is associated with magnetic memory effects, such as isothermal remanence and hysteresis. Hysteresis in an alternating applied field produces a phase lag of the magnetisation behind the field and introduces odd harmonics into the susceptibility signal (Chikazumi and Charap, 1978, p.296-298). The phase lag introduces a quadrature component into the fundamental frequency component of the measured susceptibility (Worm et al., 1993), as seen by Bloemendal et al. (1985) in basalts that contain titaniferous magnetite, which can masquerade as an effect due to magnetic viscosity. The phase lag also implies that measured susceptibility depends on the method of measurement. Phase-sensitive detection of the induced magnetisation signal yields the in-phase susceptibility component, whereas detection of signal amplitude at the excitation frequency, without regard to phase, yields the total susceptibility (square root of the summed squares of the in-phase and quadrature signals).

FIELD-DEPENDENT MAGNETIC SUSCEPTIBILITY - THEORY

When a weak magnetic field (one that is small compared to the coercive field H_c) is applied to an initially demagnetised material, the magnetisation M_i of the material often exhibits a quadratic dependence on the applied field H:

$$M_i(H) = \chi H + \eta H^2, \quad (H << H_c) \tag{1}$$

where χ is the initial susceptibility and η is the Rayleigh parameter. If the applied field increases to a maximum H_m and then is decreased, the magnetisation M_d on the concave-downward descending arc obeys

$$M_d(H) = M_m + \chi(H - H_m) - \frac{1}{2}\eta(H - H_m)^2 = (\chi + \eta H_m)H - \frac{1}{2}\eta(H^2 - H_m^2), \tag{2}$$

where $M_m = M(H_m)$ is the maximum magnetisation attained and is given by

$$M_m = \chi H_m + \eta H_m^2. \tag{3}$$

If, after the field decreases to H_1 (but not to less than $-H_m$) and the corresponding magnetisation decreases to M_1, the field is increased again, the magnetisation M_a on the concave-upward ascending branch of the hysteresis loop obeys

$$M_a(H) = M_1 + \chi(H - H_1) + \frac{1}{2}\eta(H - H_1)^2. \tag{4}$$

From (2)-(4), therefore, if the field is cycled between $\pm H_m$ the magnetisation M_a on the ascending arc obeys

$$M_a(H) = -M_m + \chi(H + H_m) + \frac{1}{2}\eta(H + H_m)^2 = (\chi + \eta H_m)H + \frac{1}{2}\eta(H^2 - H_m^2). \tag{5}$$

Equations (2) and (5) describe two parabolic arcs that together comprise a hysteresis loop, known as a Rayleigh loop. Relations (1)-(5) are known as Rayleigh's Laws, as they were first formulated by Lord Rayleigh (1882). Néel gave the first physically convincing model of these hitherto purely phenomenological laws, in terms of displacement of domain walls over multiple energy barriers produced by random fluctuations in material properties (Néel, 1988). Figure 1 illustrates the general form of the initial magnetisation curve and Rayleigh loop for a magnetic material that exhibits pronounced hysteresis in weak fields. From (2) and (3), the isothermal remanent magnetisation (IRM) obtained after reducing the field from H_m to zero is given by

$$M_{IRM} = \frac{1}{2}\eta H_m^2. \tag{6}$$

Equation (6) can in principle to be used for determining the Rayleigh parameter of a magnetic material by measuring the IRM imparted by applying a field H_m, which is then reduced to zero, to an initially demagnetised sample.

The total differential susceptibility k_{diff} is the slope of the magnetisation curve, and includes contributions from both reversible and irreversible processes. From (1), for the initial magnetisation curve this is given by

$$k_{diff} = \frac{dM_i}{dH} = \chi + 2\eta H. \quad (H << H_c) \tag{7}$$

From (2) and (5) the differential susceptibility around the Rayleigh loop is

$$k_{diff} = \frac{dM_d}{dH} = \chi - \eta(H - H_m), \ \text{(descending branch)}; \ k_{diff} = \frac{dM_a}{dH} = \chi + \eta(H + H_m), \ \text{(ascending branch)}. \tag{8}$$

We can also define an average susceptibility for the initial magnetisation curve as the ratio of the maximum magnetisation attained to the maximum applied field. From (3) this is given by

$$k_{av} = \frac{M_m}{H_m} = \chi + \eta H_m < k_{diff}(H_m). \quad (9)$$

It follows from (8) that the differential susceptibility at the loop turning points, immediately after each field reversal, is equal to the initial susceptibility χ, as shown in Figure 1. At all other points of the loop, $k_{diff} > \chi$. At the points where the loop crosses the magnetisation axis, $k_{diff}(M = \pm M_{IRM}, H = 0) = k_{av} = \chi + \eta H_m$. For each point ($M$, H) on the Rayleigh loop a reversible susceptibility k_{rev} can be measured by superimposing a small oscillating field $\Delta H << H_m$ on the steady applied field. According to (4), the resultant magnetisation describes a very thin minor hysteresis loop that essentially parallels the initial magnetisation curve near the origin. The average slope of this minor loop represents the reversible susceptibility and is given by $k_{rev} = \chi + \eta\Delta H \approx \chi$. From the above discussion, it follows that in general susceptibility is not uniquely defined. Furthermore, measured susceptibility depends on the field in which it is measured, on the past magnetic history of the sample (including remanence it has acquired), and on the measurement method.

The initial susceptibility χ represents reversible magnetisation processes. These processes include, for single domain and multidomain ferromagnetic mineral grains, rotation of magnetisations within magnetic domains away from the easy magnetisation axis in response to the perpendicular component of the applied field, as well as the contribution of paramagnetic and diamagnetic minerals. In multidomain ferromagnetic grains, bowing of domain walls between pinning sites and reversible lateral displacement of domain walls within local energy minima, away from their equilibrium positions, are important contributors towards the reversible susceptibility. In the Rayleigh region, irreversible jumps of domain walls over potential barriers between local energy minima, in response to the applied field, account for the Rayleigh coefficient.

EFFECTS OF SELF-DEMAGNETISATION ON FIELD-DEPENDENT SUCEPTIBILITY

The theory discussed above relates to the response of intrinsic properties to the internal field experienced by the magnetic material. It therefore applies to magnetisations measured in a closed magnetic circuit (e.g. a toroidal sample in a toroidal coil or a sample tightly fitting a narrow gap in a ring of high permeability material); or measured along the axis of a long thin sample or in the plane of a thin disc-like sample; for which the internal field is essentially equal to the external applied field. More generally, however, the internal field H' is the resultant of the external applied field H and the demagnetising field produced by the magnetisation of the sample. The modified internal field is given by

$$H' = H - NM, \quad (10)$$

where N is the demagnetising factor of the sample along the axis of measurement. In SI, $0 \leq N \leq 1$ and, for a uniformly magnetised sample, the sum of the demagnetising factors along three orthogonal directions (principal axes of the volume-averaged demagnetising tensor) is one (Brown, 1962). Rearranging (19) gives a useful expression for the magnetisation in terms of the external and internal fields:

$$M = (H - H')/N. \quad (11)$$

Substituting (10) into (1), the modified internal field obeys

$$H' = H - N(\chi H' + \eta H'^2) \Rightarrow \eta N H'^2 + (1 + N\chi)H' - H = 0. \quad (12)$$

The physically meaningful root of the quadratic equation (12) gives the following expression for the internal field in terms of the external applied field:

$$H' = \frac{-(1+N\chi) + \sqrt{(1+N\chi)^2 + 4\eta N H}}{2\eta N}. \quad (13)$$

Substituting (13) into (1) or (10) gives alternative, equivalent, expressions for the initial magnetisation curve, subject to self-demagnetisation:

$$\begin{aligned} M_i'(H) &= \chi H' + \eta H'^2 = \frac{\chi}{2\eta N}\left[\sqrt{(1+N\chi)^2 + 4\eta NH} - (1+N\chi)\right] + \frac{1}{4\eta N^2}\left[\sqrt{(1+N\chi)^2 + 4\eta NH} - (1+N\chi)\right]^2 \\ &= \frac{H - H'}{N} = \frac{H}{N} - \frac{(1+N\chi)}{2\eta N^2}\left[\sqrt{1 + \frac{4\eta NH}{(1+N\chi)^2}} - 1\right]. \end{aligned} \quad (14)$$

Equation (14) does not accord with Rayleigh's relation for the initial magnetisation curve, so it can be seen immediately that the general applicability of the Rayleigh laws is compromised by self-demagnetisation. For the important case where $\eta H_m << \chi$, however, equation (14) reduces to

$$M_i'(H) = \frac{-\chi(1+N\chi)}{2\eta N}\left(1 - \left[1 + \frac{4\eta NH}{(1+N\chi)^2}\right]^{1/2}\right) + \frac{(1+N\chi)^2}{4\eta N^2}\left(1 - \left[1 + \frac{4\eta NH}{(1+N\chi)^2}\right]^{1/2}\right)^2 \approx \frac{\chi H}{(1+N\chi)} + \frac{\eta H^2}{(1+N\chi)^3}, \quad (15)$$

where cubic and higher order terms in H have been neglected. Thus, provided the maximum applied field is not too strong, the initial magnetisation curve of the sample conforms to Rayleigh's laws, to a good approximation:

$$M_i(H) \approx \chi' H + \eta' H^2, \quad (0 \le H \le H_m;\ \eta H_m << \chi) \tag{16}$$

where χ' and η' are the apparent or effective (demagnetisation-limited) initial susceptibility and Rayleigh coefficient, given by

$$\chi' = \frac{\chi}{(1+N\chi)} \le \frac{1}{N}, \quad \eta' = \frac{\eta}{(1+N\chi)^3} = \frac{\eta}{(\chi/\chi')^3}. \tag{17}$$

Note that $\chi' < \chi$ and $\eta' < \eta$. The relations (10)-(17) are applicable to measurements made on a macroscopic homogeneous sample, or to the magnetisation of a mineral grain in a rock, soil or ore. Néel (1955) gave expressions equivalent to (17) for the demagnetisation-limited susceptibility and Rayleigh coefficient, and stated that the Rayleigh laws still apply to magnetic particles when self-demagnetisation is considered. As can be seen from (14), this conclusion is not strictly correct, but is reasonable in many circumstances.

For magnetic materials with high intrinsic susceptibility, equation (17) implies, firstly, the well-known suppression of the apparent susceptibility by self-demagnetisation and, secondly, an even stronger suppression of the Rayleigh coefficient. The shielding factor for the Rayleigh coefficient is equal to the cube of the shielding factor for the intrinsic susceptibility. As the intrinsic susceptibility increases without limit, such that $N\chi >>1$, $\chi' \to 1/N$ and η'/η, $\eta' H_m/\chi$, $\eta' H_m/\chi' \to 0$. In these circumstances, the magnetisation becomes essentially proportional to the applied field, hysteresis becomes negligible, the Rayleigh loop closes, and the susceptibility is field-independent. On the other hand, if $N\chi << 1$, either because N is very small or $\chi << 1$, then $\chi' \approx \chi$ and $\eta' \approx \eta$. The intrinsic properties can be determined from measured properties of a sample that is subject to self-demagnetisation by inverting (17):

$$\chi = \frac{\chi'}{(1-N\chi')}, \quad \eta = \eta'(1+N\chi)^3 = \frac{\eta'}{(1-N\chi')^3} \quad (\eta H_m << \chi). \tag{18}$$

Beyond the range of validity of the Néel approximation, the intrinsic properties may be estimated by using (10) to calculate H' and fitting a straight line to M/H' (= $\chi + \eta' H'$) versus H'. Markert and Lehmann (1996) give an expression equivalent to $\eta \approx \eta'(1+3N\chi')$ for the relationship between the intrinsic and extrinsic (demagnetisation-affected) Rayleigh coefficients, but their approximation is only valid for $N\chi << 1$. The self-demagnetisation corrections of (18) are difficult to apply when $N\chi >>1$, because the denominators $(1-N\chi')$ become very small, so that measurement errors become amplified. For this reason, measurements of highly magnetic samples are usually made with a geometry that ensures that N is very small. From (14), the maximum magnetisation attained in an applied field of H_m is

$$M'_m = \chi H'_m + \eta H'^2_m = \frac{H_m - H'_m}{N} = \frac{H_m}{N} - \frac{(1+N\chi)}{2\eta N^2}\left[\sqrt{1+\frac{4\eta N H_m}{(1+N\chi)^2}} - 1\right]. \tag{19}$$

Provided the maximum field is not too large

$$M'_m \approx \frac{\chi H_m}{(1+N\chi)} + \frac{\eta H_m^2}{(1+N\chi)^3} = \chi' H_m + \eta' H_m^2, \quad (\eta H_m << \chi), \tag{20}$$

The effect of self-demagnetisation on the descending and ascending arcs of the Rayleigh loop can be obtained similarly. The results are

$$M'_d(H) = \frac{H}{N} - \frac{(1+N\chi)}{2\eta N^2}\left[1+\sqrt{1+\frac{4\eta N H_m}{(1+N\chi)^2}} - 2\sqrt{1-\frac{2\eta N(H-H_m)}{(1+N\chi)^2}}\right] \approx M'_m + \chi'(H-H_m) - \frac{1}{2}\eta'(H-H_m)^2, \tag{21}$$

$$M'_a(H) = \frac{H}{N} + \frac{(1+N\chi)}{2\eta N^2}\left[1+\sqrt{1+\frac{4\eta N H_m}{(1+N\chi)^2}} - 2\sqrt{1+\frac{2\eta N(H+H_m)}{(1+N\chi)^2}}\right] \approx -M'_m + \chi'(H+H_m) + \frac{1}{2}\eta'(H+H_m)^2, \tag{21}$$

$$M'_{IRM} = \frac{(1+N\chi)}{2\eta N^2}\left[2\sqrt{1+\frac{2\eta N H_m}{(1+N\chi)^2}} - \sqrt{1+\frac{4\eta N H_m}{(1+N\chi)^2}} - 1\right] \approx \frac{\eta H_m^2}{2((1+N\chi)^3} = \frac{1}{2}\eta' H_m^2, \tag{22}$$

where the approximate expressions on the RHS of these equations apply if the nonlinearity is not too great, up to the maximum field. Thus, under the conditions of the Néel approximation ($\eta H_m << \chi$), and to the second order in the applied field, the Rayleigh laws are still obeyed throughout the hysteresis loop even when self-demagnetisation is considered. However, considerable departures from the Rayleigh relations occur when $\eta H_m > \chi$. Figure 2 shows the field-dependence of $k_{av} = M/H$, calculated using the exact expression

(14), for a spherical sample or for an equidimensional magnetic grain, with intrinsic properties χ = 0.1 SI, η = 0.001 m/A, in fields up to 1000 A/m. The corresponding Néel approximation, calculated using (16) and (17), is also shown for comparison. The Néel approximation implies a linear increase of susceptibility with field over the full range of applied fields. The exact expression yields a line with significant curvature and shows that the suppression of the Rayleigh coefficient by self-demagnetisation is even stronger than predicted by the Néel approximation. The two curves are in good agreement up to 100 A/m ($\eta H/\chi = 1$), but diverge significantly thereafter. Figure 3 illustrates the effect of self-demagnetisation on low-field hysteresis loops of equidimensional samples or magnetic grains. Self-demagnetisation reduces the slope of the loop axis and narrows the loop. These effects are very substantial when the initial susceptibility is high (~1 SI or higher). The Néel approximation is clearly grossly in error for the case $\eta H/\chi = 10$ and noticeably differs from the exact calculation even for $\eta H/\chi = 1$. Figure 4 shows the predicted effects for differing demagnetising factors, reflecting sample or grain shape, on measured susceptibility versus applied field curves, for several different combinations of intrinsic properties. Self-demagnetisation flattens the slope of the susceptibility-field curve, especially for higher intrinsic susceptibility, and produces concave-down curvature. Since the Rayleigh relationships imply a linear increase of susceptibility with increasing field, this curvature on the plots that use a linear scale for the field indicates departures from the Rayleigh relationships. Because field-dependence of susceptibility is often measured over several decades of field strength, a logarithmic scale for the field is often used for convenience of display in published studies. The plots on the RHS of Figure 4 show that this form of display gives the impression of constancy of susceptibility throughout the lower end of the field range, which then increases substantially at higher fields. The pronounced curvature of the log-scale plots tends to obscure the conformity, or otherwise, of the data with the Rayleigh laws.

FIELD-DEPENDENT SUSCEPTIBILITY OF ROCKS AND MINERALS

The Rayleigh relations predict that susceptibility, however measured, should increase linearly with applied field. Experimentally it is found that rocks, ores and soils obey the Rayleigh laws over a limited range of applied fields. In very low fields, much less than the geomagnetic field, susceptibility is often found to be almost field-independent or to increase much more slowly than is predicted by measurements made in moderate fields, or even to decrease slightly with increasing field (Smith and Banerjee, 1987; Hrouda et al., 2006). The Rayleigh laws usually provide a quite good fit to the measured field- dependent susceptibility over a limited range of applied fields usually, but not always, spanning the geomagnetic and common instrumental field range, in some cases up to several decades of field strength. In all cases, the Rayleigh relationships break down for sufficiently high fields, generally several times to many times higher than the geomagnetic field.

Studies of rocks and ores containing nearly pure magnetite as the only significant magnetic mineral (Worm et al., 1993; Hrouda, 2002; Hrouda et al., 2006), as well as measurements on synthetic samples with dispersed magnetite grains (Jackson et al., 1988; de Wall, 2000; Hrouda, 2002; Vahle and Kontny, 2005; Hrouda et al., 2006), consistently show negligible field dependence of susceptibility for applied fields ranging from less than the geomagnetic field up to the fields commonly used in laboratory measurements, which can be an order of magnitude higher than the geomagnetic field. This is somewhat comforting, because magnetite is the most common magnetic mineral in rocks and ores that are associated with magnetic anomalies. However, similar studies of samples that contain titaniferous magnetite (Bloemendal et al., 1985; Smith and Banerjee, 1987; Worm, 1991; Worm et al., 1993; Markert and Lehmann, 1996; Jackson et al., 1998; de Wall, 2000; Hrouda , 2002; de Wall and Nano, 2004; Vahle and Kontny, 2005; Hrouda et al., 2006), monoclinic pyrrhotite (Clark, 1983, 1984; Worm, 1991; Worm et al., 1993; Markert and Lehmann, 1996; Martin-Hernandez et al., 2008; Guerrero-Suarez and Martin-Hernandez, 2012), or large crystals of hematite (Guerrero-Suarez and Martin-Hernandez, 2012), which can also be important sources of magnetic anomalies, consistently show significant field-dependence of susceptibility.

Jackson et al.(1998) showed that the measured susceptibility of a pure magnetite single crystal in the form of a sphere ~1.5 mm in diameter was essentially indistinguishable, within the accuracy of the measurements, from the theoretical upper limit of $1/N$ = 3 SI. This means that the intrinsic susceptibility is so large that it is indistinguishable from infinity, using this method of measurement, and that the field dependence must be negligible, because the observed low field susceptibility has already attained its ceiling in low fields. With increasing titanium content the observed susceptibility in fields smaller than, and comparable to, the geomagnetic field decreases to values well below the $1/N$ limit, so the characteristic increase of susceptibility with increasing field can be observed. Furthermore, equation (18) can be applied to calculate the intrinsic initial susceptibility and Rayleigh coefficient for these samples. Several studies have confirmed the correlation between the degree of field-dependence of susceptibility of titanomagnetite-bearing rocks and measured Curie temperatures, which serve as a proxy for titanomagnetite composition (de Wall, 2000; de Wall and Nano, 2004; Vahle and Kontny, 2005). Table 2 lists measured initial susceptibilities and Rayleigh coefficients for well-characterised large crystals of titanomagnetite, sized monoclinic pyrrhotite grains and large hematite crystals, along with estimated intrinsic properties, calculated using the Néel approximation. The parameter $\eta H/\chi$, calculated for an applied field of 300 A/m, which characterises the reliability of the Néel approximation and the applicability of equation (18) is also given. Table 2 also gives a standard measure of the observed field-dependence of susceptibility for these samples, k_{Hd}, given by the percentage increase in susceptibility from 30 A/m to 300 A/m. k_{Hd} ranges from < 1% for pure magnetite to ~20% for moderately Ti-rich titanomagnetite, from ~4% for 30 μm pyrrhotite to 45% for 250 μm pyrrhotite, and falls in the range 62%-73% for large hematite crystals. Hrouda et al. (2006) report measurements of field-dependence of susceptibility (in this case over the range <10 A/m to 450 A/m) for titanomagnetite-bearing rocks of up to 70%-80%; and in pyrrhotite-bearing rocks of up to 150% and occasionally more. These data show clearly that failing to account for field-dependence of susceptibility can lead to substantial errors in estimating induced magnetisation in the geomagnetic field.

CONCLUSIONS

Field-dependence of susceptibility is negligible for most rocks and ores for which nearly pure magnetite is the only significant magnetic mineral. Susceptibility is strictly field-independent for weakly magnetic rocks that contain only paramagnetic and/or diamagnetic minerals. Many rocks and ores that contain titaniferous magnetite, monoclinic pyrrhotite, or very coarse-grained hematite exhibit pronounced field-dependence of susceptibility. The non-linearity of the initial magnetisation, which translates into susceptibility that varies with the applied field, is more pronounced for larger grain sizes that contain many magnetic domains. For samples of this type, measurements of susceptibility in fields that are substantially larger than the geomagnetic field, can significantly overestimate the geophysically applicable susceptibility of the sampled lithology. Measurements of the field-dependence of susceptibility can provide information on magnetic mineralogy and granulometry, as well as allowing estimation of the susceptibility in the geomagnetic field of the study area.

Many commercially available susceptibility instruments use fields that are several times larger, or even an order of magnitude larger, than the geomagnetic field. This means that we should exercise caution when applying susceptibility measurements made with such instruments. Information on the magnetic mineralogy of the samples, from rock magnetic methods or otherwise, can be used to determine if there is likely to be a problem with the measurements. Susceptibility measurements made at least two different field strengths can be used to estimate the geophysically applicable susceptibility, by applying Rayleigh's relationships. Ideally, a complete characterisation of the field-dependence of susceptibility by measurements over a substantial range of fields, will give the most reliable estimate of the susceptibility in the ambient field of the study area. Since not many laboratories have the capability to fully characterise field-dependent susceptibility, susceptibility instruments that use relatively low fields, as close as possible to the geomagnetic field range, are preferable to instruments that use relatively strong fields.

The Rayleigh laws are modified by self-demagnetisation. This means that, for strongly magnetic samples the measured initial susceptibility and *a fortiori* the measured Rayleigh coefficient are suppressed with respect to the intrinsic properties of the sample by an amount that depends on the sample shape, via the demagnetising factor along the applied field. Within the Rayleigh domain intrinsic properties can be recovered by using equation (18) when the Néel approximation is applicable or, for stronger fields, by estimating χ and η from a linear regression of $M/H' = M/(H-NM)$ on $H' = (H-NM)$. Self-demagnetisation also strongly affects the low-field properties of multidomain magnetic mineral grains dispersed within the weakly magnetic matrix of a rock, soil or ore. Field-dependence of susceptibility is strongly suppressed by self-demagnetisation for dispersed mineral grains that have high intrinsic susceptibility, such as end-member magnetite or nearly pure magnetite. Field-independence of susceptibility has also been confirmed over a wide range of fields for single domain magnetite (Hrouda et al., 2006). As the titanium content of titanomagnetite increases, the intrinsic susceptibility decreases, and field-dependence of susceptibility becomes important. Field-dependence of susceptibility is important for pyrrhotite grains larger than ~10 μm, and becomes more prominent with increasing grain size. Multidomain hematite grains, i.e. those larger than 100 μm, also exhibit pronounced field-dependence of susceptibility.

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Instrument	Field (A/m)	Type of field	Company/Institute
CSIRO susceptibility bridge[1,4]	~80	Peak	CSIRO
KLY1[2]	150	Effective	AGICO
KLY2[2]	300	Effective	AGICO
KLY3[2]	300	Effective	AGICO
KLY4S[3]	2-450	Effective	AGICO
KLF4A[3]	5-300	Effective	AGICO
MFK1[2]	2-700	Effective	AGICO
SI2/2B[2]	60	Peak	Sapphire
Lakeshore 7130[2]	0.1-2000	Peak	Sapphire
SI2 HF1[2]	1-1800	Peak	Sapphire
SI2 HF2[2]	400-40,000	Peak	Sapphire
SI2 HF3[2]	500-80,000	Peak	Sapphire
MS2/3[2]	~200	Peak	Bartington
MPMS[2]	0-1600	Effective	Quantum Designs
Magnon VFSM[2]	20-400	Peak?	Magnon GmbH
"Roly-Poly" magnetic anisotropy bridge[2]	8-800	Effective	Institute for Rock Magnetism (University of Minnesota)
Low field hysteresis loop tracer[4,5]	0-1200	Peak	Arun Electronics

*Data sources: [1]Ridley and Brown, (1980); [2]Guerrero-Suarez and Martin-Hernandez (2012); [3]Hrouda et al. (2006); [4]Clark (1983); [5]Radhakrishnamurty and Sastry (1970).

Table 1. Applied fields used by susceptibility instruments*

Mineral	χ'	η'	χ	η	$\eta H/\chi$ (H = 300 A/m)	k_{Hd} (%)
TM0 (1-2 mm)[1,2]	2.99	$\sim10^{-4}$	> 50	-	-	0.7
TM5 (1-2 mm)[1,2]	2.84	$\sim4\times10^{-4}$	~50	~2.5	~15	3.8
TM28 (1-2 mm)[1,2]	2.64	0.00152	22	0.84	12	13.3
TM41 (1-2 mm)[1,2]	2.23	0.00264	8.6	0.15	5.3	23.6
TM55 (1-2 mm)[1,2]	1.93	0.00148	5.4	0.032	1.8	16.7
Pyrrhotite[2,3] (250 μm)	0.174	5.82E-04	0.182	0.00066	1.00	45.1
Pyrrhotite[2,3] (150 μm)	0.155	4.04E-04	0.161	0.000452	0.78	39.5
Pyrrhotite[2,3] (100 μm)	0.132	2.61E-04	0.136	0.000287	0.59	33.5
Pyrrhotite[2,3] (75 μm)	0.107	1.00E-04	0.110	0.000108	0.28	19.7
Pyrrhotite[2,3] (55 μm)	0.087	7.00E-05	0.089	0.0000745	0.24	17.5
Pyrrhotite[2,3] (40 μm)	0.072	4.00E-05	0.073	0.0000421	0.17	12.9
Pyrrhotite[2,3] (30 μm)	0.059	1.00E-05	0.060	1.04E-05	0.05	4.4
Hematite[2]	0.173	0.00255	0.180	0.00289	4.4	73.4
Hematite[2]	0.29	0.00328	0.312	0.00407	3.4	69.5
Hematite[2]	0.163	0.00119	0.170	0.00134	2.2	61.8

[1]Jackson et al. (1998); [2]Hrouda (2002); [3]Worm et al. (1993). TMx = titanomagnetite with x% ulvospinel content

k_{Hd} = field dependence of susceptibility = $[k(H = 300 \text{ A/m}) - k(H = 30 \text{ A/m})]/ k(H = 300 \text{ A/m}) \times 100\%$; $N = 1/3$ assumed

Table 2. Field-dependent susceptibility properties of common magnetic minerals

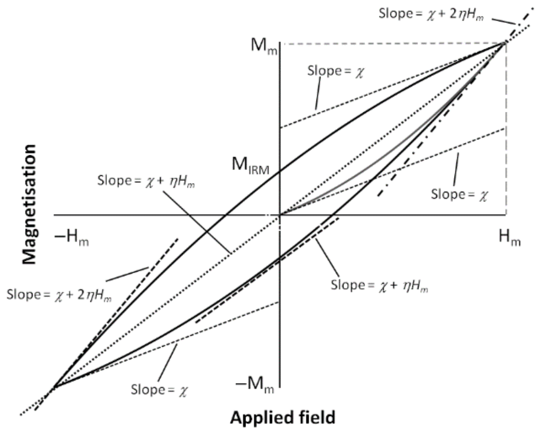

Figure 1. Intrinsic initial magnetisation curve (solid grey line) and Rayleigh hysteresis loop (black) for a magnetic material with initial susceptibility χ and Rayleigh parameter η, for the case where $\chi = \eta H_{max}$. Note that the total differential susceptibility at each turning point of the loop, immediately after a field reversal, is equal to the initial susceptibility. The total differential susceptibility at all other points of the loop and along the initial magnetisation curve, and the average susceptibility over the interval $[0,H_m]$ of the initial magnetisation curve, are greater than χ.

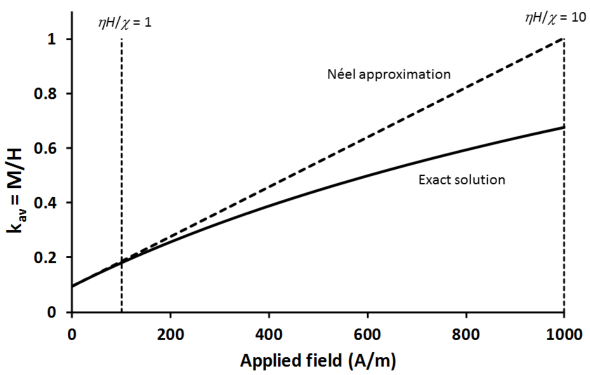

Figure 2. Comparison of the exact expression for the field-dependent average susceptibility $k_{av} = M/H$ with the Néel approximation $k_{av} = \chi' + \eta' H$, as a function of applied field, for a spherical specimen or magnetic particle (N = 1/3). The intrinsic properties used for this calculation are χ = 0.1 SI and η = 0.001 m/A. Note the significant divergence of the exact solution from the Néel approximation for $\eta H/\chi > 1$.

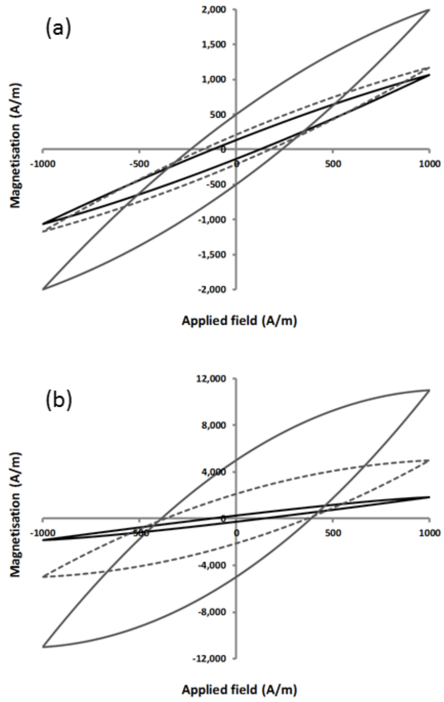

Figure 3. Effect of self-demagnetisation on the intrinsic Rayleigh loop. A spherical sample or mineral grain, for which $N = 1/3$ SI, is assumed. (a) $\chi = 1$ SI, $\eta = .001$ m/A, $\eta H_m/\chi = 1$; (b) $\chi = 1$ SI, $\eta = .01$ m/A, $\eta H_m/\chi = 10$. The intrinsic Rayleigh loop is shown in grey; the measured loop, accounting for self-demagnetisation, is in black; and the Néel approximation to the measured loop is shown by the dashed line.

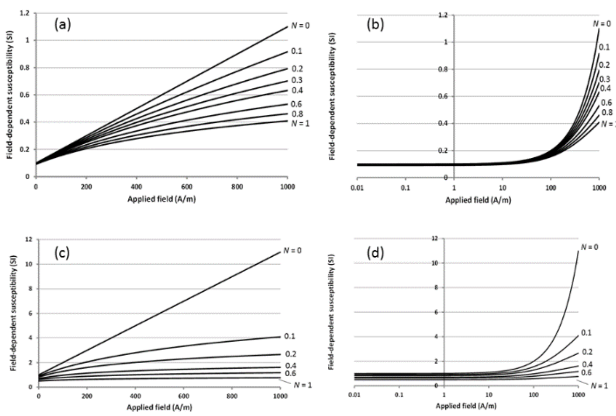

Figure 4. Effect of self-demagnetisation on field-dependent susceptibility. The horizontal axis is the external applied field H; the vertical axis is $k_{av} = M/H$, where M is calculated using the exact treatment of self-demagnetisation. The horizontal scale is linear on the LHS and logarithmic on the RHS. For (a) and (b) the assumed intrinsic properties are $\chi = 0.1$ SI, $\eta = 0.001$ m/A; for (c) and (d) they are $\chi = 1$ SI, $\eta = 0.01$ m/A. In both cases $\eta H_m/\chi = 10$ for a maximum applied field of 1000 A/m. Values of the demagnetising factor range from $N = 0$ (e.g. a sample in a closed magnetic circuit, or a needle-like grain with its long axis aligned with the field) to $N = 1$ (a thin disc-like sample or grain, perpendicular to the field), and are indicated next to the corresponding k_{av} vs H curve.