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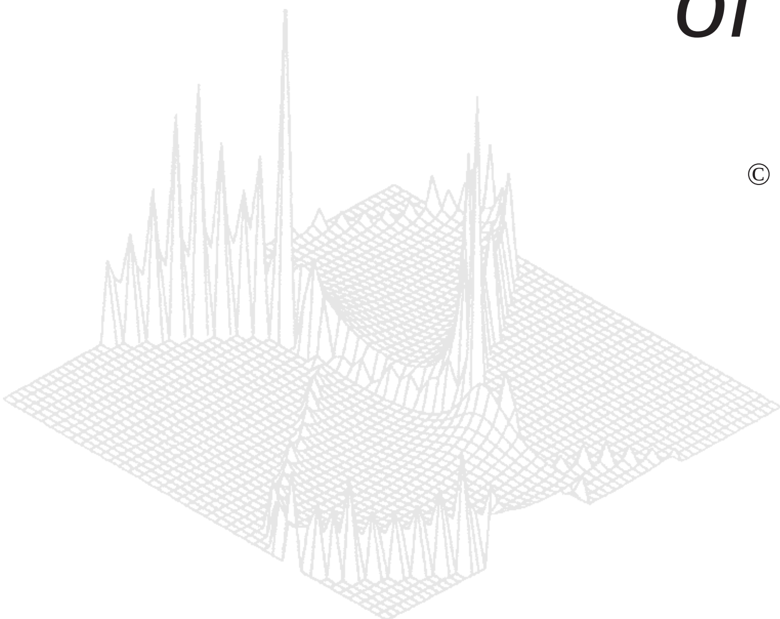

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Effect of Negative Ions on the Instability of Ion-acoustic Waves in a Relativistic Plasma

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Abstract

The dispersion relation of an ion-acoustic wave propagating through a collisionless, unmagnetised plasma, having warm isothermal electrons and cold positive and negative ions has been derived. It is seen that the ion-acoustic wave will be unstable in the presence of streaming of ions. Instability of the wave is graphically analysed for the plasma having (H^+, O^-) ions, (H^+, O_2^-) ions, (H^+, SF_5^-) ions, (He^+, Cl^-) ions and (Ar^+, O^-) ions with different negative ion concentration and relativistic velocity.

1. Introduction

Wave propagation through a plasma in the presence of negative ions has been studied by various authors. Angelo *et al.* (1966) showed that ion waves have two modes of propagation in a plasma having negative ions, one of these is a 'slow ion mode' observed by Wong *et al.* (1975). Uberoi and Das (1972) and Uberoi (1973) showed that the presence of negative ions in the lower ionosphere has significant effects on the diagnosis of the plasma. Later Sur *et al.* (1989) studied the propagation of ion-cyclotron whistlers in the atmosphere by considering the effects of negative ions. They calculated the group travel time of whistlers at mid-latitude and equator and also investigated the damping of whistlers in the presence of negative ions. Subsequently Paul *et al.* (1989) and Kashyapi *et al.* (1993) showed that the negative ions introduce a contribution to the instability of the waves and to the shift of wave parameters. However, the work by these authors is for the propagation of waves in a non-relativistic plasma. Recently, Chakraborty *et al.* (1992, 1994) considered the relativistic effect in a study of ion-acoustic solitary waves in plasma. They extended the work of Das and Tagare (1975), Watanabe (1984), Nakamura *et al.* (1985), Verheest (1988) and Singh and Das (1989) and showed that both the relativistic effect and the negative ions have a significant contribution to the formation of solitary waves. However, the stability of an ion-acoustic wave in a multicomponent plasma having negative ions and stream velocity of the ions has not yet been studied. In our present study, we have derived the dispersion relation of an ion-acoustic wave propagating through a relativistic plasma in the presence of negative ions and have studied the stability of the wave under various conditions.

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2. Assumptions and Basic Equations

We assume that the plasma is unmagnetised, cold and collisionless. It consists of electrons, positive ions and negative ions. We further assume that the velocity of the ions is weakly relativistic. Therefore, the basic equations which govern the dynamics of such a plasma system are

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0\,, \tag{1}$$

$$\frac{\partial \bar{u}_\alpha}{\partial t} + u_\alpha \frac{\partial \bar{u}_\alpha}{\partial x} = -\frac{\partial \phi}{\partial x}\,, \tag{2}$$

for the positive ions and

$$\frac{\partial n_\beta}{\partial t} + \frac{\partial}{\partial x}(n_\beta u_\beta) = 0\,, \tag{3}$$

$$\frac{\partial \bar{u}_\beta}{\partial t} + u_\beta \frac{\partial \bar{u}_\beta}{\partial x} = \frac{1}{Q}\,\frac{\partial \phi}{\partial x}\,, \tag{4}$$

for the negative ions.

The electrostatic potential ϕ satisfies the relation

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\mathrm{e}} + n_\beta - n_\alpha\,, \tag{5}$$

where

$$\bar{u}_\alpha = u_\alpha(1 - u_\alpha^2/c^2)^{-\frac{1}{2}}, \quad \bar{u}_\beta = u_\beta(1 - u_\beta^2/c^2)^{-\frac{1}{2}}, \quad n_{\mathrm{e}} = \exp\,\phi\,. \tag{6}$$

Here n_{e}, n_α and n_β are the number densities of electrons, positive ions and negative ions, u_α and u_β are the velocities of positive ions and negative ions, Q is the ratio of negative ion mass m_β and positive ion mass m_α, ϕ is the electrostatic potential, and c is the velocity of light.

3. Dispersion Relation

We assume that the variable parameters in the plasma are perturbed as

$$\begin{bmatrix} n_\alpha \\ n_\beta \\ u_\alpha \\ u_\beta \\ \phi \end{bmatrix} = \begin{bmatrix} n_{\alpha_0} \\ n_{\beta_0} \\ u_{\alpha_0} \\ u_{\beta_0} \\ 0 \end{bmatrix} + \epsilon \begin{bmatrix} n_\alpha^{(1)} \\ n_\beta^{(1)} \\ u_\alpha^{(1)} \\ u_\beta^{(1)} \\ \phi^{(1)} \end{bmatrix} + \epsilon^2 \begin{bmatrix} n_\alpha^{(2)} \\ n_\beta^{(2)} \\ u_\alpha^{(2)} \\ u_\beta^{(2)} \\ \phi^{(2)} \end{bmatrix} + \ldots, \tag{7}$$

and that the ion-acoustic wave has the form $\exp[\mathrm{i}(kx - \omega t)]$, where k is the wave number and ω is the wave frequency. Therefore, using (7) in (1)–(5) we get

$$n_\alpha^{(1)} = \frac{n_{\alpha_0}\, u_\alpha^{(1)}}{\omega - k u_{\alpha_0}}, \qquad n_\beta^{(1)} = \frac{n_{\beta_0}\, u_\beta^{(1)}}{\omega - k u_{\beta_0}}\,, \tag{8}$$

$$u_\alpha^{(1)} = \frac{k\phi^{(1)}}{\gamma_\alpha(\omega - k u_{\alpha_0})}, \qquad u_\beta^{(1)} = \frac{-k\phi^{(1)}}{Q\gamma_\beta(\omega - k u_{\beta_0})}\,, \tag{9}$$

$$(k^2+1)\phi^{(1)} = n_\alpha^{(1)} - n_\beta^{(1)}\,, \tag{10}$$

where

$$\gamma_\alpha = 1 + \frac{3u_{\alpha_0}^2}{2c^2}, \qquad \gamma_\beta = 1 + \frac{3u_{\beta_0}^2}{2c^2}\,.$$

From (8)–(10) the dispersion relation of the ion-acoustic wave is obtained as

$$k + \frac{1}{k} = \frac{n_{\alpha_0}}{\gamma_\alpha(\omega - k u_{\alpha_0})^2} + \frac{n_{\beta_0}}{Q\gamma_\beta(\omega - k u_{\beta_0})^2}\,. \tag{11}$$

Simplifying (11) we get

$$A_6\, k^6 + A_5\, k^5 + A_4\, k^4 + A_3\, k^3 + A_2\, k^2 + A_1\, k + A_0 = 0\,, \tag{12}$$

where

$$A_6 = Q\gamma_\alpha\gamma_\beta\, u_{\alpha_0}^2\, u_{\beta_0}^2\,,$$

$$A_5 = -\,2\omega Q\gamma_\alpha\,\gamma_\beta\, u_{\alpha_0}\, u_{\beta_0}(u_{\alpha_0} + u_{\beta_0})\,,$$

$$A_4 = u_{\alpha_0}^2\, u_{\beta_0}^2\, Q\gamma_\alpha\,\gamma_\beta + \omega^2 Q\gamma_\alpha\,\gamma_\beta(u_{\alpha_0}^2 + u_{\beta_0}^2) + 4\omega^2 Q\gamma_\alpha\,\gamma_\beta\, u_{\alpha_0}\, u_{\beta_0}\,,$$

$$\begin{aligned} A_3 = &-\,2\omega Q\gamma_\alpha\,\gamma_\beta\, u_{\alpha_0} u_{\beta_0}(u_{\alpha_0} + u_{\beta_0}) - 2\omega^3 Q\gamma_\alpha\,\gamma_\beta(u_{\alpha_0} + u_{\beta_0}) \\ &- (\eta_{\beta_0}\,\gamma_\alpha\, u_{\alpha_0}^2 + Q n_{\alpha_0}\gamma_\beta u_{\beta_0}\,\gamma_\beta\, u_{\beta_0}^2)\,, \end{aligned}$$

$$\begin{aligned} A_2 = &\,\omega^2 Q\gamma_\alpha\,\gamma_\beta(u_{\alpha_0}^2 + u_{\beta_0}^2) + 4\omega^2 Q\gamma_\alpha\,\gamma_\beta\, u_{\alpha_0}\, u_{\beta_0} + 2\omega(\eta_{\beta_0}\,\gamma_\alpha\, u_{\alpha_0} + Q n_{\alpha_0}\,\gamma_\beta\, u_{\beta_0}) \\ &+ \omega^4 Q\gamma_\alpha\gamma_\beta\,, \end{aligned}$$

$$A_1 = -\,2\omega^3 Q\gamma_\alpha\,\gamma_\beta(u_{\alpha_0} + u_{\beta_0}) - \omega^2(\gamma_\alpha\, n_{\beta_0} + Q\gamma_\beta\, n_{\alpha_0})\,,$$

$$A_0 = Q\gamma_\alpha\,\gamma_\beta\,\omega^4\,.$$

4. Results and Discussion

From (12) we observe that the ion-acoustic wave has six modes corresponding to six roots, some of which may be complex, i.e. $k = k_r + ik_i$, where k_r and k_i are both real. Here $k_r > 0$ implies propagation in the $+x$ direction, and $k_r < 0$ implies propagation in the $-x$ direction. If k_r and k_i are both positive, the wave will be damped, but if k_r is positive and k_i is negative, the wave will grow exponentially. From (12) it is interesting to see that the relativistic effect on the wave does not have any role in the instability in the absence of streaming of ions, i.e. $u_{\alpha_0} = u_{\beta_0} = 0$. In order to get an idea of the effects of negative ion concentration and relativistic velocity on the propagation of waves, equation (12) was solved using 'MATHEMATICA' software. We take plasmas having various types of positive and negative ions, e.g. (H^+, O^-) ions, (H^+, O_2^-) ions, (H^+, SF_5^-) ions, (He^+, Cl^-) ions and (Ar^+, O^-) ions.

In order to get more insight into the role of negative ions, stream velocity and the relativistic effect on the instability of ion-acoustic waves we plot the

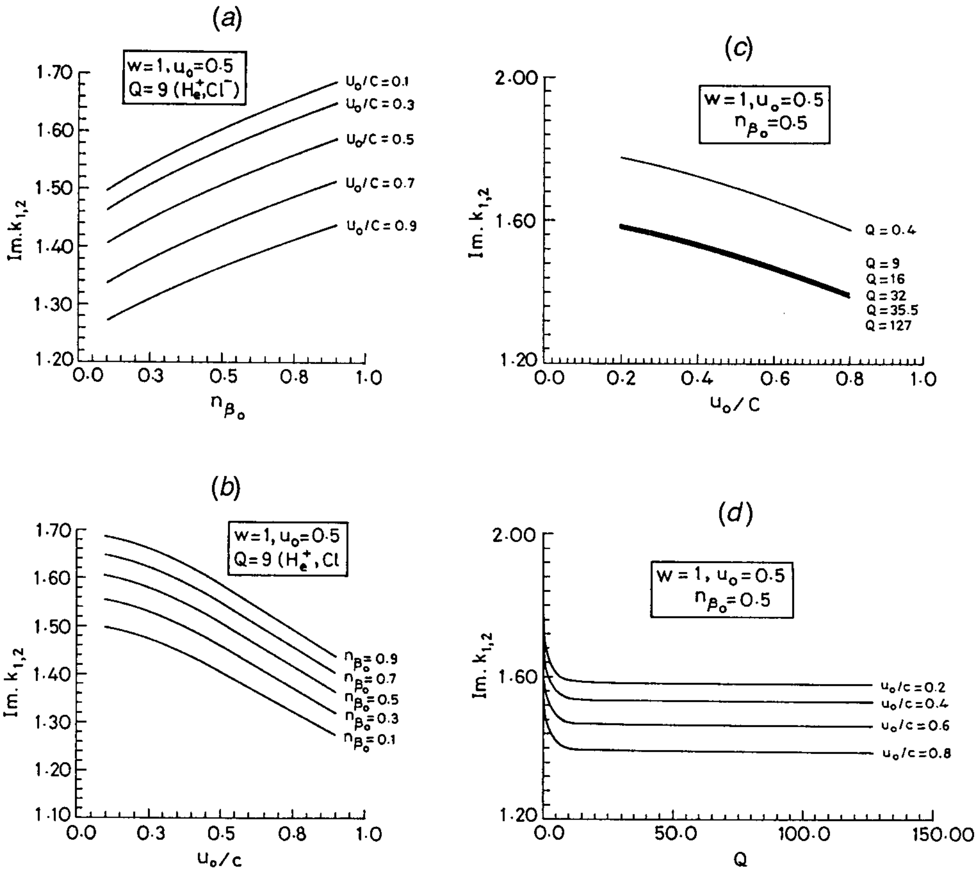

Fig. 1. (a) Variation of the instability factor $\mathrm{Im}\, k_{1,2}$ with n_{β_0}, the negative ion concentration for different values of u_0/c. (b) Change of $\mathrm{Im}\, k_{1,2}$ with u_0/c for different values of n_{β_0}. (c) Plot of $\mathrm{Im}\, k_{1,2}$ versus u_0/c for different values of Q, the ratio of negative ion mass to the positive ion mass. (d) Dependence of $\mathrm{Im}\, k_{1,2}$ on Q for different values of u_0/c.

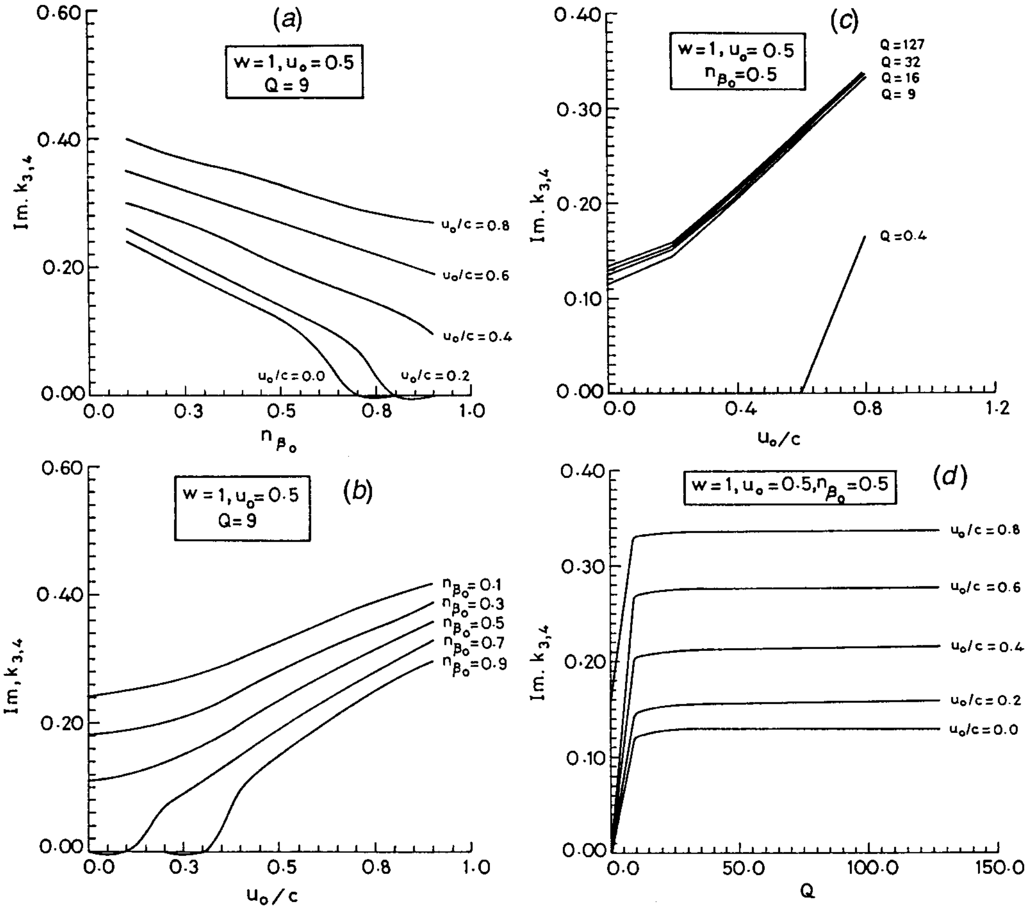

Fig. 2. (*a*) Variation of $\mathrm{Im}\, k_{3,4}$ with n_{β_0} for different values of u_0/c. (*b*) Change of $\mathrm{Im}\, k_{3,4}$ with u_0/c for different negative ion concentrations. (*c*) Plot of instability factor against u_0/c for different values of Q. (*d*) Dependence of $\mathrm{Im}\, k_{3,4}$ on Q for different values of u_0/c.

instability factor in Figs 1–3. In Fig. 1*a* we see that the first and second modes of a wave propagating through a negative ion plasma having $(\mathrm{He}^+, \mathrm{Cl}^-)$ ions will be highly unstable if the concentration of negative ions is large. From Fig. 1*b* it is observed that the wave is more stable in a highly relativistic plasma. From Figs 1*c* and 1*d* we notice the effect of negative ion mass on the instability of the waves. For heavy negative ions the instability is less than that of the lighter negative ions. On the other hand, Fig. 2*a* shows that for the third and fourth modes of the wave the instability decreases with an increase of negative ion concentration. For these modes the instability increases with an increase of relativistic velocity (Fig. 2*b*). Figs 2*c* and 2*d* indicate the role of negative ion mass on the instability. For the plasma with $(\mathrm{Ar}^+,\ \mathrm{O}^-)$ ions there exists a limiting value of the relativistic velocity $(u_0/c \approx 0{\cdot}6)$ for an unstable wave. For the other two modes the effects of negative ion concentration, mass and relativistic velocity are depicted in Fig. 3. The nature of the dependence of the instability for the above parameters is similar to that of the first two modes shown in Fig. 1. From the above analysis we observe that the characteristics of

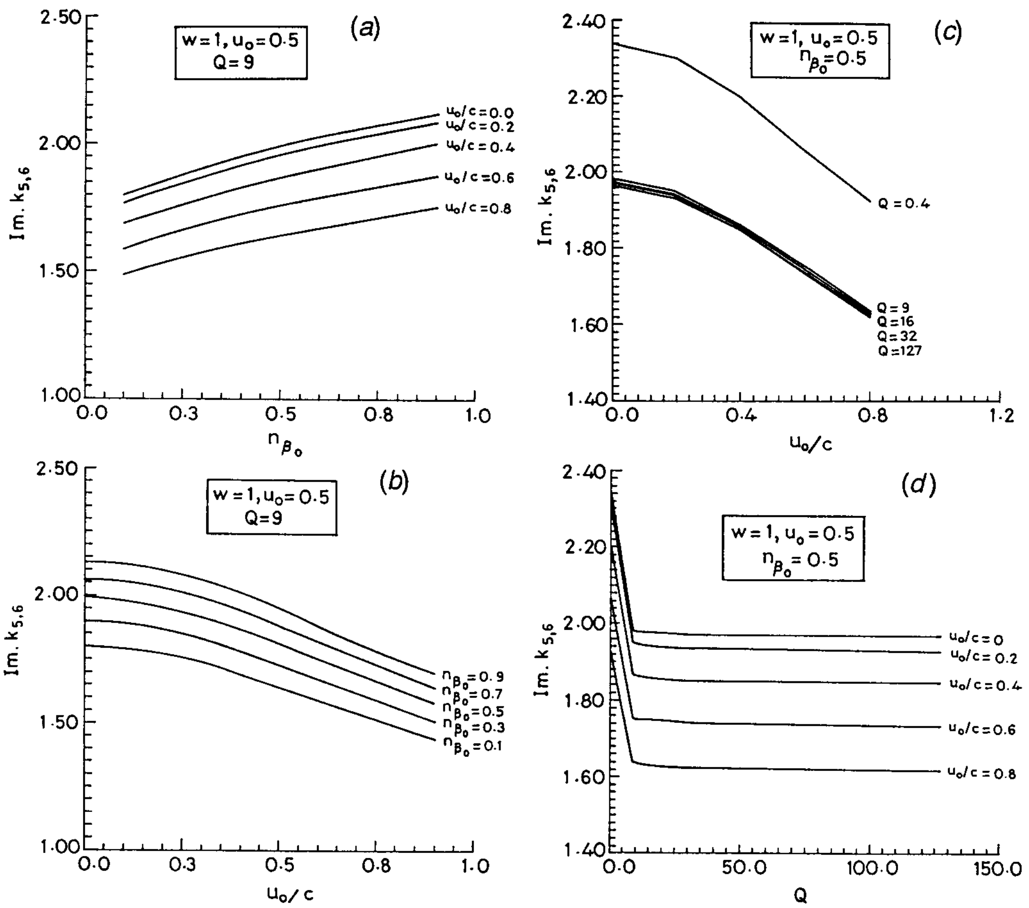

Fig. 3. (*a*) Variation of $\mathrm{Im}\, k_{5,6}$ with n_{β_0} for different values of u_0/c. (*b*) Change of $\mathrm{Im}\, k_{5,6}$ with u_0/c for different negative ion concentrations. (*c*) Plot of instability factor against u_0/c for different values of Q. (*d*) Dependence of $\mathrm{Im}\, k_{5,6}$ on Q for different values of u_0/c.

the third and fourth modes of the ion-acoustic wave are more interesting than those of the first, second, fifth and sixth modes. Moreover, we see that for the heavy negative ions, for example $(\mathrm{H}^+, \mathrm{O}_2^-)$, $(\mathrm{He}^+, \mathrm{Cl}^-)$ and $(\mathrm{H}^+, \mathrm{SF}_5^-)$, the results are very similar and so they are very difficult to identify. For experimental observations, it will be convenient to use a plasma containing $(\mathrm{He}^+, \mathrm{Cl}^-)$ ions and $(\mathrm{Ar}^+, \mathrm{O}^-)$ ions.

From Figs 1–3, we get an idea of the role of negative ions and the stream velocities on the instability of the wave, but we cannot get any concept of the six modes of the ion-acoustic wave in the plasma. From a numerical estimation for the $(\mathrm{He}^+, \mathrm{Cl}^-)$ plasma with $\omega = 1$, $u_0 = 0\cdot5$, $n_{\beta_0} = 0\cdot1$ to $0\cdot9$ and $u_0/c = 0\cdot1$ to $0\cdot9$, it is observed that the first and second modes of the ion-acoustic wave are reflected (as k_r is negative) and the other modes, i.e. third, fourth, fifth and sixth modes will propagate through the medium as k is positive. But for the above plasma with $\omega = 10$, there is no reflecting mode (as all the k_r are positive). Moreover, we observe that the phase velocities of the wave are different for the

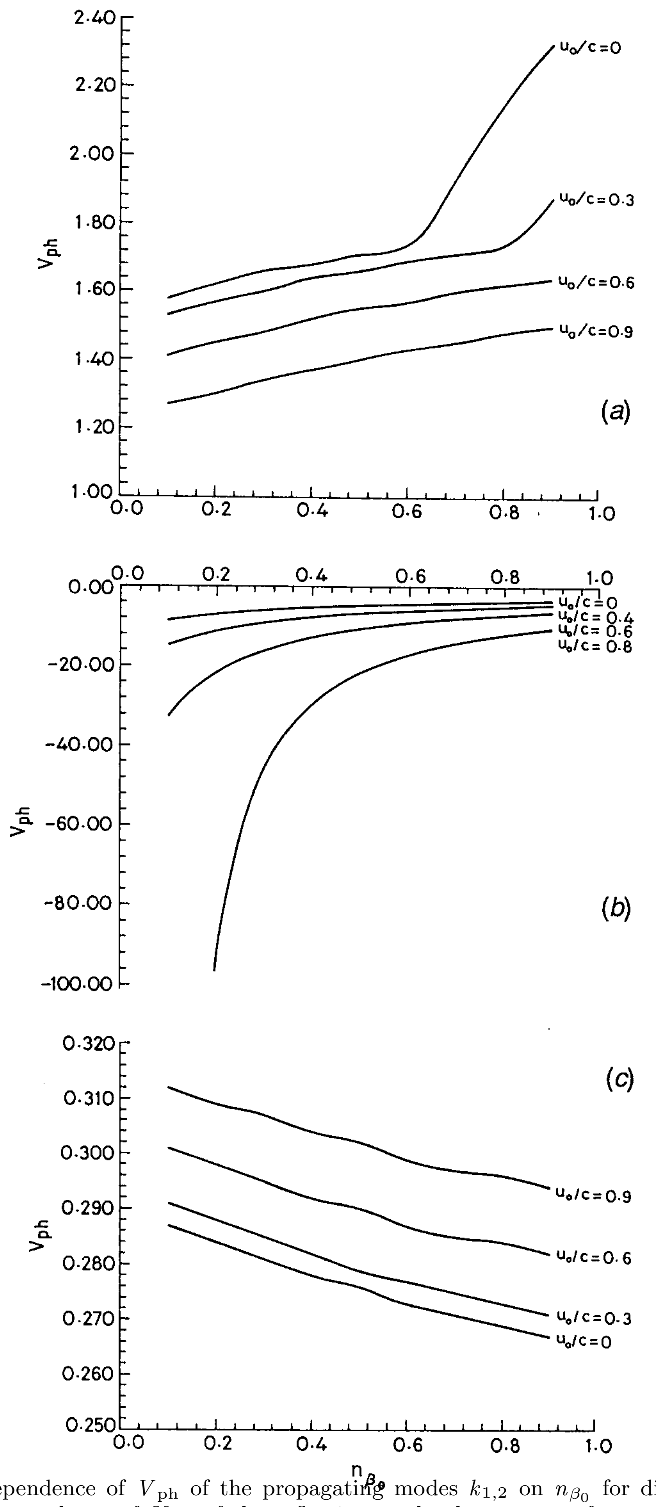

Fig. 4. (*a*) Dependence of V_{ph} of the propagating modes $k_{1,2}$ on n_{β_0} for different values of u_0/c. (*b*) Dependence of V_{ph} of the reflecting modes $k_{3,4}$ on n_{β_0} for various values of u_0/c. (*c*) Dependence of V_{ph} of of V_{ph} the propagating modes $k_{5,6}$ on n_{β_0} for different values of u_0/c.

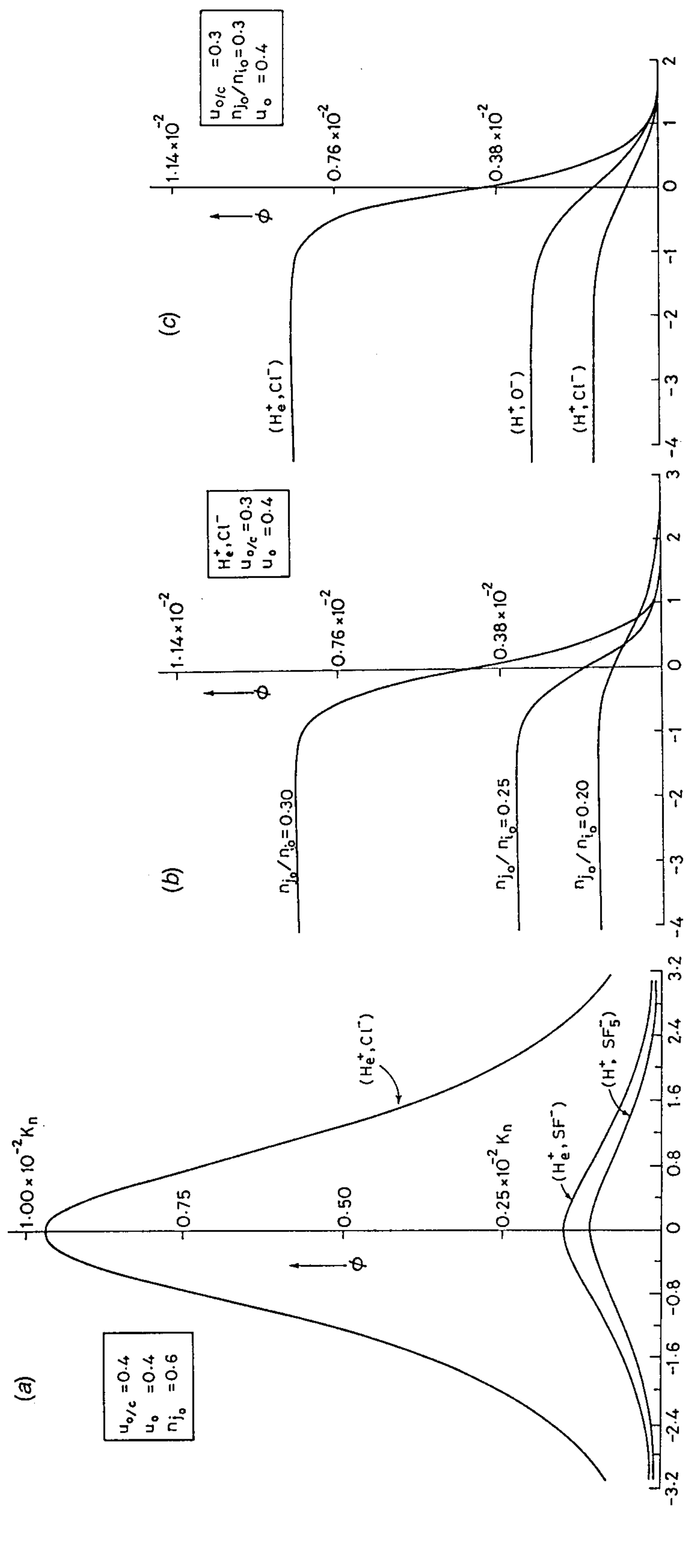

Fig. 5. (*a*) Profiles of solitary waves for various negative ions. (*b*) Profiles of ion-acoustic shocks of (He^+, Cl^-) plasma with various concentrations. (*c*) Profiles of ion-acoustic double layers for different negative ions.

reflecting and propagating modes. It is to be noted that the phase velocities depend on the species of negative ions in the plasma. In Fig. 4 the phase velocities of six modes of the ion-acoustic wave are shown. It is seen that for the reflecting modes (first and second) phase velocities are larger for large Q, i.e. the ratio of negative ion mass to positive ion mass. But for the third and fourth modes (propagating) phase velocities are large for small Q, whereas for the fifth and sixth modes the phase velocities are large for large Q. Again we observe that streaming of ions, as well as the relativistic velocities, has an important contribution to the phase velocities. For large values of u_0/c, the reflecting modes have large phase velocities. However, for the third and fourth modes the relativistic velocities reduce the phase velocities, whereas the phase velocities of the fifth and sixth modes are increased by the effect of relativistic streaming.

5. Remarks

From a study of the propagation of ion-acoustic waves it is found that the wave will be unstable in a negative-ion plasma in the presence of streaming of ions. The mass and the concentration of negative ions and the relativistic velocity are the important factors on which the instability of the wave mainly depends. Following the work of Roychowdhury *et al.* (1994) and Chakraborty *et al.* (1992), propagation of the ion-acoustic solitions and shocks can be investigated in the negative-ion relativistic plasma. The profiles of the solitary wave and shocks would be as shown in Figs 5*a* and 5*b* (Paul *et al.* 1996).

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