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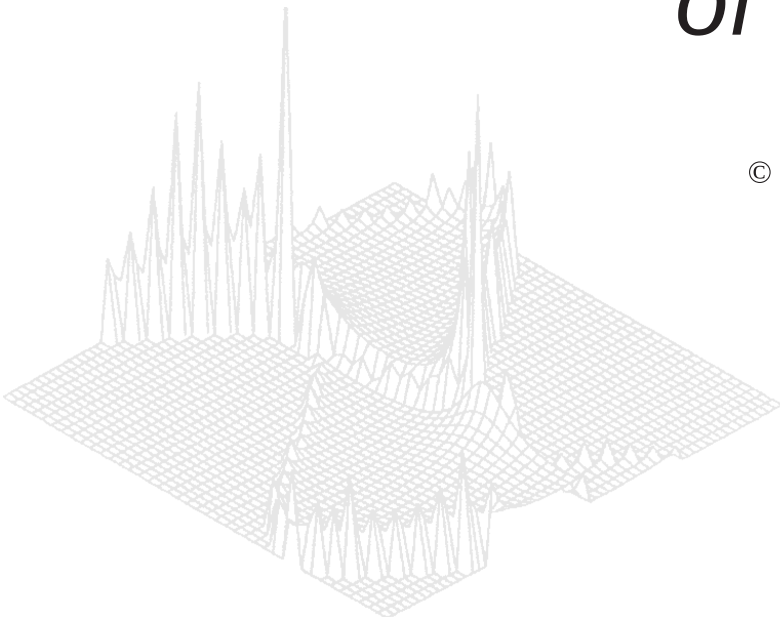

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Tensor Charge of the Nucleon on a Lattice*

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Abstract

The tensor charge of the nucleon, which will be measured in Drell–Yan processes in polarized proton–proton collisions at the RHIC, is studied in a quenched lattice QCD simulation. On the $16^3\times$ 20 lattice with $\beta = 5{\cdot}7$, connected parts of the tensor charge are determined with small statistical error, while the disconnected parts are found to be small with relatively large error bars. The flavour-singlet tensor charge ($\delta\Sigma = \delta u + \delta d + \delta s$) is not suppressed, as opposed to the flavour-singlet axial charge ($\Delta\Sigma = \Delta u + \Delta d + \Delta s$).

1. Introduction

The parton structure of the nucleon in the twist 2 level is known to be characterized by three structure functions $f_1(x,\mu)$, $g_1(x,\mu)$ and $h_1(x,\mu)$ with x being the Bjorken variable and μ being the renormalization scale (see e.g. [1]). Here f_1 and g_1, which represent the quark-momentum distribution and quark-spin distribution respectively, can be measured by deep inelastic lepton–hadron scattering (DIS). On the other hand, h_1, which represents the quark-transversity distribution, can only be measured in the polarized Drell–Yan processes, since it is related to the matrix element of the chiral-odd quark operator. Although such experiments are not yet available, they are planned for the Relativistic Heavy Ion Collider at Brookhaven National Laboratory. Therefore, theoretical prediction of h_1 has some importance. Also, whether there is a 'transversity crisis' in the first moment of h_1, as in the case of 'spin crisis' in g_1, is an interesting question to be examined. In this paper, we will concentrate on the first moment of $h_1(x)$ and report our recent studies using lattice QCD simulations [2].

2. Tensor Charge versus Axial Charge

The first moments of $g_1(x,\mu)$ and $h_1(x,\mu)$ are called $\Delta q(\mu)$ and $\delta q(\mu)$ respectively. They are related to the nucleon matrix elements of the axial current and tensor current as follows:

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$$\langle ps \mid \bar{q}\gamma_\mu\gamma_5 q \mid ps\rangle = 2Ms_\mu \ \Delta q\,, \tag{1}$$

$$\langle ps \mid \bar{q}i\sigma_{\mu\nu}\gamma_5 q \mid ps\rangle = 2(s_\mu p_\nu - s_\nu p_\mu) \ \delta q\,, \tag{2}$$

where p_μ is the nucleon's four momentum, M is the nucleon's rest mass, and s_μ is the nucleon's covariant spin-vector.

In the light cone frame, Δq is interpreted as the total quark-helicity in the nucleon, while δq is a total quark-transversity in the nucleon [1]:

$$\begin{aligned}\Delta q(\mu) &= \int_0^1 [g_1(x,\mu) + \bar{g}_1(x,\mu)]dx \\ &= \int_0^1 [N_+(x,\mu) - N_-(x,\mu) + \overline{N}_+(x,\mu) - \overline{N}_-(x,\mu)]dx\,,\end{aligned} \tag{3}$$

and

$$\begin{aligned}\delta q(\mu) &= \int_0^1 [h_1(x,\mu) - \bar{h}_1(x,\mu)]dx \\ &= \int_0^1 [N_\uparrow(x,\mu) - N_\downarrow(x,\mu) - \overline{N}_\uparrow(x,\mu) + \overline{N}_\downarrow(x,\mu)]dx\,.\end{aligned} \tag{4}$$

Here $N_+(x)$ $[N_-(x)]$ denotes the momentum distribution of quarks having the same (opposite) helicity with the nucleon, while $N_\uparrow(x)$ $[N_\downarrow(x)]$ denotes that having the same (opposite) transverse polarization with the nucleon. The quantity with bar is the distribution for anti-quarks.

On the other hand, in the rest frame of the nucleon, Δq (δq) denotes the quark-spin + anti-quark-spin (quark-spin–anti-quark-spin), which can be seen by taking $\mu = i$ $(\mu = 0, \nu = i)$ in equations $(1, 2)$. In this frame, estimates by using hadron models are possible. For example, relativistic quark models which can reproduce $g_A = 1\cdot 25$ correctly give simple inequalities;

$$\mid \delta u \mid \ > \mid \Delta u \mid \ , \qquad \mid \delta d \mid \ > \mid \Delta d \mid \ . \tag{5}$$

The lower component of the Dirac spinor of the confined quarks plays an essential role for the above inequalities.

Drawbacks of such model calculations are (i) the renormalization scale where the matrix elements are evaluated is not clear, and (ii) the strange quark contribution, which originates from the OZI violating processes, is hard to estimate in a reliable manner. Lattice QCD simulations can overcome these problems even within the quenched approximation. In particular, Δu, Δd and Δs have been studied by two groups [3, 4] and their results are consistent with the recent experimental data on the spin structure of the nucleon [5]. At the quenched level, two kinds of diagrams arise: one is the connected amplitude (Fig. 1a) in which the external operator is connected to one of the valence nucleon lines, and another is the disconnected amplitude (Fig. 1b) where the quark line coming from the external operator is closed by itself. The latter gives an OZI violating amplitude leading to the strangeness content in the nucleon.

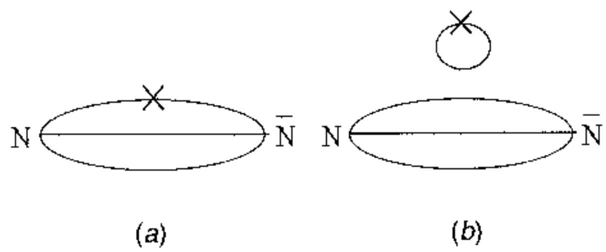

Fig. 1. (a) Connected matrix element of the nucleon where the cross denotes the operator insertion. (b) Disconnected matrix element of the nucleon.

3. Matrix Elements on the Lattice

(3a) Mass and Matrix Elements

We use the standard Wilson action in our simulation in which two basic parameters read $\beta \equiv 6/g^2$ (g being the bare gauge coupling) and the hopping parameter K. In the following, instead of K, we use the 'quark–mass' $ma \equiv (1/K - 1/K_c)/2$ where a is the lattice spacing and K_c is the critical hopping parameter at which the pion becomes massless.

Hadron masses are obtained by the correlation function of composite operators for large time t. For example, the nucleon mass is obtained from

$$\langle N(t)\overline{N}(0)\rangle \to \text{const.} \times e^{-m_N t}\,, \tag{6}$$

with $N(t)$ being the spatially integrated interpolating operator for the nucleon $N(t) = \int d^3x(qC^{-1}\gamma_5 q)q$. On the other hand, the matrix element of the local operator is obtained as

$$R(t) \equiv \frac{\langle N(t)\sum_{t',x} O(t',x)\overline{N}(0)\rangle}{\langle N(t)\overline{N}(0)\rangle} \to \text{const.} + \langle N \mid O \mid N\rangle\ t\,. \tag{7}$$

Namely, the linear slope of $R(t \to \text{large})$ gives matrix elements defined on the lattice.

(3b) Some Remarks

In our actual simulation, the following points have been considered [3]. (i) To get a large overlap of $N(0)$ with the real nucleon, we use a wall source at initial time slice $t = 0$. To do this, we made Coulomb gauge fixing only at $t = 0$. (ii) To avoid a mirror source at the final time slice $t = t_f$, we set the Dirichlet boundary condition at t_f. (iii) To compare the matrix element on the lattice with that in the $\overline{MS}$ scheme, we multiplied the data obtained by a renormalization factor $Z(\mu a)$. For the tensor charge, Z calculated for $\mu = 1/a$ using tadpole improved perturbation theory [6] reads

$$Z = \left(1 - \frac{3K}{4K_c}\right)[1 - 0{\cdot}44\ \alpha_s(1/a)]\,. \tag{8}$$

4. Results

We have done simulations in three cases: $12^3 \times 20$ ($\beta = 5\cdot 7$), $16^3 \times 20$ ($\beta = 5\cdot 7$) and $16^3 \times 20$ ($\beta = 6\cdot 0$). In this paper we show the results of the second simulation. We have used a Fujitsui VPP500 computer and analysed 550 gauge configurations. Each configuration is taken after every 1000 sweeps. Three different values of the quark masses $K = 0\cdot 160, 0\cdot 164$ and $0\cdot 1665$ are adopted to extract physical quantities in the chiral limit. The statistical errors are estimated by the jackknife procedure.

Hadron masses are extracted by the χ^2 fitting of the data in the interval $5 \le t \le 10$. By using the physical hadron masses $m_{\pi,\rho,K} = 135, 770$ and 498 MeV, one obtains $a^{-1} = 1\cdot 42$ GeV ($a = 0\cdot 14$ fm), $m = 4\cdot 8$ MeV and $m_s = 125$ MeV. This also predicts the nucleon mass as $m_N = 1\cdot 13$ GeV. The physical volume of the lattice is $V = (2\cdot 24)^3 \times 2\cdot 8\ \mathrm{fm}^4$.

A χ^2 fitting in the interval $5 \le t \le 10$ is also applied for the connected and disconnected part of the matrix elements. Shown in Fig. 2 are the data for the correlation function $R(t)$ for $K = 0\cdot 164$. For connected u, d contributions ($\delta u_{con.}, \delta d_{con.}$), one can see a clear non-vanishing linear slope in $5 \le t \le 10$, while the disconnected $u - d$ contribution ($\delta u_{dis.}, \delta d_{dis.}$) has a very small slope, if at all.

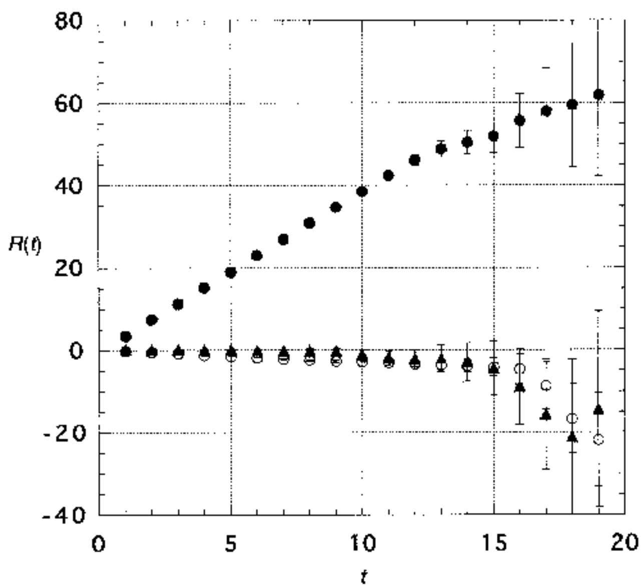

Fig. 2. $R(t)$ as a function of t for a medium–heavy quark mass $K = 0\cdot 164$. The black (white) circles denote the connected amplitude $\delta u_{con.}(\delta d_{con.})$, while the black triangles denote the disconnected amplitude $\delta u_{dis.} = \delta d_{dis.}$.

Table 1 shows the results of the data extrapolated down to the chiral limit. The results compared with that for Δq in [3] with the same lattice size and β.

Table 1. Comparison of the tensor and axial charges measured on the lattice

$\delta u \equiv \delta u_{con.} + \delta u_{dis.}$, $\delta d \equiv \delta d_{con.} + \delta d_{dis.}$, $\delta s \equiv \delta s_{dis.}$, and the same definitions also hold for Δq

Tensor charge δq (this work) $16^3 \times 20$, $\beta = 5{\cdot}7$ 550 gauge configurations ($\mu^2 = 3$ GeV^2)			Axial charge Δq (ref. [3]) $16^3 \times 20$, $\beta = 5{\cdot}7$ 260 gauge configurations ($\mu^2 = 3$ GeV^2)		
$\delta u_{con.}$	=	$+0{\cdot}89$ (2)	$\Delta u_{con.}$	=	$+0{\cdot}76$ (4)
$\delta u_{dis.}$	=	$-0{\cdot}05$ (6)	$\Delta u_{dis.}$	=	$-0{\cdot}12$ (4)
δu	=	$+0{\cdot}84$ (6)	Δu	=	$+0{\cdot}64$ (5)
$\delta d_{con.}$	=	$-0{\cdot}18$ (1)	$\Delta d_{con.}$	=	$-0{\cdot}23$ (2)
$\delta d_{dis.}$	=	$-0{\cdot}05$ (6)	$\Delta d_{dis.}$	=	$-0{\cdot}12$ (4)
δd	=	$-0{\cdot}23$ (6)	Δd	=	$-0{\cdot}35$ (5)
δs	=	$-0{\cdot}05$ (10)	Δs	=	$-0{\cdot}11$ (3)
$\delta\Sigma$	=	$+0{\cdot}56$ (13)	$\Delta\Sigma$	=	$+0{\cdot}18$ (10)

5. Summary and Discussion

In our simulation, we have found the following.

(1) For the connected part of the tensor/axial charge, the following inequalities hold:

$$|\,\delta u\,| \;>|\,\Delta u\,|\,, \qquad |\,\delta d\,| \;<|\,\Delta d\,|\;. \tag{9}$$

This is different from the prediction of relativistic quark models which have the universal inequality eq. (5).

(2) The disconnected part still has large statistical error and one cannot make definite conclusions from our simulation. Nevertheless, there is an indication that (i) the disconnected part is flavour independent, i.e. $\delta u_{dis.} \sim \delta d_{dis.} \sim \delta s_{dis.}$, and (ii) they are small but slightly negative.

(3) Flavour-singlet tensor charge $\delta\Sigma = \delta u + \delta d + \delta s$ is not suppressed as opposed to $\Delta\Sigma$:

$$\delta\Sigma(3\mathrm{GeV}^2) = 0{\cdot}56\ (13) \quad \leftrightarrow \quad \Delta\Sigma(3\mathrm{GeV}^2) = 0{\cdot}18(10)\,, \tag{10}$$

which implies that there is no 'transversity crisis' for the tensor charge.

Now, what we need to understand is the origin of the smallness of the disconnected part of the tensor charge. Since the operator $\bar{q}\sigma_{\mu\nu}\gamma_5 q$ ($\bar{q}\gamma_\mu\gamma_5 q$) is a charge conjugation odd (even) operator, δq (Δq) can be interpreted as q-spin $-$ $\bar{q}$-spin (q-spin + $\bar{q}$-spin). This indicates that there is a large cancellation of the quark-spin content and the anti-quark-spin content of the nucleon. Also, the disconnected tensor charge is zero in any order of perturbation theory in massless QCD, which might have some relation to its smallness in the non-perturbative regime. To clarify the above issue, we are currently collecting more data on $\delta q_{dis.}$ and $\Delta q_{dis.}$ simultaneously with a $16^3 \times 20$ lattice ($\beta = 5{\cdot}7$).

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