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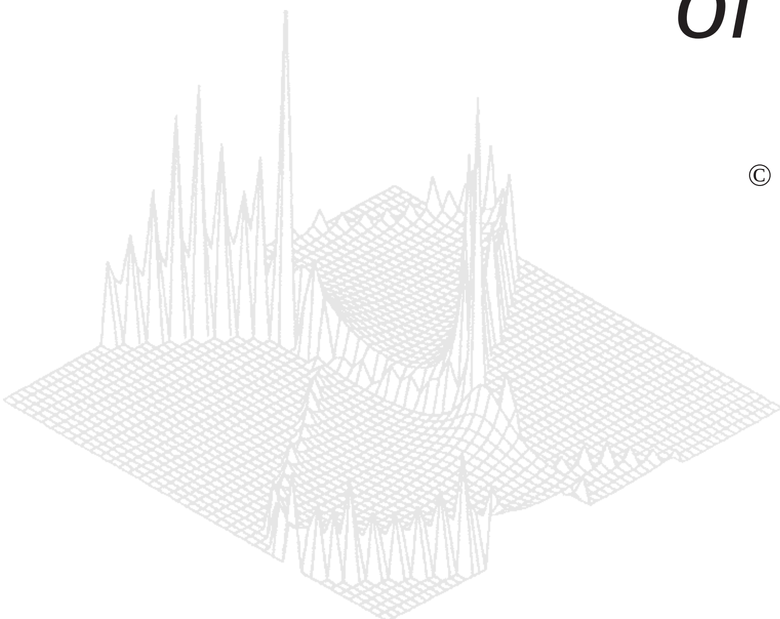

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Spectroscopy of Heavy Mesons Expanded in $1/m_Q$*

Takayuki Matsuki[A] *and Toshiyuki Morii*[B]

[A] Tokyo Kasei University, 1-18-1 Kaga,
Itabashi, Tokyo 173, Japan.
email: matsuki@tokyo-kasei.ac.jp or matsuki@ins.u-tokyo.ac.jp

[B] Faculty of Human Development, Kobe University,
3-11 Tsurukabuto, Nada, Kobe 657, Japan.
email: morii@kobe-u.ac.jp

Abstract

Operating just once with the naive Foldy–Wouthuysen–Tani transformation on the Schrödinger equation for $Q\bar{q}$ bound states, described by the Hamiltonian which includes Coulomb-like as well as confining scalar potentials, we have calculated the heavy meson spectrum of D, D^*, B and B^*. Based on a formulation recently proposed, their masses and wave functions are expanded in $1/m_Q$, with a heavy quark mass m_Q, up to the second order. The lowest order equation is examined carefully to obtain a complete set of eigenfunctions for the Schrödinger equation.

1. Introduction

Hadrons are composed of quarks and anti-quarks and are considered to be governed by quantum chromodynamics, at least in principle. Since QCD describes a strong coupling interaction, the perturbative calculation of physical properties of hadrons is not so reliable, other than the deep inelastic region due to asymptotic freedom, and hence other methods like lattice gauge theory have been developed to take into account nonperturbative effects. However, the situation changed dramatically when it was discovered that heavy mesons, composed of heavy quark Q and light anti-quark $\bar{q}$, can be systematically expanded in $1/m_Q$ with a heavy quark mass, m_Q.

This theory, known as heavy quark effective theory (HQET) [1], has been applied to many aspects of high energy physics and many kinds of physical quantities of QCD which can be perturbatively calculated in $1/m_Q$. Especially those regarding B meson physics, e.g. the lowest order form factor (which is now called the Isgur–Wise function) of semileptonic weak decay processes $B \to D\ell\nu$ and the Kobayashi–Maskawa matrix element V_{cb}, have been calculated by many people [3]. However, applications of HQET to higher order perturbative calculations are very restricted so that only forms of higher order operators are

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obtained, whose coefficients should be obtained so that results can be fitted with experimental data [2]. This is because most of the calculations based on HQET do not introduce heavy meson wave functions and hence there is no way to determine those coefficients within the model.

In former paper [4], using the Foldy–Wouthuysen–Tani transformation [5] we have developed a formulation where the Schrödinger equation for a $Q\bar{q}$ bound state can be expanded in terms of $1/m_Q$, that is, the resulting eigenvalues as well as wave functions are obtained order by order in $1/m_Q$. In this paper, as one of the applications of our formulation, we will calculate the heavy meson spectrum of D, D^*, B and B^*. In order to do so, we would like to start by introducing phenomenological dynamics, i.e. we assume Coulomb-like vector and confining scalar potentials to $Q\bar{q}$ bound states (heavy mesons), expand the Hamiltonian in $1/m_Q$ and then perturbatively solve the Schrödinger equation in $1/m_Q$. The angular part of the lowest order wave function is solved exactly. After extracting the asymptotic forms of the lowest order wave function at both $r \to 0$ and $r \to \infty$ and adopting the variational method, we numerically obtain the radial part of the trial wave function which is expanded in polynomials of the radial variable r. Then fitting the smallest eigenvalues of a Hamiltonian with masses of the D and D^* mesons, a strong coupling α_s and other parameters included in the scalar and vector potentials are determined uniquely. Using parameters obtained this way, other mass levels are calculated and fitted with experimental data for D/B mesons at the second order of perturbation. Meson wave functions obtained thereby and expanded in $1/m_Q$ may be used to calculate ordinary form factors as well as Isgur–Wise functions for semileptonic weak decay processes.

All the above calculations are calculated up to $1/m_Q^2$ and analysed order by order in $1/m_Q$ to determine parameters, as well as to compare with results of the HQET. The final goal of this paper is to obtain higher orders of Isgur–Wise functions, decay constants of heavy mesons, and the Kobayashi–Maskawa matrix element V_{cb}, by using wave functions of heavy mesons obtained after calculating the heavy meson spectrum.

Below we first give a formulation of this study and next give a qualitative discussion of the results obtained.

2. The Hamiltonian

The Hamiltonian density for our problem is given by

$$\mathcal{H}_0 = \int dx^3 \left[q^{\dagger\, c}(x) \left(\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q\right) q^c(x) + Q^\dagger(x)(\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) Q(x) \right], \quad (1)$$

$$\mathcal{H}_{\rm int} = \int\int dx^3 dx'^3\, \overline{q^c}(x)\, O_i\, q^c(x)\, V_i(x-x')\, \overline{Q}(x')\, O_i\, Q(x')\,, \quad (2)$$

where we consider only a scalar confining potential, $O_s = 1, V_s = S(r)$, and a vector potential, $O_v = \gamma_\mu, V_v = V(r)$, with a relative radial variable r, which we think the best choice to phenomenologically describe the meson mass levels. The state of $Q\bar{q}$ is defined by

$$|\psi\rangle = \int d^3x \int d^3y\ \psi_{\alpha\beta}(x-y)\ q^{c}_{\alpha}{}^{\dagger}(x)\ Q^\dagger_\beta(y)|0\rangle\,, \quad (3)$$

where $q^c(x)$ is the charge conjugate field of a light quark q, and the conjugate state of $Q\bar{q}$ is defined by $\langle\psi| = |\psi\rangle^\dagger$ with $\langle 0| \equiv |0\rangle^\dagger$. From these definitions, we obtain the Schrödinger equation

$$H\,\psi = (m_Q + \tilde{E})\,\psi\,, \tag{4}$$

where the bound state mass E is split into two parts, m_Q and $\tilde{E}$ ($E = m_Q + \tilde{E}$), so that it expresses the fact that the heavy quark mass is dominant in the bound state $Q\bar{q}$, and ψ is nothing but the wave function which appears in the rhs of Eq. (3).

Operating with the FWT transformation and a charge conjugation operator only on a heavy quark sector in this equation at the centre of mass system of a bound state, one can modify the Schrödinger equation to be

$$(H_{FWT} - m_Q) \otimes \psi_{FWT} = \tilde{E}\psi_{FWT}\,, \tag{5}$$

where the notation $\otimes$ is introduced to denote that gamma matrices of a light anti-quark are multiplied from the left, while those of a heavy quark from the right, and

$$H_{FWT} = U_c U_{FWT}({p'}_Q)\ H\ U_{FWT}^{-1}(p_Q) U_c^{-1}, \quad \psi_{FWT} = U_c U_{FWT}(p_Q)\psi\,, \tag{6}$$

$$U_{FWT}(p) = \exp(W(p)\vec{\gamma}_Q \cdot \hat{\vec{p}}) = \cos\ W + \vec{\gamma}_Q \cdot \hat{\vec{p}}\ \sin\ W\,, \tag{7}$$

$$\hat{\vec{p}} = \frac{\vec{p}}{p}, \quad \tan W(p) = \frac{p}{m+E}\,, \tag{8}$$

$$U_c = \gamma_Q^0 \gamma_Q^2\,. \tag{9}$$

As described first in this section, interaction terms are given by a confining scalar potential and a Coulomb vector potential with transverse interaction [6, 7] and a total Hamiltonian is given by

$$\begin{aligned} H = (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_q \beta_Q\, S \\ + \{1 - \tfrac{1}{2}[\,\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})\,]\}V\,, \end{aligned} \tag{10}$$

the where scalar and vector potentials are given by

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\tfrac{4}{3}\frac{\alpha_s}{r} \quad \text{and} \quad \vec{n} = \frac{\vec{r}}{r}\,. \tag{11}$$

The transformed Hamiltonian is expanded in $1/m_Q$ and is given by

$$H_{FWT} - m_Q = H_{-1} + H_0 + H_1 + H_2 + \cdots\,, \tag{12}$$

where

$$H_{-1} = -(1+\beta_Q)m_Q\,, \tag{13}$$

$$H_0 = \vec{\alpha}_q\cdot\vec{p} + \beta_q m_q - \beta_q\beta_Q\,S + \{1+\tfrac{1}{2}[\,\vec{\alpha}_q\cdot\vec{\alpha}_Q + (\vec{\alpha}_q\cdot\vec{n})(\vec{\alpha}_Q\cdot\vec{n})\,]\}V, \tag{14}$$

$$\begin{aligned} H_1 = &-\frac{1}{2\,m_Q}\beta_Q\,\vec{p}^{\,2} + \frac{1}{m_Q}\vec{\alpha}_Q\cdot(\vec{p}+\tfrac{1}{2}\vec{q})\,S + \frac{1}{2m_Q}\vec{\gamma}_Q\cdot\vec{q}\,V \\ &-\frac{1}{2m_Q}[\vec{\alpha}_q + (\vec{\alpha}_q\cdot\vec{n})\vec{n}\,]\cdot[\beta_Q(\vec{p}+\tfrac{1}{2}\vec{q}) + i\,\vec{q}\times\beta_Q\,\vec{\Sigma}_Q]V\,, \end{aligned} \tag{15}$$

$$\begin{aligned} H_2 = &\frac{1}{2m_Q^2}\,\beta_q\beta_Q\,(\vec{p}+\tfrac{1}{2}\vec{q})^2 S - \frac{i}{4m_Q^2}\vec{q}\times\vec{p}\cdot\beta_q\beta_Q\,\vec{\Sigma}_Q\,S \\ &-\frac{1}{8m_Q^2}\vec{q}^{\,2}V - \frac{i}{4m_Q^2}\vec{q}\times\vec{p}\cdot\vec{\Sigma}_Q V \\ &-\frac{1}{8m_Q^2}[\vec{\alpha}_q + (\vec{\alpha}_q\cdot\vec{n})\vec{n}\,]\cdot[(\vec{p}+\vec{q})\,(\vec{\alpha}_Q\cdot\vec{p}) + \vec{p}(\vec{\alpha}_Q\cdot(\vec{p}+\vec{q})\,) \\ &+ i\,\vec{q}\times\vec{p}\,\gamma_Q^5]\,V, \end{aligned} \tag{16}$$

$$\vdots$$

Here H_i stands for the ith order expanded Hamiltonian terms and since a bound state is at rest,

$$\vec{p} = \vec{p}_q = -\vec{p}_Q, \quad \vec{p}\,' = \vec{p}_q\,' = -\vec{p}_Q\,', \quad \vec{q} = \vec{p}\,' - \vec{p}, \tag{17}$$

are defined, where primed quantities are final momenta. Details of the derivation of equations in this section are given in the paper [8].

3. Perturbation

Using the Hamiltonian obtained in the last section, we give in this section the Schrödinger equation order by order in $1/m_Q$. Details of the derivation in this section will be given in a future paper [8]. First we introduce projection operators:

$$\Lambda_\pm = \frac{1\pm\beta_Q}{2}\,, \tag{18}$$

which correspond to positive- and negative-energy projection operators for a heavy quark sector at the rest frame of a bound state. These are given by $(1\pm\not{v})/2$ in the moving frame of a bound state with v^μ the four-velocity of a bound state. Then we expand the mass and wave function of a bound state in $1/m_Q$ as

$$\tilde{E} = E_0^\ell + E_1^\ell + E_2^\ell + \cdots\,, \tag{19}$$

$$\psi_{FWT} = \psi_0^\ell + \psi_1^\ell + \psi_2^\ell + \cdots\,, \tag{20}$$

where ℓ stands for a set of quantum numbers that distinguish independent eigenfunctions of the lowest order Schrödinger equation, and a subscript i of E_i^ℓ and ψ_i^ℓ for the order of $1/m_Q$.

(3a) Leading Order

The leading order Schrödinger equation in $1/m_Q$ gives

$$\psi_0^\ell = \Lambda_- \otimes \psi_0^\ell, \tag{21}$$

whose explicit form is given by

$$\psi_0^\ell = \Psi_\ell^+ = (0 \quad \Psi_{j\,m}^k(\vec{r}))\,, \tag{22}$$

with

$$\Psi_{j\,m}^k(\vec{r}) = \frac{1}{r}\begin{pmatrix} u_k(r) \\ -i\,v_k(r)(\vec{\sigma}\cdot\vec{n}) \end{pmatrix} y_{j\,m}^k(\Omega)\,, \tag{23}$$

where j is the total angular momentum of a meson, m is its z component, k is a quantum number which takes only values $k = \pm j$, $\pm(j+1)$ and $\neq 0$, and $u_k(r)$ and $v_k(r)$ are polynomials of a radial variable r. $y_{j\,m}^k(\Omega)$ are functions of angles and spinors of a total angular momentum, $\vec{j} = \vec{l} + \vec{s}_q + \vec{s}_Q$. The operator for the quantum number k is given by $-\beta_q(\vec{\Sigma}_q\cdot\vec{\ell}+1)$ when it operates on $(0\ \Psi_{j\,m}^k(\vec{r}))$.

Note that since charge conjugation is operated on the heavy quark sector the Λ_- projection operator appears in Eq. (21), i.e. positive components of Q correpond to negative components of $U_c\,Q$.

(3b) Zeroth Order

The zeroth order equations are given by

$$[\,\vec{\alpha}_q\cdot\vec{p} + \beta_q(m_q+S) + V\,] \otimes \psi_0^\ell = E_0^\ell\,\psi_0^\ell\,, \tag{24}$$

$$-2m_Q\Lambda_+ \otimes \psi_1^\ell + \tfrac{1}{2}\Lambda_-[\,\vec{\alpha}_q\cdot\vec{\alpha}_Q + (\vec{\alpha}_q\cdot\vec{n})(\vec{\alpha}_Q\cdot\vec{n})\,]V \otimes \psi_0^\ell = 0\,. \tag{25}$$

Eq. (24) gives the lowest non-trivial Schrödinger equation with a solution given by Eq. (22) and $\vec{n}$ is defined in Eq. (11). The Λ_+ components of wave functions can be expanded in terms of the eigenfunctions,

$$\Psi_\ell^- = (\Psi_{j\,m}^k(\vec{r})\ \ 0)\,. \tag{26}$$

Expanding $\Lambda_+ \otimes \psi_1^\ell$ in terms of this set of eigenfunctions, one can obtain the solution as

$$\Lambda_+ \otimes \psi_1^\ell = \sum_{\ell'} c_{1-}^{\ell\,\ell'}\Psi_{\ell'}^-\,, \tag{27}$$

with the coefficients

$$c_{1-}^{\ell\,\ell'} = \frac{1}{4m_Q}\langle\Psi_{\ell'}^{-}|[\,\vec{\alpha}_q\cdot\vec{\alpha}_Q + (\vec{\alpha}_q\cdot\vec{n})(\vec{\alpha}_Q\cdot\vec{n})\,]V|\Psi_\ell^{+}\rangle\,. \tag{28}$$

Here the inner product is defined to be

$$\langle\Psi_{\ell'}^{-}|O|\Psi_{\ell'}^{+}\rangle = \int d^3r\,\mathrm{tr}(\Psi_\ell^{-\,\dagger}(O\otimes\Psi_\ell^{+}))\,, \tag{29}$$

and the zeroth order wave functions are normalized to be 1,

$$\langle\Psi_\ell^{\alpha}|\Psi_{\ell'}^{\beta}\rangle = \delta_{\ell\,\ell'}\delta^{\alpha\,\beta} \quad \text{for} \quad \alpha,\,\beta = + \text{ or } - \,. \tag{30}$$

(3c) First Order

The first order equation is given by

$$-2m_Q\Lambda_+\otimes\psi_2^\ell + H_0\otimes\psi_1^\ell + H_1\otimes\psi_0^\ell = E_0^\ell\psi_1^\ell + E_1^\ell\psi_0^\ell\,. \tag{31}$$

Multiplying projection operators $\Lambda_\pm$ from the right in the above equation, and expanding ψ_1^k in terms of Ψ_k^+ and Ψ_k^- as

$$\psi_1^\ell = \sum_\ell (c_{1+}^{\ell\,\ell'}\Psi_{\ell'}^{+} + c_{1-}^{\ell\,\ell'}\Psi_{\ell'}^{-})\,, \tag{32}$$

one obtains

$$E_1^\ell = \sum_{\ell'} c_{1-}^{\ell\,\ell'}\langle\Psi_\ell^{+}|\Lambda_+H_0\Lambda_-|\Psi_{\ell'}^{-}\rangle + \langle\Psi_\ell^{+}|\Lambda_-H_1\Lambda_-|\Psi_\ell^{+}\rangle\,, \tag{33}$$

which gives the first order perturbation correction to the mass when one calculates matrix elements of the rhs among eigenfunctions, and

$$c_{1+}^{\ell\,k} = \frac{1}{E_0^\ell - E_0^k}[\sum_{\ell'} c_{1-}^{\ell\,\ell}\langle\Psi_k^{+}|\Lambda_+H_0\Lambda_-|\Psi_{\ell'}^{-}\rangle + \langle\Psi_k^{+}|\Lambda_-H_1\Lambda_-|\Psi_\ell^{+}\rangle], \quad \text{for } k\neq\ell \tag{34}$$

$$c_{1+}^{k\,k} = 0\,. \tag{35}$$

This completes the solution for ψ_1^ℓ since $\Lambda_-\psi_1^\ell$, or $c_{1-}^{\ell\,\ell'}$, was obtained in the previous subsection. Here we have used the normalization for the total wave function ψ^ℓ as

$$\langle\psi^\ell|\psi^{\ell'}\rangle = \delta_{\ell\,\ell'}. \tag{36}$$

This definition is allowed because here we are not calculating the absolute value of the form factors. The appropriate normalization will be determined in future papers in which we will give several kinds of form factors. This way of solving Eq. (31) is unique and we use this method below to solve similar equations appearing Section $3d$ as well. One can obtain $\Lambda_+\otimes\psi_2^\ell$ as

$$\Lambda_+ \otimes \psi_2^\ell = \sum_{\ell'} c_{2-}^{\ell\,\ell'} \Psi_{\ell'}^- \,, \tag{37}$$

with the coefficients

$$c_{2-}^{\ell\,\ell'} = \frac{1}{2m_Q} \langle \Psi_{\ell'}^- | ((H_0 - E_0^\ell)\Lambda_+ \otimes \psi_1^\ell + H_1 \Lambda_+ \otimes \psi_0^\ell) \rangle \,. \tag{38}$$

(3d) Second Order

The second order equation is given by

$$-2m_Q \Lambda_+ \otimes \psi_3^\ell + H_0 \otimes \psi_2^\ell + H_1 \otimes \psi_1^\ell + H_2 \otimes \psi_0^\ell = E_0^\ell \psi_2^\ell + E_1^\ell \psi_1^\ell + E_2^\ell \psi_0^\ell \,. \tag{39}$$

As in the above case (first order), we obtain

$$\begin{aligned} E_2^\ell = & \sum_{\ell'} c_{2-}^{\ell\,\ell'} \langle \Psi_\ell^+ | \Lambda_+ H_0 \Lambda_- | \Psi_{\ell'}^- \rangle + \langle \Psi_\ell^+ | H_1 \Lambda_- \otimes \psi_1^\ell \rangle \\ & + \langle \Psi_\ell^+ | \Lambda_- H_2 \Lambda_- | \Psi_\ell^+ \rangle \,, \end{aligned} \tag{40}$$

which gives the second order perturbation corrections to the mass and

$$\begin{aligned} c_{2+}^{\ell\,k} = & \frac{1}{E_0^k - E_0^\ell} [\sum_{\ell'} c_{2-}^{\ell\,\ell'} \langle \Psi_k^+ | \Lambda_+ H_0 \Lambda_- | \Psi_{\ell'}^- \rangle + \langle \Psi_k^+ | \Lambda_+ H_1 \otimes \psi_1^\ell \rangle \\ & + \langle \Psi_k^+ | \Lambda_- H_2 \Lambda_- | \Psi_\ell^+ \rangle - E_1^\ell \, c_{1+}^{\ell\,k}], \qquad \text{for } k \neq \ell \end{aligned} \tag{41}$$

$$c_{2+}^{k\,k} = -\tfrac{1}{2} \sum_\ell (|c_{1+}^{k\,\ell}|^2 + |c_{1-}^{k\,\ell}|^2) \,. \tag{42}$$

This completes the solution for ψ_2^k since $\Lambda_- \psi_2^k$, or $c_{2-}^{\ell\,\ell'}$, is obtained in the previous subsection.

Although we do not need to, one can obtain $\Lambda_+ \otimes \psi_3^\ell$ as

$$\Lambda_+ \otimes \psi_3^\ell = \sum_{\ell'} c_{3-}^{\ell\,\ell'} \Psi_{\ell'}^- \,, \tag{43}$$

with the coefficients

$$c_{3-}^{\ell\,\ell'} = \frac{1}{2m_Q} \langle \Psi_{\ell'}^- | ((H_0 - E_0^\ell)\Lambda_+ \otimes \psi_2^\ell + (H_1 - E_1^\ell)\Lambda_+ \otimes \psi_1^\ell + H_2 \Lambda_+ \psi_0^\ell) \rangle \,. \tag{44}$$

4. Numerical Analysis

In this section we give a numerical analysis of the calculations obtained by applying our formulation, i.e. perturbatively expanding the Hamiltonian given by Eq. (4) in $1/m_Q$ and computing all the matrix elements among eigenfunctions, $\Psi_\ell^\pm$. In order to solve Eq. (24), we have to numerically solve an explicit form of the wave function, $\Psi_\ell^+ = (0 \;\; \Psi_{j\,m}^k)$, as

$$\Psi^k_{j\,m}(\vec{r}) = \frac{1}{r}\begin{pmatrix} u_k(r) \\ -i\,v_k(r)(\vec{\sigma}\cdot\vec{n})\;y^k_{j\,m}(\Omega)\end{pmatrix},$$

some properties of which are described in the paper [8]. As described in that paper, the Schrödinger equation is reduced to

$$\begin{pmatrix} m_q + S + V & -\partial_r + k/r \\ \partial_r + k/r & -m_q - S + V \end{pmatrix}\begin{pmatrix} u_k(r) \\ v_k(r)\end{pmatrix} = E^k_0\begin{pmatrix} u_k(r) \\ v_k(r)\end{pmatrix},$$

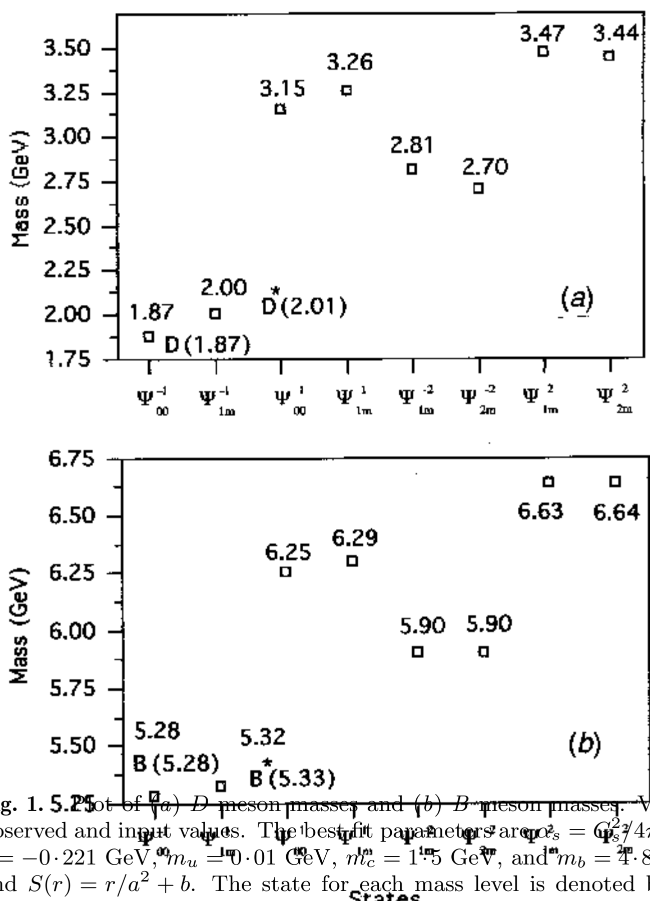

Fig. 1. Plot of (*a*) D meson masses and (*b*) B meson masses. Values in parentheses are the observed and input values. The best fit parameters are $\alpha_s = G_s^2/4\pi = 0\cdot 321$, $a = 1\cdot 44\ \mathrm{GeV}^{-1}$, $b = -0\cdot 221$ GeV, $m_u = 0\cdot 01$ GeV, $m_c = 1\cdot 5$ GeV, and $m_b = 4\cdot 84$ GeV with $V(r) = -4\alpha_s/3$ and $S(r) = r/a^2 + b$. The state for each mass level is denoted by the corresponding lowest eigenstate without any correction. In (*b*) the value of the B meson mass is the input.

which is solved numerically by taking into account the asymptotic behaviours at both $r \to 0$ and $r \to \infty$ and their forms are given by

$$u_k(r),\ v_k(r) \sim w(r)\, r^\gamma\, \exp\left[-(m_q+b)\, r - \tfrac{1}{2}\left(\frac{r}{a}\right)^2\right], \tag{45}$$

where

$$\gamma = \sqrt{k^2 - \left(\frac{4\alpha_s}{3}\right)^2} \tag{46}$$

and $w(r)$ is a finite series of a polynomial r. In the case of a hydrogen atom, for instance, only the potential V survives and a radial function, $w(r)$ becomes a hypergeometric function and its finite series of a polynomial gives discrete energy levels. In our case, since the potential includes a scalar term we cannot analytically solve the above reduced Schrödinger equation (5). If we are forced to make the functions $u_k(r)$ and $v_k(r)$ finite series and relate the coefficients of those functions via recursive equations, it leads to inconsistency among the coefficients of each term, r^i, of a polynomial. The results of our numerical calculations using Eqs (4), (19), (33) and (40) are depicted in Fig. 1.

One can easily see degeneracy among the lowest lying pseudoscalar and vector states in our formulation as follows. Define

$$|P\rangle = U_c^{-1}(0\ \ \Psi_{0\,0}^{-1}), \quad |V,\ \lambda\rangle = U_c^{-1}(0\ \ \Psi_{1\,\lambda}^{-1})\,,$$

where $\Psi_{j\,m}^{k}$ is an eigenfunction obtained in the last chapter and quantum number k can take only $\pm j$, or $\pm(j+1)$. Assigning those states to D mesons, one can have

$$|P\rangle = |D^\pm\rangle \ \text{ or } \ |D^0\rangle, \quad |V,\ \lambda\rangle = |D^*\rangle\,.$$

Since these states have the same quantum number $k = -1$, these have the same masses as well as the same wave functions up to the zeroth order calculation in $1/m_Q$, which is nothing but the spin-flavour symmetry.

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