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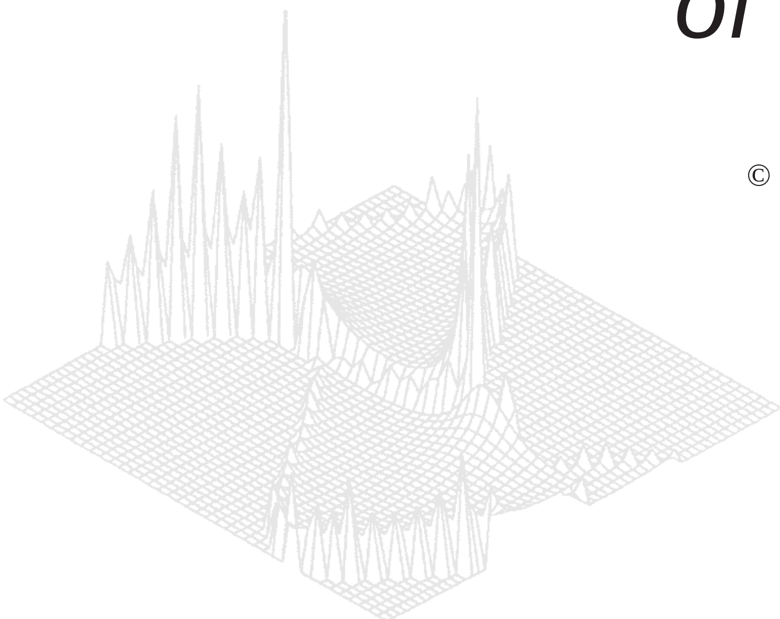

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Hadron-Nucleon Scattering Lengths from QCD Sum Rules*

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Abstract

Hadron–nucleon scattering lengths are studied by the QCD sum rule. First we explain our motivation and present the formulation for calculating hadron–nucleon scattering lengths by the QCD sum rule, where the relation between the hadron mass in the nuclear medium and the hadron–nucleon scattering length is also clarified. Secondly we discuss two applications, the pion–nucleon scattering lengths and the nucleon–nucleon scattering lengths. In the case of the pion-nucleon scattering length we show that the results of the QCD sum rule are consistent with the low-energy theorem. In the case of the nucleon–nucleon scattering lengths we show that the results of the QCD sum rule are in qualitative agreement with experiment.

1. Introduction

Let us start with reviewing the QCD sum rule , which was invented by Shifman, Vainshtein and Zakharov in 1979 in order to describe resonance physics taking into account non-perturbative effects [1]. In this formalism the basic object is the correlation function

$$\Pi^H(q) = -i \int d^4x e^{iqx} \langle T(\eta_H(x)\eta_H^\dagger(0))\rangle \,, \tag{1}$$

where η_H is the interpolating field for the hadron H, i.e. a quark–gluon composite operator which creates H. The correlation function satisfies the following dispersion relation, or equivalently the Lehmann representation,

$$\Pi^H(\omega, \mathbf{q}) = \int_{-\infty}^{\infty} \frac{\rho^H(\omega', \mathbf{q})}{\omega - \omega'} d\omega' \,, \tag{2}$$

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where ρ^H is the spectral function defined in terms of the correlation function as

$$\rho^H(\omega, \mathbf{q}) = \frac{i}{2\pi}\left\{\Pi^H(\omega + i\eta, \mathbf{q}) - \Pi^H(\omega - i\eta, \mathbf{q})\right\} = -\frac{1}{\pi}\mathrm{Im}\Pi^H(\omega + i\eta, \mathbf{q})\,. \quad (3)$$

In the physical region, $q^2 \sim M_H^2$, Π^H is non-perturbative and difficult to calculate. However, in the deep-Euclid region, $q^2 \to -\infty$, Π^H is perturbative and can be approximated by the operator product expansion (OPE),

$$-i\int d^4x e^{iqx} T(\eta_H(x)\eta_H^\dagger(0)) = C_I(q)I + C_{\bar{q}q}(q)\bar{q}q + C_{G^2}(q)G_{\mu\nu}G^{\mu\nu} + \cdots\,. \quad (4)$$

Therefore, by letting $q^2 \to -\infty$ in eq. (3), evaluating the l.h.s. by the OPE and expressing the r.h.s. in terms of physical quantities of hadrons such as the mass and the decay constant, we obtain relations between observables of hadrons and the quark–gluon condensates as

$$M_H \propto C_{\bar{q}q}\langle\bar{q}q\rangle_0 + C_{G^2}\langle G^2\rangle_0 \cdots\,. \quad (5)$$

These relations are the QCD sum rules.

Recently it was pointed out that the application of the QCD sum rule can be extended to the study of properties of hadrons at finite temperature [3, 4] and/or density [5–7]. Since the OPE is an operator equality, the OPE at finite temperature and/or density is the same as the OPE at zero temperature and density. The difference lies in the condensates of the operators. Thus, we obtain relations between observables of hadrons and the quark–gluon condensates at finite temperature and/or density as

$$(M_H)_{\rho,T} \propto C_{\bar{q}q}\langle\bar{q}q\rangle_{\rho,T} + C_{G^2}\langle G^2\rangle_{\rho,T} \cdots\,, \quad (6)$$

where $\langle\mathcal{O}\rangle_{\rho,T}$ is the condensate of the operator $\mathcal{O}$ at finite density and/or temperature.* Eq. (6) shows that the change of observables of hadrons at finite temperature and/or density can be understood by the change of the condensates there. Therefore, the question is how to understand the change of the condensates at finite temperature and/or density.

From now on we concentrate on the QCD sum rule at finite density. The crucial step made by Drukarev and Levin [5] was to notice that when $\langle\mathcal{O}\rangle_\rho$ is expanded in the baryon number density ρ, the $O(\rho)$ term is given by the expectation value of the operator with respect to the one-nucleon state: $\langle\mathcal{O}\rangle_\rho = \langle\mathcal{O}\rangle_0 + \langle\mathcal{O}\rangle_N\rho + o(\rho)$. This implies that when the correlation function of the hadron in nuclear matter Π_ρ^H is expanded in a similar way,

$$\Pi_\rho^H = \Pi_0^H + \Pi_N^H\,\rho + o(\rho)\,, \quad (7)$$

* Eq. (6) should include not only scalar operators but also non-scalar operators which have non-vanishing condensates at finite temperature and/or density.

then Π_N^H is the new information obtained by $\langle\mathcal{O}\rangle_N$. Since Π_N^H is the correlation function of H in the presence of the nucleon,

$$\Pi_N^H(q) = -i\int d^4x e^{iqx}\langle N|T(\eta_H(x)\eta_H^\dagger(0))|N\rangle\,, \tag{8}$$

it is natural to expect that Π_N^H is related to the hadron–nucleon scattering. This expectation can be easily confirmed. The LSZ reduction formula tells us that the correlation function has a second order pole at $q^2 = M_H^2$ with the coefficient being the hadron–nucleon T-matrix, T_{HN}:

$$\Pi_N^H(q) \sim \frac{1}{\not{q} - M_H} T_{HN} \frac{1}{\not{q} - M_H}\,, \tag{9}$$

where the hadron is assumed to be a spin $\frac{1}{2}$ particle, for definiteness. In particular for $\mathbf{q} = 0$ the coefficient becomes essentially the hadron–nucleon scattering length.

Now, what is the relation between the hadron mass at finite density and the hadron–nucleon scattering length? We expand the correlation function of the hadron at finite density in ρ around $q^2 = M_H^2$,

$$\begin{aligned}\Pi_\rho^H &= \frac{1}{\not{q} - \Sigma(\omega,\mathbf{q}) - M_H} \\ &\sim \frac{1}{\not{q} - M_H} + \frac{1}{\not{q} - M_H}\frac{\partial\Sigma(\omega,\mathbf{q})}{\partial\rho}\frac{1}{\not{q} - M_H}\rho + o(\rho) \quad (q^2 \sim M_H^2)\,,\end{aligned} \tag{10}$$

where Σ denotes the self-energy of the hadron at finite density. The $O(\rho)$ term of the hadron mass at finite density is given by

$$\delta M_H = \frac{\partial\Sigma(M_H,\mathbf{q}=0)}{\partial\rho}\rho\,. \tag{11}$$

From eqs (9), (10) and (11) we obtain the following relation between the baryon mass shift in nuclear matter and the baryon–nucleon scattering lengths:

$$\delta M_H = \begin{cases} -2\pi\dfrac{M_H + M_N}{M_H M_N}\left(\frac{3}{4}a_{HN}^{(S=1)} + \frac{1}{4}a_{HN}^{(S=0)}\right)\rho + o(\rho) \quad (H \neq N), \\ -3\pi\dfrac{1}{M_N}\left(\frac{1}{2}a_{NN}^{(S=1)} + \frac{1}{2}a_{NN}^{(S=0)}\right)\rho + o(\rho) \quad (H = N), \end{cases} \tag{12}$$

which is a famous relation in the context of the multiple scattering theory [8]. This clearly shows that the $O(\rho)$ term of the hadron mass in the nuclear medium should be identified with the hadron–nucleon scattering length.

Therefore, we start from Π_N^H and study hadron–nucleon interactions, in particular hadron–nucleon scattering lengths. In the following two sections, we explain two examples of such applications, the pion–nucleon scattering lengths [9] and the nucleon–nucleon scattering lengths [10]. In the case of the pion–nucleon scattering lengths there exists a low energy theorem that the scattering lengths are determined by the chiral symmetry. Therefore, we can check if the results of the QCD sum rule are consistent with those of the low energy theorem. On the other hand, in the case of the nucleon–nucleon scattering lengths there is no such theorem and the scattering lengths are supposed to reflect more detailed underlying dynamics. Therefore, it would be a real challenge.

In these applications we use a variation of the QCD sum rule, the Borel sum rule, which is derived as follows. Splitting the correlation function into even and odd parts as $\Pi(\omega, \mathbf{q}) = \Pi_{\rm even}(\omega^2, \mathbf{q}) + \omega \Pi_{\rm odd}(\omega^2, \mathbf{q})$, and applying the Borel transformation defined by

$$L_{\rm Borel} \equiv \lim_{\substack{n\to\infty \\ -\omega^2\to\infty \\ -\omega^2/n \to M^2_{\rm Borel}}} \frac{(\omega^2)^n}{(n-1)!} \left(-\frac{d}{d\omega^2} \right)^n , \tag{13}$$

where $M^2_{\rm Borel}$ is the square of the Borel mass, we obtain

$$L_{\rm Borel}[\Pi_{\rm even}(\omega^2, \mathbf{q})] = - \int_{-\infty}^{\infty} \rho(\omega', \mathbf{q}) \frac{\omega'}{M^2_{\rm Borel}} \exp\left(-\frac{{\omega'}^2}{M^2_{\rm Borel}} \right) d\omega',$$

$$L_{\rm Borel}[\Pi_{\rm odd}(\omega^2, \mathbf{q})] = - \int_{-\infty}^{\infty} \rho(\omega', \mathbf{q}) \frac{1}{M^2_{\rm Borel}} \exp\left(-\frac{{\omega'}^2}{M^2_{\rm Borel}} \right) d\omega' . \tag{14}$$

By evaluating the l.h.s. with the OPE and parametrizing the r.h.s. in terms of physical observables, we obtain relations between matrix elements of the quark–gluon operators with respect to the one-nucleon state and the physical quantities. These relations are our new Borel sum rules.

Before closing this section we summarize here the parameters used in the following calculations: the quark masses are $m_u = m_d = 7$ MeV, $m_s = 170$ MeV, the condensates of quark–gluon operators are [2] $\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = -(225\ \text{MeV})^3$, $\langle \bar{s}s \rangle_0 = -(217\ \text{MeV})^3$, $\langle \frac{\alpha_s}{\pi} G^2 \rangle_0 = (340\ \text{MeV})^4$, and the expectation values of quark–gluon operators with the nucleon are [5–7] $\langle u^\dagger u \rangle_p = \langle d^\dagger d \rangle_n = 2$, $\langle u^\dagger u \rangle_n = \langle d^\dagger d \rangle_p = 1$, $\langle s^\dagger s \rangle_n = \langle s^\dagger s \rangle_p = 0$, $\langle \bar{u}u \rangle_p = \langle \bar{d}d \rangle_n = 3{\cdot}46$, $\langle \bar{u}u \rangle_n = \langle \bar{d}d \rangle_p = 2{\cdot}96$, $\langle \bar{s}s \rangle_p = \langle \bar{s}s \rangle_n = 0{\cdot}77$, $i\langle \mathcal{S}[\bar{u}\gamma_\mu D_\nu u] \rangle_p = i\langle \mathcal{S}[\bar{d}\gamma_\mu D_\nu d] \rangle_n = 222$ MeV, $i\langle \mathcal{S}[\bar{d}\gamma_\mu D_\nu d] \rangle_p = i\langle \mathcal{S}[\bar{u}\gamma_\mu D_\nu u] \rangle_n = 95$ MeV, $i\langle \mathcal{S}[\bar{s}\gamma_\mu D_\nu s] \rangle_p = i\langle \mathcal{S}[\bar{s}\gamma_\mu D_\nu s] \rangle_n = 18$ MeV, $\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle_N = -738$ MeV and $\langle \frac{\alpha_s}{\pi} \mathcal{S}[G_{\mu 0} G^{\mu 0}] \rangle_N = -50$ MeV.

2. Pion–Nucleon Scattering Lengths

First we discuss the application to the pion–nucleon scattering lengths [9]. For the pion interpolating field, we take the axial-vector current

$$A_\mu(x) = \bar{q}_1(x) \gamma_\mu \gamma_5 q_2(x) , \tag{15}$$

which has a property required for the interpolating field, $\langle 0 | A_\mu(0) | \pi(k) \rangle = i\sqrt{2} f_\pi k_\mu$, with f_π being the pion (kaon) decay constant.

The OPE for the correlation function can be obtained by the standard procedure and is available up to dimension-six in ref. [9]. However, important terms can be obtained without explicitly performing the calculation by the following Ward–Takahashi identity:

$$
\begin{aligned}
-i\int d^4x e^{ikx} k^\mu k^\nu \langle T(A_\mu(x)A_\nu^\dagger(0))\rangle \\
&= k^\mu \langle \bar{q}_1\gamma_\mu q_1 - \bar{q}_2\gamma_\mu q_2\rangle - (m_1+m_2)\langle \bar{q}_1 q_1 + \bar{q}_2 q_2\rangle \\
&\quad -(m_1+m_2)^2 i\int d^4x e^{ikx}\langle T\left(\varphi(x)\varphi^\dagger(0)\right)\rangle\,, \qquad (16)
\end{aligned}
$$

where $\varphi(x) = i\bar{q}_1(x)\gamma_5 q_2(x)$. On the r.h.s. of eq. (16), the dimensions of the operators in the first, second and third terms are three, four and five or higher, respectively. (The OPE for the correlation function of φ has at least dimension-three.) Moreover, their quark mass dependence is constant, linear and quadratic, respectively. Therefore, the first term is the most important, the second is next and the third is the least important, not only in the sense of the OPE but also in the sense of the chiral symmetry breaking expansion. Thus, we first concentrate on the first two terms in eq. (16). The effect of the higher dimension operators in the last term will be discussed later.

Hereafter we take $\mathbf{k}=0$. Then only the $\mu=\nu=0$ component of $\Pi_{\mu\nu}$ becomes relevant. Therefore, we simplify our notation as follows: $\Pi(\omega)=\Pi_{00}(\omega,\mathbf{k}=0)$, $\rho(\omega)=\rho_{00}(\omega,\mathbf{k}=0)$.

In ref. [1] Shifman, Vainshtein and Zakharov showed that the form of the OPE in the vacuum requires a massless pion in the chiral limit, if the chiral symmetry is spontaneously broken, and that the leading order term in the OPE is identified with the contribution of the pion state. Let us briefly review their discussion:

$$
\begin{aligned}
\Pi_0(\omega) &= \sum_P \frac{m_P^2 f_P^2}{\omega^2 - m_P^2} \\
&= -\frac{(m_1+m_2)\langle 0|\bar{q}_1 q_1 + \bar{q}_2 q_2|0\rangle}{\omega^2} + \mathcal{O}(m_q^2) \qquad (17)
\end{aligned}
$$

holds only if there exists a pseudoscalar state satisfying the conditions $m_P^2 = O(m_q)$, $f_P = O(m_q^0)$, while all the states with a non-vanishing mass decouple in the chiral limit, $f_P = O(m_q)$ if $m_P = O(m_q^0)$. The Gell-Mann–Oakes–Renner relation [11] is just the $O(m_q)$ term in eq. (17).

Similarly we can demonstrate that the form of the OPE, in which the matrix elements are taken with respect to the one nucleon state, determines the low energy behaviour of the pion–nucleon interaction:

$$
\begin{aligned}
\Pi_N(\omega) &= \sum_n \left\{ \frac{|\langle n|A_0(0)|N\rangle|^2}{\omega-(E_n-M_N)} - \frac{|\langle n|A_0^\dagger(0)|N\rangle|^2}{\omega+(E_n-M_N)} \right\} \\
&= \frac{\langle N|q_1^\dagger q_1 - q_2^\dagger q_2|N\rangle}{\omega} - \frac{(m_1+m_2)\langle N|\bar{q}_1 q_1 + \bar{q}_2 q_2|N\rangle}{\omega^2} + O(m_q^2) \qquad (18)
\end{aligned}
$$

holds only if their exists a state satisfying the condition $\langle n|A_0^\dagger(0)|N\rangle = O(m_q^0)$ if $E_n - M_N = o(m_q^0)$, while all other states decouple in the chiral limit, $\langle n|A_0^\dagger(0)|N\rangle = O(m_q)$ if $E_n - M_N = O(m_q^0)$. The state which survives is that of the pion–nucleon at the threshold and the matrix element of the pion–nucleon intermediate state with proper normalization has the following structure:

$$\langle\pi(\mathbf{k})N(-\mathbf{k})|A_0^\dagger(0)|N(\mathbf{0})\rangle = i\sqrt{2}m_\pi f_\pi(2\pi)^3\delta^3(\mathbf{k}) - i\frac{1}{\sqrt{2}}f_\pi T_{\pi N}\delta(\omega_{\mathbf{k}} - m_\pi) + \theta(\omega_{\mathbf{k}} - m_\pi)F(\omega_{\mathbf{k}})\,, \quad (19)$$

where $\omega_{\mathbf{k}} = \sqrt{m_\pi^2 + \mathbf{k}^2} + \sqrt{M_N^2 + \mathbf{k}^2} - M_N$, $F(\omega) = O(m_q)$ and $T_{\pi N}/m_\pi = O(m_q^0)$, as can be explicitly seen later. Splitting the correlation function into even and odd parts and taking the combination, $\tilde{\Pi}_N(\omega^2) = \Pi_N(\omega^2) + (\omega^2 - m_\pi^2)d\Pi_N(\omega^2)/d\omega^2$, for both parts, we obtain

$$-\frac{2m_\pi f_\pi^2 T^{(-)}}{(\omega^2 - m_\pi^2)^2} = m_\pi^2\frac{\langle u^\dagger u - d^\dagger d\rangle_p}{\omega^4} + O(m_q^2)\,, \quad (20)$$

$$-\frac{2m_\pi^2 f_\pi^2 T^{(+)}}{(\omega^2 - m_\pi^2)^2} = -m_\pi^2\frac{(m_u + m_d)\langle\bar{u}u + \bar{d}d\rangle_p}{\omega^4} + O(m_q^2)\,, \quad (21)$$

where $T^{(\pm)} = \frac{1}{2}\left(T_{\pi^- p} \pm T_{\pi^+ p}\right) = \frac{1}{2}\left(T_{\pi^+ n} \pm T_{\pi^- n}\right)$. Therefore, we obtain the following expressions for the scattering lengths, which are related to the T-matrices as $a^{(\pm)} = -\frac{1}{4\pi}\left(1 + \frac{m_\pi}{M_N}\right)^{-1} T^{(\pm)}$,

$$a_{\pi N}^{(-)} = \frac{1}{4\pi}\left(1 + \frac{m_\pi}{M_N}\right)^{-1}\frac{m_\pi}{2f_\pi^2}\langle u^\dagger u - d^\dagger d\rangle_p + O(m_q^{\frac{3}{2}})\,, \quad (22)$$

$$a_{\pi N}^{(+)} = -\frac{1}{4\pi}\left(1 + \frac{m_\pi}{M_N}\right)^{-1}\frac{1}{f_\pi^2}(m_u + m_d)\langle\bar{u}u + \bar{d}d\rangle_p + O(m_q)\,. \quad (23)$$

On the r.h.s. of eq. (22) the term proportional to $\langle u^\dagger u - d^\dagger d\rangle_p$, which is called the Tomozawa–Weinberg term, is $O(m_q^{\frac{1}{2}})$, while the rest is $O(m_q^{\frac{3}{2}})$. Therefore, this term dominates the isospin-odd scattering length. This is nothing but the Tomozawa–Weinberg relation [12, 13]. On the other hand, on the r.h.s. of eq. (23) the term proportional to $(m_u + m_d)\langle\bar{u}u + \bar{d}d\rangle_p$, which is called the sigma term, is $O(m_q)$, but the rest is also $O(m_q)$. Therefore, in contrast to the isospin-odd scattering length the sigma term does not necessarily dominate the isospin-even scattering length. These results are consistent with the low energy theorem and can also be numerically confirmed: the Tomozawa–Weinberg term is $0{\cdot}11$ fm and the experimental isospin-odd scattering length is $0{\cdot}13$ fm, while the sigma term is $-0{\cdot}07$ fm but the isospin-even scattering length is $-0{\cdot}01$ fm. We explicitly calculated the higher order terms of the OPE up to dimension-six.

It turns out that higher order terms are small, which is consistent with the consideration based on the order of m_q.

Up to this point we have shown that we can reproduce the low energy theorem for the pion–nucleon scattering lengths by just looking at the OPE for the axial-vector current, where matrix elements are taken with respect to the one-nucleon state. However, the OPE does not explain the origin of the difference between the sigma term and the observed isospin-even scattering length. As we discussed above, in the isospin-even correlation function the contribution of the pion-nucleon states above the threshold is of the same order as the pion–nucleon state at the threshold. Therefore, in the following we estimate phenomenologically the contribution of the pion–nucleon continuum above the threshold and check if it is consistent with the difference between the sigma term and the observed isospin-even scattering length. We define the off-shell pion–nucleon T-matrix by

$$T(\nu, t, q^2, q^{'2}) = -i\frac{(q^2 - m_\pi^2)(q^{'2} - m_\pi^2)}{2f_\pi^2 m_\pi^4}\int d^4x e^{iqx}\langle N(p)|T(\partial^\mu A_\mu(x)\partial^\nu A_\nu^\dagger(0))|N(p')\rangle, \tag{24}$$

where $\nu = \omega + t/4M_N$, $t = (q - q')^2$ and $q + p = q' + p'$. Then the spectral function in the nucleon becomes

$$\begin{aligned}\rho_N^\pi(\omega = \; & -\tfrac{1}{2}f_\pi^2\bigg[\delta'(\omega - m_\pi)\mathrm{Re}T_{\pi N} - \delta(\omega - m_\pi)\mathrm{Re}\bigg({T'}_{\pi N} - \frac{3}{m_\pi T_{\pi N}}\bigg) \\ & + \delta'(\omega + m_\pi)\mathrm{Re}T_{\bar{\pi}N} + \delta(\omega + m_\pi)\mathrm{Re}\bigg({T'}_{\bar{\pi}N} - \frac{3}{m_\pi}T_{\bar{\pi}N}\bigg)\bigg] \\ & + \frac{4m_\pi^4}{\omega^2}\mathrm{Re}\frac{1}{(\omega^2 - m_\varpi^2})^2\frac{1}{\pi}\mathrm{Im}T(\omega, O, \omega^2, \omega^2)\bigg],\end{aligned} \tag{25}$$

where $T_{\binom{\pi N}{\bar{\pi}N}} = T(\pm m_\pi, 0, m_\pi^2, m_\pi^2)$, $T'_{\binom{\pi N}{\bar{\pi}N}} = \pm\dfrac{\partial}{\partial\omega}T(\omega, 0, \omega^2, \omega^2)|_{\omega=\pm m_\pi}$. In eq. (25) the term proportional to $\mathrm{Im}\,T$ represents the continuum contribution above the pion–nucleon threshold. We estimate this continuum contribution employing the non-linear sigma model [14], which is known to describe the low-energy pion–nucleon scatterings well. The relevant interaction Lagrangian density of the non-linear sigma model is given by

$$\mathcal{L}_{\rm int} = \frac{1}{4f_\pi^2}\bar{\psi}i\gamma^\mu\tau\psi\cdot(\phi\times\partial_\mu\phi) + \frac{g}{2M_N}\bar{\psi}\gamma_5\gamma^\mu\tau\psi\cdot\partial_\mu\phi\,, \tag{26}$$

where ψ is the nucleon field, ϕ is the pion field and g is the πNN coupling constant ($g = 13{\cdot}5$). In order to obtain $\mathrm{Im}\,T$ we use the optical theorem, $\mathrm{Im}\,T_{ii} = -\frac{1}{2}\sum_n(2\pi)^4\delta^4(p_n + k_n + p + k)|\mathcal{T}_{ni}|^2$, and calculate the off-shell T-matrix, $\mathcal{T}_{ni}$, at the tree-level for the interaction Lagrangian density, eq. (26), as shown in Fig. 1.

The calculated results are $-0{\cdot}06$ fm and $-0{\cdot}02$ fm for the isospin-even and isospin-odd channels, respectively, where the Borel mass is taken to be 1 GeV.

If we … scattering lengths, the results agree

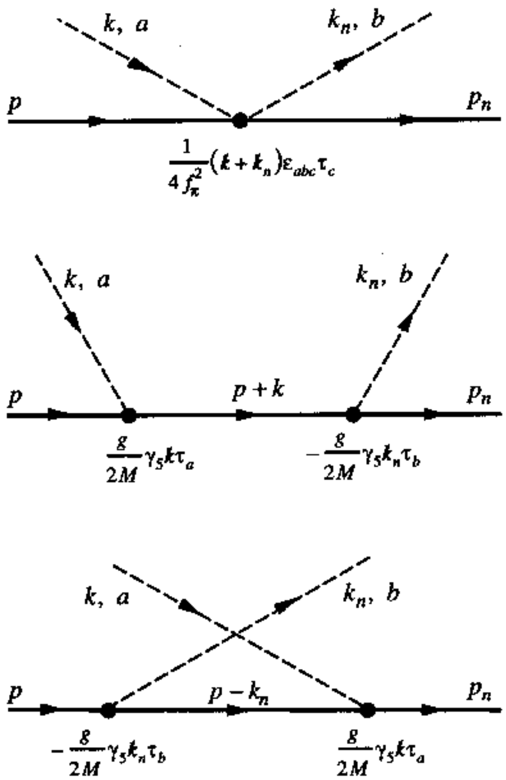

Fig. 1. Tree-level Feynmann diagrams for the pion–nucleon T-matrix in the non-linear sigma model.

3. Nucleon–Nucleon Scattering Lengths

Next we discuss the application to nucleon–nucleon scattering lengths [10]. Following ref. [15], we take the interpolating field for the nucleon (proton) as

$$\eta_p(x) = \epsilon_{abc}\left(u^{Ta}(x) C \gamma_\mu u^b(x)\right)\gamma_5 \gamma^\mu d^c(x)\,, \tag{27}$$

where C denotes the charge conjugation and a, b, and c are colour indices.

The OPE for the correlation function is given in ref. [10], where we calculated the OPE for the correlation function taking into account all the terms up to dimension-four and the dimension-six four-quark operators, since four-quark operators are known to give the largest contribution among higher order operators [2, 3]. We approximated the expectation values of the four-quark operators with respect to the nucleon (vacuum) by saturating the one-nucleon (vacuum) intermediate states.

The spectral function in the nucleon is assumed to be saturated by the nucleon pole terms as

$$\rho_N^B(\omega) = \lambda^2 \left\{ -T_+ \delta'(\omega - M) + T'_+ \delta(\omega - M) \right\} P_+ \\ + \lambda^2 \left\{ -T^- \delta'(\omega + M) - T'_- \delta(\omega + M) \right\} P_- \,, \tag{28}$$

where $P_\pm = (1 \pm \gamma^0)/2$ and $\mathbf{q}$ is taken to be 0. The continuum part is neglected for simplicity.

In the isospin-singlet, $T = 0$, nucleon-nucleon channel, the contribution of the deuteron to the spectral function has to be taken into account. Assuming that the deuteron consists of two non-relativistic nucleons, we can take into account its contribution by the replacement, $a_{NN}^{T=0} \to \tilde{a}_{NN}^{T=0} = a_{NN}^{T=0} + 2\pi^2 M_N B_D |\tilde{f}_D(0)|^2$, where B_D is the binding energy of the deuteron and $\tilde{f}_D(\mathbf{p})$ is the relative wave function of two nucleons in momentum space. Taking this into account, the nucleon–nucleon scattering lengths are obtained as:

$$a_{NN}^{T=1} = \frac{4}{\pi} M_p \frac{A_p M_{\rm Borel}^4 - M_p \big(B_p M_{\rm Borel}^2 + C_p \big)}{M_{\rm Borel}^6 + \pi^2 (8 m_d \langle \bar{d} d \rangle_0 + \langle \dfrac{\alpha_s}{\pi} G^2 \rangle_0) M_{\rm Borel}^2 + \frac{64}{3} (\pi^2 \langle \bar{u} u \rangle_0)^2},$$

$$\tfrac{1}{4} a_{NN}^{T=1} + \tfrac{3}{4} \tilde{a}_{NN}^{T=0}$$

$$= \frac{4}{\pi} \frac{M_p M_n}{M_p + M_n} \frac{A_n M_{\rm Borel}^4 - M_p \big(B_n M_{\rm Borel}^2 + C_n \big)}{M_{\rm Borel}^6 + \pi^2 (8 m_d \langle \bar{d} d \rangle_0 + \langle \dfrac{\alpha_s}{\pi} G^2 \rangle_0) M_{\rm Borel}^2 + \frac{64}{3} (\pi^2 \langle \bar{u} u \rangle_0)^2}, \tag{29}$$

where $A_N = \pi^2 \langle \bar{d} d \rangle_N + 3\pi^2 \langle u^\dagger u \rangle_N + \pi^2 \langle d^\dagger d \rangle_N$, $B_N = m_d \pi^2 (2 \langle u^\dagger u \rangle_N - \langle \bar{d} d \rangle_N) - 2\pi^2 i \langle \bar{d} D_0 d \rangle_N - \frac{1}{8} \pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle_N - \frac{4}{3} \pi^2 (4 i \langle \mathcal{S}[\bar{u} \gamma_0 D_0 u] \rangle_N + i \langle \mathcal{S}[\bar{d} \gamma_0 D_0 d] \rangle_N)$ and $C_N = -\frac{16}{3} \pi^4 (\langle \bar{d} d \rangle_0 \langle u^\dagger u \rangle_N + \langle \bar{u} u \rangle_0 \langle \bar{u} u \rangle_N)$.

Let us first concentrate on the leading order terms of the OPE. In the leading order of the OPE the scattering lengths are obtained as

$$a_{NN}^{T=1} = \frac{4\pi}{M_{\rm Borel}^2} M_N \left(\langle \bar{d} d \rangle_p + 3 \langle u^\dagger u \rangle_p + \langle d^\dagger d \rangle_p \right) = 23 \cdot 2 \text{ fm},$$

$$\tilde{a}_{NN}^{T=0} = \frac{4\pi}{3 M_{\rm Borel}^2} M_N \left(2 \langle \bar{u} u \rangle_p - \langle \bar{d} d \rangle_p - \langle u^\dagger u \rangle_p + 5 \langle d^\dagger d \rangle_p \right) = 5 \cdot 4 \text{ fm}\,, \tag{30}$$

where the Borel mass, $M_{\rm Borel}$, is taken to be 1 GeV. Experimental scattering lengths are $a_{NN}^{T=1}(exp) = 23 \cdot 7$ fm and $a_{NN}^{T=0}(exp) = -5 \cdot 4$ fm. We evaluated the deuteron pole contribution by employing the Paris potential [16] and found that $2\pi^2 M_N B_D |\tilde{f}(0)|^2 = 9 \cdot 4$ fm. If we add this contribution to $a_{NN}^{T=0}(exp)$, we obtain $\tilde{a}_{NN}^{T=0}(\text{‘}exp\text{’}) = 4 \cdot 0$ fm. The calculated scattering lengths are surprisingly close to these values. It is also interesting that the scalar and vector densities of quarks in the nucleon induce attraction between two nucleons.

We next take into account higher order terms of the OPE. The calculated scattering lengths are plotted as a function of the Borel mass squared, M_{Borel}^2, in Fig. 2.

Fig. 2. NN scattering lengths as a function of the square of the Borel mass, $M^2{}_{\mathrm{Borel}}$. The solid curve is for a_{NN}^1 and the dashed curve for $\tilde{a}_{NN}^3$.

The scattering lengths change very little in the region from $M_{\mathrm{Borel}}^2 \sim 1\ \mathrm{GeV}^2$ to $1\cdot 5\ \mathrm{GeV}^2$. By taking the maximum values, we obtain $a_{NN}^{T=1} = 11\cdot 6$ fm and $\tilde{a}_{NN}^{T=0} = 2\cdot 8$ fm. These values are in qualitative agreement with the experimental ones.* Even though the leading-order results are closer to the experimental values than the full results, we do not take it too seriously because of the crude approximations used in the present calculation. It should be also noted that $a_{NN}^{T=1}$ is sensitive to the change of the interaction strength since there is almost a bound state in the spin-singlet nucleon–nucleon channel.

4. Summary

What we have shown in this paper can be summarized as follows:

- The hadron–nucleon scattering lengths, which characterize the hadron–nucleon interaction at low energy, can be studied by QCD sum rules.
- The $O(\rho)$ term of the hadron effective mass in the nuclear medium should be identified as the hadron–nucleon scattering length.
- In the pion–nucleon channel the results of the QCD sum rule are consistent with the low energy theorem.

* There is, however, a controversy concerning the validity and interpretation of the application of the sum rules to the hadron–nucleon scatterings, and consensus on this issue has not yet been achieved [17, 18].

- In the nucleon–nucleon channel the results of the QCD sum rule are in qualitative agreement with experiment but need more study in order to come to a final conclusion.

In conclusion, the QCD sum rule seems to be a promising approach in order to study hadronic interactions. But clearly, much work has to be done in order to confirm the present results and clarify existing discrepancies.

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