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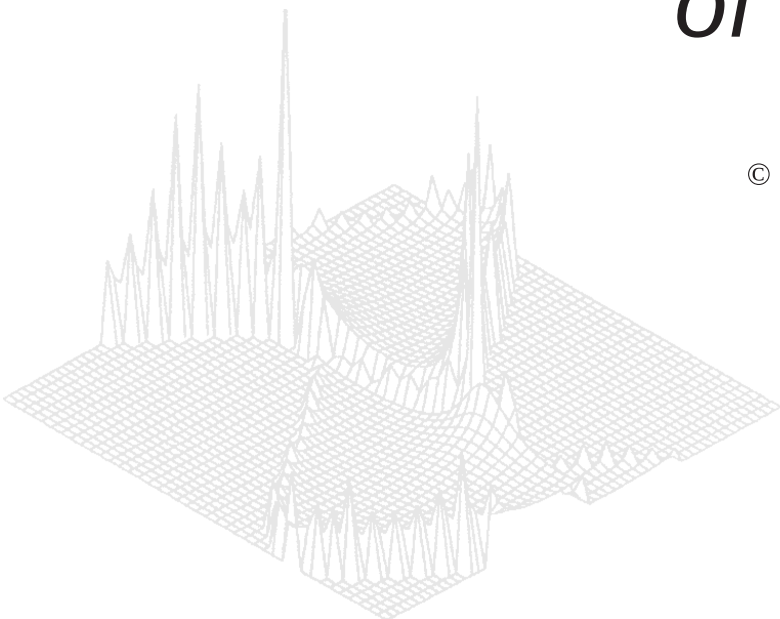

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Higher Twist Structure Functions*

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Abstract

I discuss the importance of some of the higher twist structure functions, and then calculate the twist-two, three and four parton distribution functions involving two quark unpolarized correlations using the wavefunction of the MIT bag model.

1. Introduction

Deep inelastic lepton–nucleon scattering (DIS) has been an important tool in particle physics for more than twenty-five years. Recent developments such as high precision measurements by the NMC [1], SMC [2] and SLAC's E142 [3] and El143 [4] groups are testing the limits of our theoretical knowledge of the structure of the nucleon. While these groups have focussed on the twist-two structure functions, there have been some investigations of the first moments of some higher twist structure functions [5]. In the near future we can also expect the HERMES experiment to investigate the spin dependent structure functions $G_{1,2}$ in some detail. Also the development of polarized beams at RHIC will lead to measurements of the chiral-odd structure functions $h_{1,L}$ which are twist-two and twist-three respectively [6].

Higher twist structure functions will also be important at the energies of CEBAF and the proposed ELFE accelerator because of the phenomenon of hadron–parton duality [8]. The first observation of hadron–parton duality was by Bloom and Gilman [7], who noticed that the structure function $F_2(\omega')$, where the scaling variable $\omega' = 1 + W^2/Q^2$, in the resonance region $W < 2$ GeV roughly averages to $F_2(\omega')$ in the deep inelastic region. This duality appears to be local in that it exists for each interval of ω' where there is a prominent nucleon resonance. De Rujula and collaborators [9] offered an explanation of this phenomenon, which they called 'precocious scaling', in QCD. They argued that the nth moment of F_2 (the argument can be extended to other structure functions) has a twist expansion

$$M_n(Q^2) = \sum_{k=1}^{\infty} \left(\frac{nM_0^2}{Q^2}\right)^{k-1} B_{n,k}(Q^2)\,, \tag{1}$$

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where M_0 is a mass scale in the region of $0 \cdot 4$ GeV and all the $B_{n,k}(Q^2)$ are the same order of magnitude and have logarithmic dependence on Q^2. What this implies is that for $n \leq Q^2/M_0^2$ there is a region in the (n, Q^2) plane where the higher twist contribution is non-negligible, and the dominant contribution to the moments comes from the low-lying resonances. This is exactly the region that will be explored in detail by CEBAF. Hadron–parton duality may enable us to extract from CEBAF data the moments of the relevant higher-twist distributions, and hence important information about the corresponding matrix elements in the operator product expansion.

2. Higher Twist Parton Distributions

In a recent paper, Ji [10] defined the 14 possible nucleon structure functions in the standard model, ignoring CP violating effects. The structure functions are classified by their twist, which is roughly equivalent to their leading Q behaviour, i.e. twist-2 corresponds to Q^0, twist-3 to $1/Q$, twist-4 to $1/Q^2$ etc. Each structure function can be written in terms of all the possible parton distribution functions up to the level of twist-four (ignoring loop effects from QCD radiative corrections).

The parton distribution (or correlation) functions are defined in terms of Fourier transforms of matrix elements of non-local quark and gauge fields separated along the light-cone [10, 6]. At present these matrix elements cannot be calculated in QCD, however calculations of twist-two matrix elements using the wavefunction of the MIT bag model have been performed [11, 12], and these describe current data relatively well.

Consider a nucleon of mass M with momentum P^μ and polarisation vector S^μ. If we choose the nucleon to be moving along the z-axis with momentum P we have

$$P^\mu = (\sqrt{M^2 + P^2}, 0, 0, P)\,. \tag{2}$$

We introduce two orthogonal light-like vectors p and n

$$p^\mu = \tfrac{1}{2}(\sqrt{M^2 + P^2} + P)(1, 0, 0, 1), \quad n^\mu = \frac{1}{M^2}(\sqrt{M^2 + P^2} - P)(1, 0, 0, -1)\,, \tag{3}$$

satisfying $P^\mu = p^\mu + M^2 n^\mu/2$. We also decompose the polarisation vector, $S = S_\parallel + M S_\perp$, where

$$S_\parallel^\mu = p^\mu - \tfrac{1}{2} M^2 n^\mu, \quad S_\perp^\mu = (0, 1, 0, 0)\,. \tag{4}$$

A parton distribution with k light-cone momentum fractions $M(x_1, \ldots, x_k)$ is defined via the matrix element

$$\int \prod_{i=1}^{k} \frac{d\lambda_i}{2\pi} \exp(i\lambda_i x_i) \langle PS | \hat{Q}(\lambda_1 n, \ldots, \lambda_k n) | PS \rangle = M(x_1, \ldots, x_k) \hat{T}(p, n, S_\perp)\,, \tag{5}$$

where $\hat{Q}$ is a product of k quark and gluon fields, and $\hat{T}$ is a Lorentz tensor.

If the mass dimension of the parton distribution $M(x_i)$ is d_M then the distribution is called a twist-$(d_M + 2)$ distribution. As the dimension of a physical observable is fixed, the higher d_M becomes, the higher the inverse power of hard momenta must become in the observable. Thus a twist-n parton distribution can only contribute to physical observables of twist-n or higher, which behave as $Q^{2-(n+a)}$, where $a \geq 0$, in the $Q \to \infty$ limit.

For scattering processes involving two quark fields we can define a quark density matrix

$$M_{\alpha\beta}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}_\beta(0)\psi_\alpha(\lambda n)|PS\rangle \,. \tag{6}$$

From this it is possible to systematically generate the possible distributions at a given twist. The twist-2 part of the density matrix can be written as

$$M(x)|_{\mathrm{twist}-2} = \tfrac{1}{2} \not{p} f_1(x) + \tfrac{1}{2}\gamma_5 \not{p}(S_\parallel \cdot n) g_1(x) + \tfrac{1}{2}\gamma_5 \not{S}_\perp \not{p} h_1(x) \,, \tag{7}$$

where the three quark distribution functions f_1, g_1 and h_1 are defined by

$$\begin{aligned} f_1(x) &= \tfrac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{\psi}(0) \not{n} \psi(\lambda n)|P\rangle \,, \\ g_1(x) &= \tfrac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\parallel|\bar{\psi}(0) \not{n}\gamma_5 \psi(\lambda n)|PS_\parallel\rangle \,, \\ h_1(x) &= \tfrac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp|\bar{\psi}(0) \not{n}\gamma_5 \not{S}_\perp \psi(\lambda n)|PS_\perp\rangle . \end{aligned} \tag{8}$$

These represent the unpolarized quark density, the quark helicity density and the quark transversity density [6] respectively. The light-cone gauge $A^+ = 0$ has been chosen, so that the distributions are manifestly gauge invariant.

Similarly at the twist-3 level, the appropriate portion of the quark density matrix can be written as

$$M(x)|_{\mathrm{twist}-3} = \frac{\Lambda}{2}[e(x) + \tfrac{1}{2}(S_\parallel \cdot n)(\not{p}\not{n} - \not{n}\not{p})\gamma_5 h_L(x) + \gamma_5 \not{S}_\perp g_T(x)] \,, \tag{9}$$

where Λ is a soft mass scale in QCD. The three twist-3 distribution functions are given by

$$\begin{aligned} e(x) &= \frac{1}{2\Lambda} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{\psi}(0)\psi(\lambda n)|P\rangle \,, \\ h_L(x) &= \frac{1}{2\Lambda} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\parallel|\bar{\psi}(0)\tfrac{1}{2}(\not{p}\not{n} - \not{n}\not{p})\gamma_5 \psi(\lambda n)|PS_\parallel\rangle \,, \\ g_T(x) &= \frac{1}{2\Lambda} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp|\bar{\psi}(0)\gamma_5 \not{S}_\perp \psi(\lambda n)|PS_\perp\rangle \,. \end{aligned} \tag{10}$$

At twist-4 we have

$$M(x)|_{\mathrm{twist}-4} = \frac{\Lambda^2}{4}[\not{n} f_4(x) + \not{n}\gamma_5 (S_\parallel \cdot n) g_3(x) + \not{n}\gamma_5 \not{S}_\perp h_3(x)] \,, \tag{11}$$

with the twist-4 distributions

$$f_4(x) = \frac{1}{2\Lambda^2}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P|\overline{\psi}(0)\ \not{p}\psi(\lambda n)|P\rangle\,,$$

$$g_3(x) = \frac{1}{2\Lambda^2}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\parallel|\overline{\psi}(0)\gamma_5\ \not{p}\psi(\lambda n)|PS_\parallel\rangle\,,$$

$$h_3(x) = \frac{1}{2\Lambda^2}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\perp|\overline{\psi}(0)\gamma_5\ \not{S}_\perp\ \not{p}\psi(\lambda n)|PS_\perp\rangle\,. \tag{12}$$

The quark field ψ can be decomposed into 'good' and 'bad' components, ψ_+ and ψ_- respectively,

$$\psi_\pm = P^\pm\psi, \quad P^\pm = \tfrac{1}{2}\gamma^\mp\gamma^\pm, \quad \gamma^\pm = \sqrt{\tfrac{1}{2}}(\gamma^0 \pm \gamma^3)\,. \tag{13}$$

By inspection it can be seen that the twist-2 quark distributions involve only the 'good' component ψ_+, whereas the twist-3 distributions involve mixing one 'good' and one 'bad' component, and the twist-4 distributions involve only the 'bad' components.

The QCD equations of motion [13]

$$i\frac{d}{d\lambda}\psi_-(\lambda n) = \tfrac{1}{2}\ \not{n}(-i\not{D}_\perp + m_q)\psi_+(\lambda n)\,, \tag{14}$$

$$-i\frac{d}{d\lambda}\overline{\psi}_-(\lambda n) = \tfrac{1}{2}\overline{\psi}_+(\lambda n)(-i\not{D}_\perp + m_q)\ \not{n} \tag{15}$$

make it possible to eliminate the 'bad' components from the twist-three and four distributions, at the cost of introducing gluon fields into the matrix elements. Because the model wavefunctions do not include gluon fields, I will not do this here. Also note that at twist-three and twist-four there exist distributions involving the gluon field with two or three light-cone momentum fractions, such as [10]

$$E(x,y) = -\frac{1}{4\Lambda}\int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{i\lambda x}e^{i\mu(y-x)}\langle P|\overline{\psi}(0)i\not{D}_\perp(\mu n)\ \not{n}\psi(\lambda n)|P\rangle\,, \tag{16}$$

and

$$B_1(x,y,z) = \frac{1}{2\Lambda^2}\int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}\frac{d\nu}{2\pi} e^{i\lambda x}e^{i\mu(y-x)}e^{i\nu(z-y)} \times \langle P|\overline{\psi}(0)\ \not{n}i\not{D}_\perp(\nu n)i\not{D}_\perp(\mu n)\psi(\lambda n)|P\rangle\,, \tag{17}$$

which can be related to $e(x)$ and $f_4(x)$ by integrating over y or y and z respectively. However, as the MIT bag wavefunction has no explicit gluon field, these distributions will be zero in the model. Also the distributions with only one light-cone momentum fraction are of the most interest for DIS and Drell–Yan processes.

Finally there exist distributions at the twist-four level involving four quark operators, e.g.

$$U_1^s(x,y,z) = \frac{1}{4\Lambda^2}\int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}\frac{d\nu}{2\pi} e^{i\lambda x} e^{i\mu(y-x)} e^{i\nu(z-y)} \langle P|\overline{\psi}(0)\ \not{n}\psi(\nu n)\overline{\psi}(\mu n)\psi(\lambda n)|P\rangle\,, \tag{18}$$

which is a four quark light-cone correlation function. Calculating this distribution by the method below would require the evaluation of the overlap integral between the four quark fields over the bag volume, which is expected to be much smaller than the two quark overlap integral required for the two quark correlation functions. Hence I will not consider these distributions here.

3. Calculation of Quark Distributions

At present no QCD wavefunction for the nucleon can be calculated. So in order to make useful calculations of the quark distributions at any twist it is necessary to the use the wavefunction from some phenomenological model of the nucleon. I choose to use the MIT bag model [14], as it incorporates relativistic, light quarks, and also models confinement. It also has the advantage that the wavefunction is simple and analytic. Other models could also be chosen [15].

The major problem in calculating the relevant matrix elements for the quark distributions is ensuring that momentum conservation is obeyed throughout the calculation, hence ensuring that the calculated distributions have the correct support, i.e. they vanish for light-cone momentum fraction x outside the interval $[0,1]$. To guarantee momentum conservation, a complete set of intermediate states $\sum_m |m\rangle\langle m|$ can be inserted into the matrix elements of the quark distributions [eqs (8, 10, 12)]. Using translational invariance of the matrix element, all the λ dependence can go into the argument of the exponential function. Then integrating over λ gives a momentum conserving delta function. The twist-two quark distributions then become

$$\begin{aligned}
f_1(x) &= \sqrt{\tfrac{1}{2}}\sum_m \delta(p^+(1-x)-p_m^+)|\langle m|\psi_+(0)|P\rangle|^2\,,\\
g_1(x) &= \sqrt{\tfrac{1}{2}}\sum_m \delta(p^+(1-x)-p_m^+)[|\langle m|\hat{R}\psi_+(0)|PS_\parallel\rangle|^2 - |\langle m|\hat{L}\psi_+(0)|PS_\parallel\rangle|^2]\,,\\
h_1(x) &= \sqrt{\tfrac{1}{2}}\sum_m \delta(p^+(1-x)-p_m^+)[|\langle m|\hat{Q}_+\psi_+(0)|PS_\perp\rangle|^2\\
&\qquad\qquad - |\langle m|\hat{Q}_-\psi_+(0)|PS_\perp\rangle|^2]\,,
\end{aligned} \tag{19}$$

where $\hat{R}$ ($\hat{L}$) is the projection operator for right (left) handed quarks $\hat{R}$ ($\hat{L}$) $=(1\pm\gamma_5)/2$, and $\hat{Q}_\pm$ is the projection operator $\hat{Q}_\pm = (1\pm\gamma_5\ \not{S}_\perp)/2$, which projects out eigenstates of the transversely projected Pauli–Lubanski operator $\not{S}_\perp\gamma_5$ in a transversely projected nucleon.

The twist-four distributions are similar to the twist-two ones, except they involve 'bad' components of the quark wavefunction

$$f_4(x) = \sqrt{\tfrac{1}{2}} \sum_m \delta(p^+(1-x) - p_m^+) |\langle m|\psi_-(0)|P\rangle|^2 \,,$$

$$g_3(x) = \sqrt{\tfrac{1}{2}} \sum_m \delta(p^+(1-x) - p_m^+) [|\langle m|\hat{L}\psi_-(0)|PS_\parallel\rangle|^2 - |\langle m|\hat{R}\psi_-(0)|PS_\parallel\rangle|^2] \,,$$

$$h_3(x) = \sqrt{\tfrac{1}{2}} \sum_m \delta(p^+(1-x) - p_m^+) [|\langle m|\hat{Q_+}\psi_-(0)|PS_\perp\rangle|^2 - |\langle m|\hat{Q_-}\psi_-(0)|PS_\perp\rangle|^2] \,, \quad (20)$$

Here I have rescaled the distributions to make them dimensionless, and factors of $(M/\Lambda)^2$ have been absorbed in the twist-four part of the density matrix $M(x)|_{\mathrm{twist-4}}$.

Note that the twist-two and twist-four distributions have a natural interpretation in the parton model, where they are related to the probability of finding a parton carrying fraction x of the plus component of momentum of the nucleon, and in the appropriate helicity or transversity eigenstates. In the twist-three case, the distributions do not have a similar interpretation in the parton model. However, we can still guarantee momentum conservation by introducing a complete set of intermediate states $\sum_m |m\rangle\langle m|$, and then write the distributions in terms of the matrix elements between nucleon states and intermediate states:

$$e(x) = \sum_m \delta(p^+(1-x) - p_m^+) \langle P|\psi^\dagger(0)|m\rangle \gamma^0 \langle m|\psi(0)|P\rangle \,,$$

$$h_L(x) = \sum_m \delta(p^+(1-x) - p_m^+) \langle PS_\parallel|\psi^\dagger(0)|m\rangle \gamma^3\gamma^5 \langle m|\psi(0)|PS_\parallel\rangle \,,$$

$$g_T(x) = \sum_m \delta(p^+(1-x) - p_m^+) \langle PS_\perp|\psi^\dagger(0)|m\rangle \gamma^0\gamma^5 \not{S}_\perp \langle m|\psi(0)|PS_\perp\rangle \,. \quad (21)$$

Again I have rescaled the distributions so they are dimensionless, absorbing factors of M/Λ into the density matrix $M(x)|_{\mathrm{twist-3}}$.

The next step in the model calculation of the distributions is to form the momentum eigenstates $|P\rangle$ and $|m\rangle$ from the static states of the model. This can be done using either the Peierls–Yoccoz [16] projection, which gives a momentum dependent normalisation, or the Peierls–Thouless [17] projection, which leads to a more difficult calculation, but which preserves Galilean invariance of the matrix elements. The distributions can then be calculated in terms of the Hill–Wheeler overlap integrals between the quark wavefunctions [11, 12].

Using the wavefunction of a model also introduces a scale μ into the calculated distribution functions. This is the scale at which the model wavefunction is considered a good approximation to the true QCD wavefunction, which is presently unknown. The natural scale for the bag model, and most other phenomenological models employing light relativistic quarks, is around $k_T \approx 400$ MeV of the quarks. In order to compare a calculated distribution function with experiment, the calculated distribution needs to be evolved from the model scale up to the experimental scale Q. This has previously been done using leading order QCD evolution for the twist-2 distributions $f_1(x)$ and $g_1(x)$, with good agreement being obtained for a value of μ in the region of 250–500 MeV. This could be criticised on the grounds that the strong coupling constant is not small in this region, however calculations using next to leading order evolution [18] also give good agreement with experiment for values of $\mu \sim 350$ MeV.

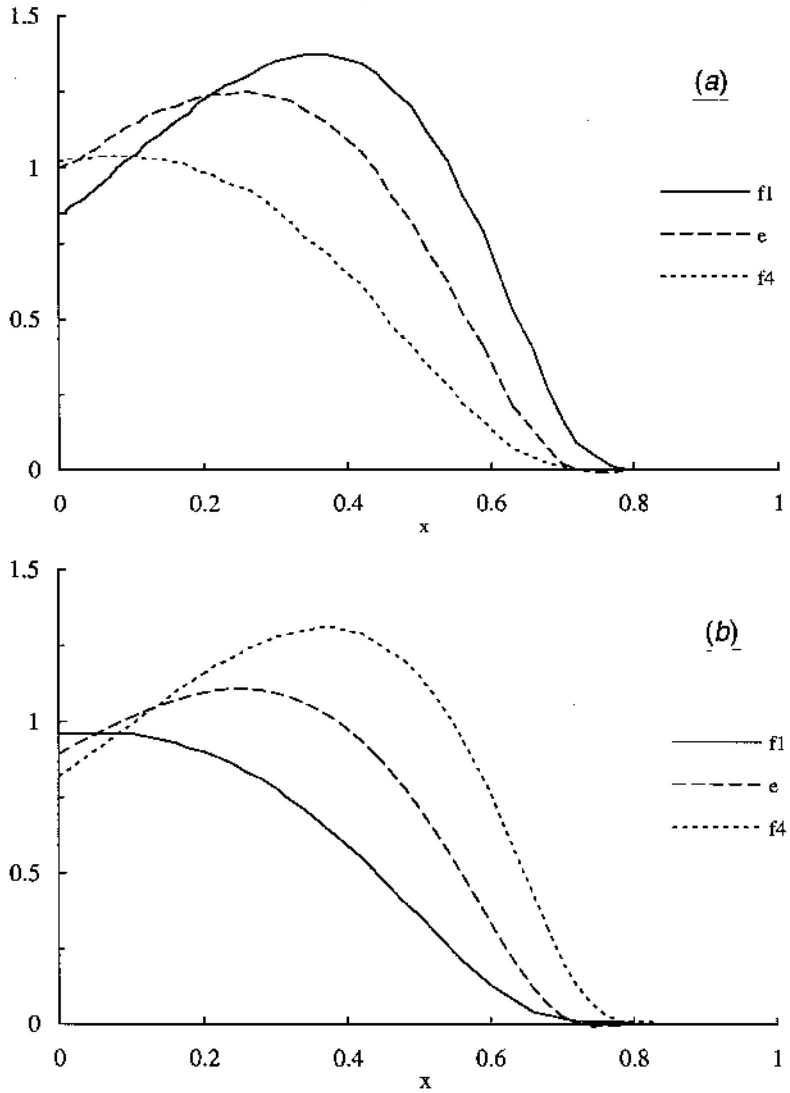

Fig. 1. Unpolarized quark distributions $f_1(x)$, $e(x)$ and $f_4(x)$ calculated at the bag scale μ^2, with bag radius $0{\cdot}8$ fm, using (a) the Peierls–Yoccoz projection and (b) the Peierls–Thouless projection for momentum eigenstates.

In Fig. 1 I show the unpolarized distributions $f_1(x)$, $e(x)$ and $f_4(x)$ calculated at the bag scale $Q^2 = \mu^2$, and for a bag radius of $0{\cdot}8$ fm, using both the Peierls–Yoccoz and Peierls–Thouless projections for the momentum eigenstates. From the figure it is clear that there is a difference between the distributions calculated using the two different projections. In earlier work [15] the difference between the two $f_1(x)$ distributions was ascribed to a difference in the appropriate bag scale μ between the Peierls–Yoccoz and Peierls–Thouless projections for the bag. However, this explanation cannot account for the difference between the $f_4(x)$ distributions. At this stage it is unclear why the two projections should give different results, and this will be the subject of further investigation.

In Fig. 2 I show the first 10 moments of the unpolarized parton distributions, defined by $M_n^i = \int_0^1 x^n q_i(x)dx$ calculated in Fig. 1. Again we see a clear difference in behaviour between the moments calculated using the Peierls–Yoccoz projection, and those calculated using the Peierls–Thouless projection.

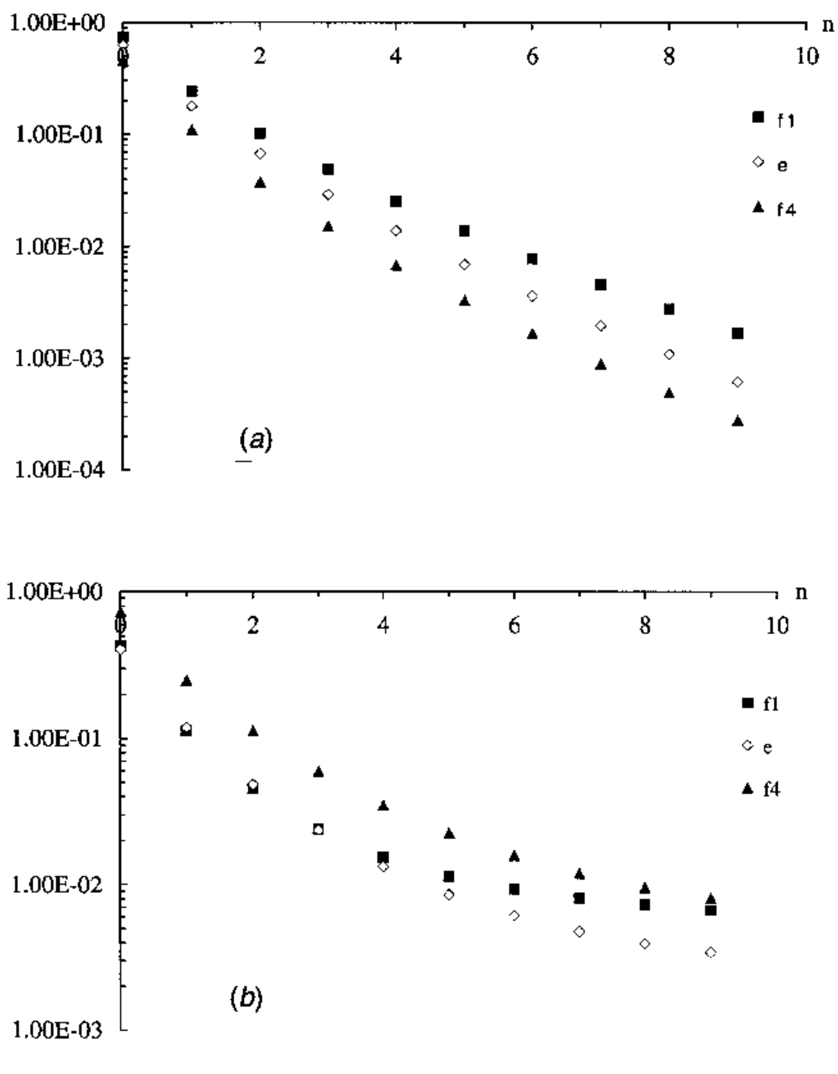

Fig. 2. First 10 moments of the unpolarized quark distributions $f_1(x)$, $e(x)$ and $f_4(x)$ shown in Fig. 1, using (a) the Peierls–Yoccoz projection and (b) the Peierls–Thouless projection for momentum eigenstates.

There still needs to be further work before we can compare these calculations with experiment and make predictions about the resonance region. For instance there are the effects of the nucleon's pion cloud, which can be treated in the context of the cloudy bag model [11], and effects of the SU(6) portion of the nucleon wavefunction which lead to a difference in u and d quark distributions. Also we need to know more about the anomalous dimensions of the moments, and the renormalization behaviour of the higher twist distributions in order to evolve them up to the experimental scales. This work is in progress.

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References

[1] NMC, P. Amaudruz *et al.*, Phys. Rev. Lett. **66** (1991) 560; M. Arneodo *et al.*, Phys. Lett. **B364** (1995) 107.
[2] SMC, D. Adams *et al.*, Phys. Lett. **B329** (1994) 399.
[3] E142, P. L. Anthony *et al.*, Phys. Rev. Lett. **71** (1993) 959.
[4] E143, K. Abe *et al.*, Phys. Lett. **B364** (1995) 61.
[5] SMC, D. Adams *et al.*, Phys. Lett. **B336** (1994) 125.
[6] R. L. Jaffe and X. Ji, Phys. Rev. Lett. **67** (1991) 552.
[7] E. D. Bloom and F. J. Gilman, Phys. Rev. **D4** (1972) 2901.
[8] X. Ji and P. Unrau, Phys. Rev. **D5** (1995) 72.
[9] A. De Rújula, H. Georgi and H. D. Politzer, Ann. Phys. **103** (1977) 315.
[10] X. Ji, Nucl. Phys. **B402** (1993) 217.
[11] A. W. Schreiber, A. I. Signal and A. W. Thomas, Phys. Rev. **D44** (1991) 2653.
[12] A. Ardekani and A. I. Signal, Phys. Lett. **B311** (1993) 281.
[13] J. Kogut and D. E. Soper. Phys. Rev. **D1** (1970) 2901.
[14] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. **D9** (1974) 3471.
[15] M. R. Bate and A. I. Signal, J. Phys. **G18** (1992) 1875.
[16] R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. London **A70** (1957) 381.
[17] R. E. Peierls and D. J. Thouless, Nucl. Phys. **38** (1962) 154.
[18] F. M. Steffens and A. W. Thomas, Adelaide Preprint ADP-94–27/T166 (1994).