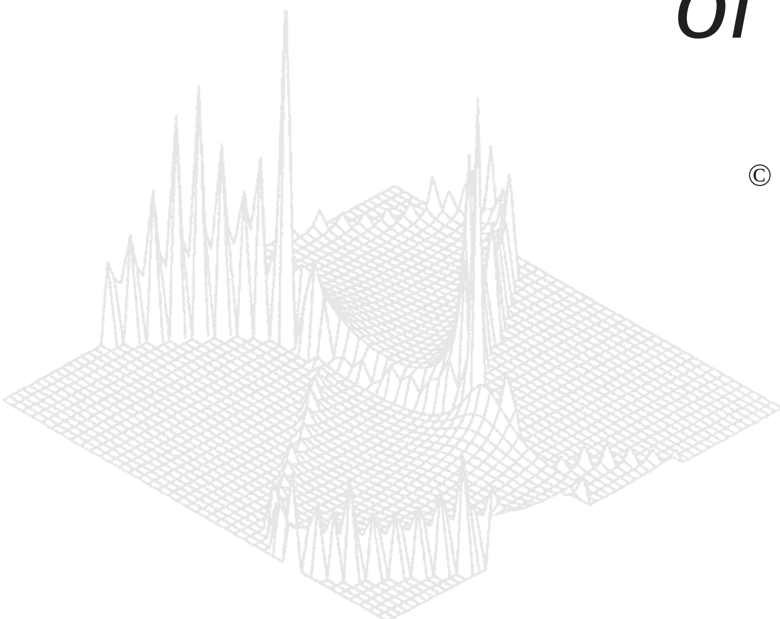
CSIRO PUBLISHING

Australian Journal of Physics

Volume 50, 1997



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia
Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: peter.robertson@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO Australia and
the Australian Academy of Science

Aust. J. Phys., 1997, **50**, 71–8

Polarized Deep Inelastic Scattering*

F. M. Steffens and A. W. Thomas

Department of Physics and Mathematical Physics,
University of Adelaide, Adelaide, SA 5005, Australia.

Abstract

We give an overview of present calculations involving the proton spin structure function. It is shown that a significant part of the discepancy between the data and the Ellis–Jaffe sum-rule may arise through the axial anomaly if the gluons within the proton are strongly polarized. While a quark model, such as the MIT bag, does not include the anomaly, and therefore cannot be expected to reproduce the spin structure function, it does give a rather good description of recent data which is anomaly free, such as the distribution of polarized, valence up-quarks in the proton.

1. Introduction

Experiments involving inclusive deep inelastic scattering have provided important information on nucleon structure. The experiments are able to tell us many things, from the confirmation of the assumption of the quark model that the nucleon is made of three valence quarks to the spin fraction carried by quarks. Of course, there are still many questions to be answered and the answers to some of these have become extremely active research topics in the last few years. The search for a complete picture of hadron structure is in this sense the goal being sought.

Much of the interest in the area was revived several years ago by the EMC experiment where the proton spin structure function was measured in a region of small x previously unexplored. From this experiment it was concluded that the spin of the proton carried by quarks was very small, being compatible with zero. This became known as the spin problem. Since then, many other polarized experiments have been performed and now there is a reasonable amount of data on the proton, neutron and deuterium spin structure functions [1]. Many other experiments are also planned or being carried out now which will give further details on the spin structure of the proton. Nevertheless, a full understanding of the current experimental results has not been achieved and, among others, the following questions remain to be answered:

• The combined experiments on the proton spin structure function seem to indicate that the quark singlet axial charge is about half of what is expected

* Refereed paper based on a contribution to the Japan–Australia Workshop on Quarks, Hadrons and Nuclei held at the Institute for Theoretical Physics, University of Adelaide, in November 1995.

10.1071/PH96044 0004-9506/97/010071$05.00

from the quark model. What is the mechanism behind such a reduction that can explain both the first moment and the x dependence of the structure function?

- Are the gluons important for the first moment of $g_1(x, Q^2)$? What is their actual contribution to the spin sum rule? Almost nothing is known about the polarization of the gluons, even its sign is not known, and its study is one of the major questions in hadron structure.
- What is the valence and sea decomposition of the polarized quark distributions? Is the polarized sea, like its unpolarized counterpart, asymmetric? Present experiments are unable to give a definite answer to these questions and only recently have data on the valence distribution become available from semi-inclusive data [2].
- The small x behaviour of polarized parton distributions is a complete unknown. A recent experiment suggests a rapid rise of g_1^p in that region. If this rise were steep enough, it could cause the saturation of the first moment of g_1 at its quark model value. Firm conclusions are not yet possible because of the large error bars.
- The role of higher twist terms, mainly in some small x data where Q^2 is small, is still to be evaluated; not only their contribution to the sum rules, which seems to be small [3], but also possible effects on the x dependence [4].

There are other topics to be addressed and those quoted above are only examples which show that there is much work to be done. We will briefly review these and some related topics in an attempt to summarise the present status of the subject. The number of subjects covered is by no means complete and follows the authors' interests.

2. The Problem

The proton spin structure function is given by

$$g_1^p(x, Q^2) = \int_x^1 \frac{dy}{y} C_q^{NS}(x/y, Q^2/\mu^2)\Delta q^{NS}(y, \mu^2) + \int_x^1 \frac{dy}{y} C_q^S(x/y, Q^2/\mu^2)\Delta q^S(y, \mu^2) + \int_x^1 \frac{dy}{y} C_g(x/y, Q^2/\mu^2)\Delta g(y, \mu^2)\,, \tag{1}$$

where the C denote the gluon and quark Wilson coefficients for singlet and nonsinglet operators and μ^2 is the renormalization scale. For three flavours, the nonsinglet charge is $\Delta q^{NS} = \frac{1}{12}(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}) + \frac{1}{36}(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\Delta s - 2\Delta\bar{s}) = \frac{1}{12}g_a + \frac{1}{36}g_a^8$, while the singlet charge is $\Delta q^S = \frac{1}{9}(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}) = \frac{1}{9}\Delta\Sigma$.

The proton spin structure function measured by the EMC, SMC and E143 experiments are at a different average Q^2. Even so, their results tend to agree if the whole data set is evolved to a common Q^2. To perform the evolution, it is usually assumed that the measured asymmetries are Q^2 independent—an assumption on which we will comment later. Meanwhile, to fix ourselves to a definite number, we will use the SMC results [5] in conjunction with earlier EMC and SLAC data:

$$\int_0^1 dx g_1^p(x,Q^2) = \int_0^1 dx \frac{A_1^p(x,Q^2)F_2^p(x,Q^2)}{2x[1+R(x,Q^2)]} = 0\cdot 142 \pm 0\cdot 008 \pm 0\cdot 011\,, \quad (2)$$

calculated at an average of 10 GeV^2. The asymmetry A_1 is what is measured and the unpolarized structure functions F_2 and R are taken from previous experiments. Eq. (2) then predicts that $\Delta\Sigma = 0\cdot 27 \pm 0\cdot 008 \pm 0\cdot 011$. On the other hand, the Ellis–Jaffe prediction for the integral of g_1^p at $10\,\mathrm{GeV}^2$ is $\int_0^1 dx g_1^p(x,Q^2) = 0\cdot 176 \pm 0\cdot 006$, where $O(\alpha_s)$ corrections were included. We remind the reader that the Ellis–Jaffe result is based on the assumption that the sea is not polarized and $\Delta\Sigma \sim g_a^8$ is expected to be of order $0\cdot 6$. This discrepancy between theory and experiment is what is known as 'the spin problem' [6, 7, 8]. In the following we discuss the various approximations made to get the quoted numbers, as well as a few possible solutions to the problem.

3. The Small x Region

Of course, experimentally it is not possible to go to $x = 1$ or $x = 0$. In the case of the SMC the limits are $0\cdot 003 < x < 0\cdot 7$. For the new E143 SLAC data [9], the limits are $0\cdot 029 < x < 0\cdot 8$ and their integral of g_1 agrees within errors with the SMC results. Extrapolations have to be made to cover the whole x interval. The large x region is well behaved and perturbation theory predicts that $A_1 \to 1$ as $x \to 1$ [10]. For small x a Regge-type behaviour [11], $g_1^p \propto x^\alpha$, $0 < \alpha < 0\cdot 5$ [12], has been assumed—with the errors in the quoted data expressing the uncertainty in α. However, the SMC data show a tendency to increase for $x < 0\cdot 02$. In their analysis of the data, the SMC did not consider that the measured tendency of the data to rise at small x was enough to motivate the use of some other extrapolation of g_1^p in that region. Close and Roberts [13] considered a few other possibilities. They are:

(a) $-Lnx$, derived using the fact that the Froissart bound [14] for the total unpolarized cross section, $\sigma \leq Log^2\ s$ with s the square of the centre of mass energy, is saturated. Then $g_1 \propto -Lnx$ follows if one calculates the behaviour of the spin asymmetry for a vector potential in which case $A \propto 1/sLogs$ and hence $g_1 \propto -Lnx$ as $x \to 0$.

(b) $1/xLn^2x$. This is derived as in (a) but in this case the potential transforms like an axial vector.

(c) $-(1+2Lnx)$. This was derived by Bass and Landshoff [15] using a model for the non-perturbative Pomeron exchange simulated through non-perturbative gluons. Their calculation affects only the singlet part of g_1.

According to the data shown by Close and Roberts, the $-Lnx$ and $-(1+2Lnx)$ forms tend to fit the data, though the last point is missed, while the very singular behaviour $1/xLn^2x$ tends to fit the data over the whole small -x region. On the other hand, the extrapolation used by the SMC appears to be somewhat below the trend of the data. Some comments on these results seem appropriate. First, in their analysis Close and Roberts extrapolated the individual data of each x to a common $Q^2 = 10$ GeV^2, using the assumption of a Q^2-independent asymmetry. This seems to be contradicted by recent studies of the QCD evolution of the spin structure functions at next-to-leading order [16, 18]. In fact, if one looks to the resulting Q^2 dependence of the asymmetry as calculated by [16], one may conclude that the data used by Close and Roberts may be overestimated at least for the last two x points. The tendency for the asymmetry to be Q^2 dependent in the region of $Q^2 \sim 1$ GeV^2 has been recently corroborated by the first experimental test on the Q^2 dependence of A_1 made by the E143 experiment [17]. Second, the rise of g_1^p at small x should be reflected also in the singlet part of the deuteron spin structure function. However, recent deuteron data show no signal of a rapid raise of g_1^d [19]. Thus a very singular g_1 can be ruled out but the question of its precise behaviour at small x is still open.

4. The Gluon Contribution

It is a fact that the spin sum rule should be satisfied:

$$\tfrac{1}{2} = \tfrac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g \,, \tag{3}$$

where Δg is the total gluon spin and L denotes the z-component of the orbital angular momentum of the quarks and gluons. Notice that, contrary to unpolarized deep inelastic scattering where measurements of F_2 alone determine the momentum carried by gluons via the momentum sum rule, measurements of g_1 alone are not able to determine Δg.*

According to Eq. (1), the integral of $g_1(x, Q^2)$, Γ_1^p, is given by

$$\Gamma_1^p \sim \tfrac{1}{9}\Delta\Sigma\left(1 - \frac{\alpha_s(Q^2)}{3\pi} + ...\right) + \tfrac{1}{9}\Delta g(\mu^2)\int_0^1 C_g(x, Q^2/\mu^2)dx \,, \tag{4}$$

where the nonsinglet part was not explicitly written, the perturbative result up to order [20] was shown and the integral over x of C_g was left undone.

In the operator product expansion for g_1, there is no gluon operator contribution to [20], which means that $\int_0^1 C_g(x, Q^2/\mu^2)dx = 0$. In this case, measurements of g_1 would determine $\Delta\Sigma$ and hence the spin carried by quarks. However, the operator, $\overline{\psi}\gamma_\mu\gamma_5\psi$, giving rise to $\Delta\Sigma$ in the OPE is gauge invariant but not conserved even in the chiral limit due to the axial anomaly [21]:

$$\partial_\mu J^\mu_{5R} = \frac{\alpha_s}{2\pi} Tr G_{\mu\nu}\tilde{G^{\mu\nu}} \,, \tag{5}$$

where $G_{\mu\nu}$ is the gluon tensor and $\tilde{G^{\mu\nu}}$ is its dual. As a first consequence, the matrix elements of $\overline{\psi}\gamma_\mu\gamma_5\psi$ *are not* the quark helicity of the naive parton model.

It is a property of renormalized operators having an anomalous dimension different from zero that they will not always have the same symmetry properties as their classical counterparts. This is a well known fact of quantum field theory where regularization can spoil some of the classical symmetries. The process of renormalization should restore such symmetries but in anomalous theories this is not what happens. Of course, the anomalous dimension beyond leading order is dependent on the renormalization scheme used, which means that one can always find a scheme where the anomalous dimension is zero and hence one would have again the identification of the axial current with spin. However, such schemes break gauge invariance. So, as the operators appearing in the operator product

* As a matter of fact, interpretations of g_1 that include gluonic contributions through the axial anomaly, can be used to constrain the total gluon spin. This will be discussed soon.

expansion are, by definition, gauge invariant, the matrix elements of the axial current are not related to the spin carried by quarks and the first moment of the gluon Wilson coefficient is zero.

With respect to the x-dependence, this would mean that when calculating the cross sections related to such quantities, one should use renormalization schemes where gauge invariance is preserved, like $\overline{MS}$, in which case it also happens that chiral symmetry is broken. For the case of the hard gluon coefficient, an extensive study was done by Bodwin and Qiu [22] where they indeed found that in schemes where gauge invariance is preserved, $\int_0^1 C_g(x, Q^2/\mu^2)dx = 0$. If one uses a scheme involving a cut-off in transverse momentum, where the parton model is often formulated, one finds that $\int_0^1 C_g(x, Q^2/\mu^2)dx \neq 0$, in which case gauge invariance is broken but the axial current is free from the anomaly. It is then possible to construct, inspired by Eq. (5), a new axial current that is conserved in the chiral limit with the price that it is gauge dependent [23].

Although, at first sight, this is a high price to pay, we notice that the decomposition of the spin into its parts in Eq. (3) *is not* gauge invariant itself [24]. Moreover, the parton model is formulated in a specific gauge, the axial gauge, in which case the gluon distribution is given by the forward matrix elements of the gluon operator in Eq. (5). Hence one can, in principle, express Γ_1^p in terms of the quark and gluon spin. This is a very fragile interpretation, valid only in the axial gauge but, again, it is this gauge that is used to define parton distributions as well. If we change gauge we no longer have a clear quark and gluon helicity decomposition but neither do we have the parton model. Finally we note that up to large gauge transformations, like the ones that change the topological number, the forward matrix elements of the conserved axial current and gluon operator are gauge invariant [6]. As a conclusion, it seems that if one wants to talk about spin carried by quarks, then one should use a renormalization scheme that allows such a thing, like the cut-off scheme, and also a gauge where such interpretations through the parton model are allowed.

In the parton model formulation where the gluons contribute to Γ_1^p, the sum rule has usually been written as [8]

$$\Gamma_1^p = (g_a + g_a^8)\left(1 - \frac{\alpha_s}{\pi}\right) + \tfrac{1}{9}\Delta\Sigma\left(1 - \frac{\alpha_s}{3\pi}\right) - \tfrac{1}{3}\frac{\alpha_s}{2\pi}\Delta g\,, \tag{6}$$

where $\Delta\Sigma$ is the spin content of the proton and three active flavours are considered. However, a recent study where the quark masses are treated consistently has shown that for the momentum transfer of the present experiments, Eq. (6) overestimates the strange quark contribution [25]. Moreover, it was also shown that there is a sizable charm contribution that should be taken into account. In practical terms it means that if we blame the discrepancy between experiment, Eq. (2), and the Ellis–Jaffe sum rule entirely on the gluon term in Eq. (6), Δg is reduced from $3{\cdot}04 \pm 1{\cdot}4$ to $2{\cdot}32 \pm 1{\cdot}06$ at 10 GeV^2 if the quark masses are treated correctly and charm is included.

The discussion of the polarised glue is far from settled. For instance, Jaffe [26] recently calculated Δg in various nucleon models. His results suggest that Δg is negative in any model for the nucleon, independent of its particular structure. Moreover, he concludes that the sign of the polarized gluon is directly related to the $N - \Delta$ splitting: a negative Δg results from the fact that the Δ heavier than the nucleon. If Δg were positive, the Δ would be lighter than the nucleon. His conclusions are supposed to be valid at the scale of the model. If QCD evolution or (in his calculations) the omission of gluon self interactions do not change the sign of Δg, one would be forced to accept a negative Δg, in which case the proton spin problem would worsen. However, even the connection between the sign of Δg and the baryon spin splitting used by Jaffe has been challenged by another recent calculation. In a study of lattice QCD, Liu [27] calculated some hadron properties in a valence approximation, where one gluon exchange is not switched off. He found that the nucleon and the Δ were degenerate, in contradiction to the many nucleon model calculations where one gluon exchange is used to justify the baryon spin splitting.

To help resolve these questions, there are a few experiments planned in the near future aimed at measuring Δg through charm production [28] or gluon fusion in *pp* or Drell–Yan processes at the RHIC [29]. The measurement of Δg would be one of the landmarks in the study of hadron structure.

5. Results from Lattice QCD

Motivated by the experimental results, a few groups [30] have engaged in the calculation of g_a, g_a^8 and $\Delta\Sigma$ in lattice QCD. So far, all the results have been obtained in quenched QCD but they are separated into connected and disconnected insertions, with the latter being identified with the sea quarks and the former containing both valence and cloud* contributions. In general, only gauge invariant operators are used which means that the $\Delta\Sigma$ calculated is directly related to the measured and previously quoted $\Delta\Sigma$ but it is not related to the quark spin content of the proton. We now quote a few results with some comments. The Kentucky group [32] calculated $\Delta\Sigma = 0\cdot25 \pm 0\cdot12$, $g_a = 1\cdot2 \pm 0\cdot1$ and $g_a^8 = 0\cdot61 \pm 0\cdot13$, where both connected and disconnected contributions are included. Interesting enough, in the connected approximation, they also found $\Delta\Sigma/g_a < 3/5$, the value expected from SU(6). However, if the valence approximation was taken (the cloud quarks in the connected graphs are disregarded), they indeed got the 3/5 from SU(6), in a remarkable result.

Other works, like the calculations by Fukugita *et al.* [31], also found similar results, with the exception of g_a: $\Delta\Sigma = 0\cdot18 \pm 0\cdot1$, $g_a = 0\cdot985 \pm 0\cdot025$ and $g_a^8 = 0\cdot509 \pm 0\cdot12$. Remarkably, g_a is around 20% smaller than the experimental value. Moreover this result seems not to depend on the quenched approximation [30]. On the other hand, the result for g_a from the Kentucky group tends to agree with the data, contrary to the world average of g_a in lattice calculations [30]. The large value of g_a obtained in [32] may be related to the small number of gauge configurations used there [33]. In any case, it seems that the origin of the problems with g_a in the lattice is open, a disappointing observation in view of the fact they tend to get a reasonable value for $\Delta\Sigma$.

6. The x Dependence

Although much attention has been paid to the first moment of g_1, not many calculations have been made for its x dependence. That is to say, besides the many parametrizations for the polarized parton distributions [16], there are very few predictions based on a genuine model of the nucleon structure. We will

* Cloud means that the connected quark lines form a connected loop.

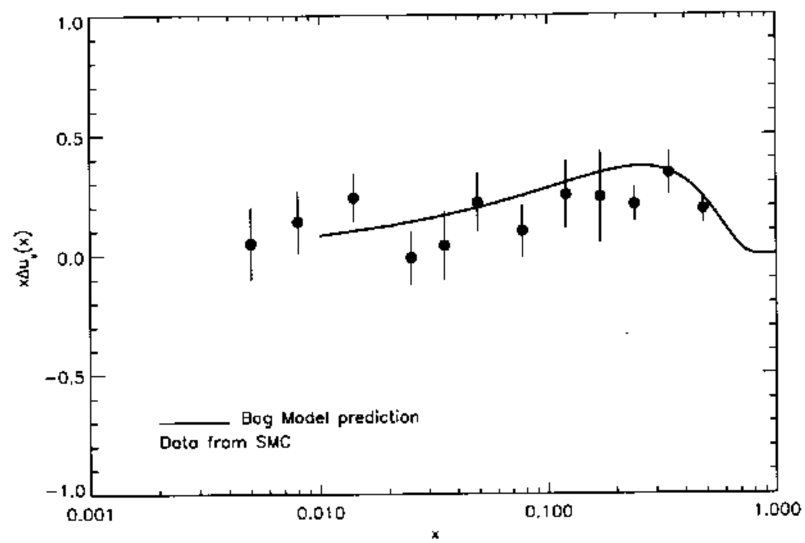


Fig. 1. Polarized valence up-quark data from the SMC against bag model predictions.

show some results from the Adelaide group [34, 35] noticing that there are others around. These calculations involved the use of bag model wave functions as an approximation for the proton wave function. The corresponding leading twist parton distributions were then evolved [35] in leading and next-to-leading order QCD evolution to the scale of the experiments. The resulting agreement between theory and experiment is very rewarding especially because of the simplicity of the model and the fact that all the free parameters were fixed in the unpolarized total valence distribution. Then all the curves for the polarized distributions shown in Ref. [35] are predictions of the model. In Fig. 1 we show one of the predictions of the model against the very recent data from the SMC [2] for the polarized valence distributions. We note also that the calculated value of the Ellis–Jaffe sum rule agrees with usual expectations with its value being $\sim 0 \cdot 17$. In these calculations, the anomaly is not present. If we then follow the work of ref. [25] where the x dependence of the anomalous gluon contribution was calculated, we see that it can affect the x dependence of g_1^p calculated in ref. [35] exactly where it overestimates the data.

Acknowledgments

We would like to thank T. Hatsuda for discussions on the lattice calculations of the spin structure functions. This work was supported by the Australian Research Council and by CAPES (Brazil).

References

[1] R. Voss, Cern Preprint CERN-PPE/95–131, August 1995, invited talk at the Workshop on Deep Inelastic Scattering and QCD, Paris, April 1995.
[2] B. Adeva *et al.* (SMC), Phys. Lett. **B369** (1996) 93.
[3] L. Mankiewicz, E. Stein and A. Schäfer, hep-ph/9510418.
[4] A. I. Signal, Aust. J. Phys. **50** (1997) 61, this issue.

[5] D. Adams *et al.* (SMC), Phys. Lett. **329B** (1994) 399; *erratum*, Phys. Lett. **339B** (1994) 332.
[6] R. L. Jaffe and A. Manohar, Nucl. Phys. **B337** (1990) 509.
[7] S. D. Bass and A. W. Thomas, J. Phys. **G19** (1993) 639.
[8] M. Anselmino, A. Efremov and E. Leader, Phys. Rep. **261** (1995) 1.
[9] K. Abe *et al.* (E143 Collaboration), Phys. Rev. Lett. **74** (1995) 346.
[10] S. J. Brodsky, M. Burkardt and I. Schmidt, Nucl. Phys. **B441** (1995) 197.
[11] R. L. Heimann, Nucl. Phys. **B64** (1973) 429.
[12] J. Ellis and M. Karliner, Phys. Lett. **213B** (1988) 73.
[13] F. E. Close and R. G. Roberts, Phys. Lett. **336B** (1994) 257.
[14] M. Froissart, Phys. Rev. **123** (1961) 1053.
[15] S. D. Bass and P. V. Landshoff, Phys. Lett. **336B** (1994) 537.
[16] M. Glück *et al.*, Phys. Rev. **D53** (1996) 4775; T. Gehrmann and W. J. Stirling, Phys. Rev. **D53** (1996) 6100.
[17] The E143 Collaboration, Phys. Lett. **364B** (1995) 61.
[18] T. Weigl and W. Melnitchouk, Munich Tech. Uni. Preprint TUM-T39–95–9, Sept. 95, Nucl. Phys. **B465** (1996) 267.
[19] D. Adams *et al.* (SMC), Phys. Lett. **357B** (1995) 248.
[20] J. Kodaira, Nucl. Phys. **B165** (1980) 129.
[21] S. L. Adler, Phys. Rev. **177** (1969) 129; J. S. Bell and R. J. Jackiw, Nuovo Cimento **51A** (1969) 47.
[22] G. T. Bodwin and J. Qiu, Phys. Rev. **D41** (1990) 2755.
[23] W. Bardeen, Nucl. Phys. **B75** (1974) 246.
[24] X. Ji, J. Tang and P. Hoodbhoy, MIT Preprint MIT-CPT-2476, hep-ph/9510304.
[25] F. M. Steffens and A. W. Thomas, Phys. Rev. **D53** (1996) 1191.
[26] R. L. Jaffe, Phys. Lett. **B365** (1996) 359.
[27] K.-F. Liu and S.-J. Dong, Proc. Conf. on High Energy Physics, Glacow 1994, Vol. 2, p. 717 and hep-lat/9411067.
[28] E. Nappi *et al.*, Letter of intent for the experiment, Cern preprint CERN/SPSLC 95–27.
[29] R. W. Robinett, Argonne Preprint ANL-HEP-CP-95–28, hep-ph/9506230.
[30] M. Okawa, KEK Preprint KEK-CP-37, July 1995, hep-lat/9510047.
[31] M. Fukugita *et al.*, Phys. Lett. **75** (1995) 2092.
[32] S. J. Dong, J.-F. Lagaë and K. F. Liu, Phys. Lett. **75** (1995) 2096.
[33] T. Hatsuda, private communication.
[34] A. W. Schreiber, A. I. Signal and A. W. Thomas, Phys. Rev. **D44** (1991) 2653.
[35] F. M. Steffens and A. W. Thomas, Prog. Theor. Phys. Suppl. **120** (1995) 145; Phys. Lett. **358B** (1995) 139.