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Monopole Dominance for Nonperturbative QCD*

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Abstract

Monopole dominance for the nonperturbative features in QCD is studied both in the continuum
and the lattice gauge theories. First, we study the dynamical chiral-symmetry breaking
(DχSB) in the dual Higgs theory using the effective potential formalism. We find that the
main driving force for DχSB is brought from the confinement part in the nonperturbative
gluon propagator rather than the short-range part, which means monopole dominance for
DχSB. Second, the correlation between instantons and QCD–monopoles is studied. In the
Polyakov-like gauge, where $A_4(x)$ is diagonalized, the QCD-monopole trajectory penetrates
the centre of each instanton, and becomes complicated in the multi-instanton system. Finally,
using the SU(2) lattice gauge theory with 16^4 and $16^3 \times 4$, the instanton number is measured
in the singular (monopole-dominating) and regular (photon-dominating) sectors, respectively.
Instantons and anti-instantons only exist in the monopole sector both in the maximally abelian
gauge and in the Polyakov gauge, which means monopole dominance for the topological charge.

1. Introduction

Nonabelian gauge theories are reduced to abelian gauge theories with monopoles in
the 't Hooft abelian gauge [1], where a gauge-dependent variable is diagonalized.
The reduced abelian group is the maximal torus subgroup of the original
nonabelian group. For instance, SU(N_c) gauge theory is reduced into U(1)$^{N_c-1}$
gauge theory. Similar to the GUT monopole, the nontrivial homotopy group
$\pi_2(\text{SU}(N_c)/\text{U}(1)^{N_c-1}) = Z_\infty^{N_c-1}$ is the topological origin of the monopole in this
gauge [1, 2, 3].

Recent lattice QCD studies show monopole condensation [4]–[6] in the confinement
phase in the abelian gauge, and strongly support abelian dominance [5]–[9] and
monopole dominance [6]–[11] for the nonperturbative QCD (NP-QCD), e.g. linear
confinement potential, dynamical chiral-symmetry breaking (DχSB) and instantons.
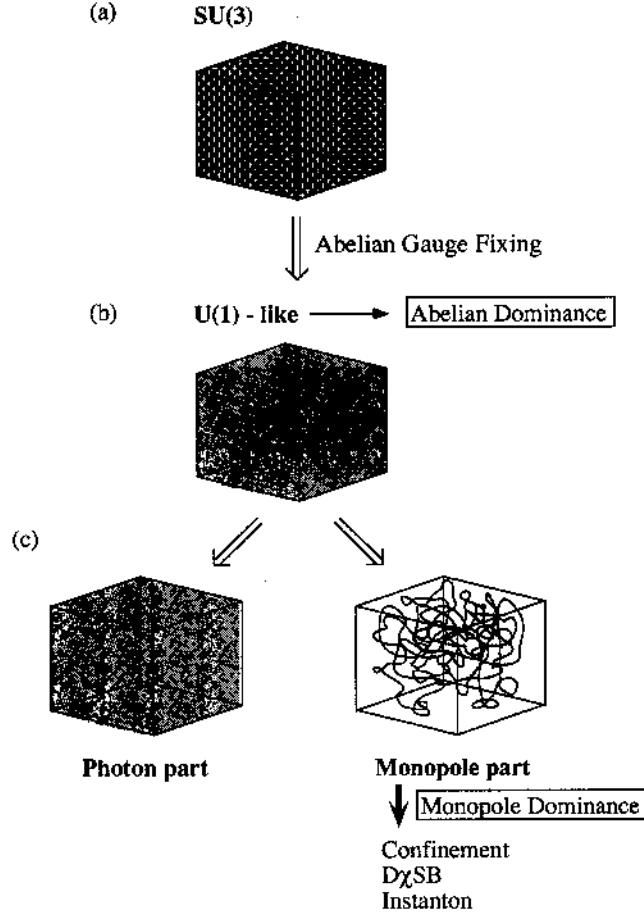Here, abelian dominance [12] means that QCD phenomena are described only by

Fig. 1. Schematic on abelian and monopole dominance in lattice QCD: (*a*) QCD system in $\mathbf{R}^4$ without gauge fixing. (*b*) QCD in the abelian gauge becomes U(1)-like: abelian dominance. A complicated QCD-monopole loop appears in $\mathbf{R}^4$. (*c*) Separation of the U(1) variable into the regular (photon) and singular (monopole) parts. The monopole part leads to NP-QCD (confinement, DχSB, instanton): monopole dominance.

abelian variables in the abelian gauge. Monopole dominance is more strict, and means that the essence of NP-QCD is described only by the singular (monopole) part of abelian variables [6]–[11]. We show in Fig. 1 a schematic figure on abelian and monopole dominance in lattice QCD (see Section 4):

(a) Without gauge fixing, it is very difficult to extract relevant degrees of freedom in NP-QCD.

(b) In the abelian gauge, only U(1) gauge degrees of freedom including monopole is relevant for NP-QCD: abelian dominance. On the other hand, off-diagonal parts scarcely contribute to NP-QCD.

(c) The U(1) variable can be separated into the regular (photon-dominating) and singular (monopole-dominating) parts [6]–[11], [13]. The monopole part leads to NP-QCD (confinement, DχSB, instanton): monopole dominance. On the other hand, the photon part is almost trivial.

Thus, as the modern picture for NP-QCD, its origin is found in the dynamics of condensed monopoles in the 't Hooft abelian gauge. In this paper, we study the role of the condensed monopole to NP-QCD. In Section 2, monopole dominance for DχSB is studied in the dual Higgs theory [2, 3]. In Section 3, we find a strong correlation between instantons and monopoles in an abelian gauge within the analytical argument [3, 10, 11]. In Section 4, monopole dominance for instantons is found using the SU(2) lattice gauge theory [8, 10, 11].

2. Monopole Dominance for Chiral-symmetry Breaking

The dual Ginzburg–Landau theory (DGL) theory [2, 3, 14] is the infrared effective theory of QCD based on the dual Higgs mechanism [15] in the abelian gauge,

$$\mathcal{L}_{\text{DGL}} = \text{tr}\hat{K}_{\text{gauge}}(A_\mu, B_\mu) + \bar{q}(i\ \not{\partial} - e\ \not{A} - m_q)q$$

$$+ \text{tr}[\hat{\mathcal{D}}_\mu, \chi]^\dagger[\hat{\mathcal{D}}^\mu, \chi] - \lambda \text{tr}(\chi^\dagger\chi - v^2)^2, \tag{1}$$

where $\hat{\mathcal{D}}_\mu \equiv \hat{\partial}_\mu + igB_\mu$ is the dual covariant derivative. The dual gauge coupling g obeys the Dirac condition $eg = 4\pi$ [2]. The diagonal gluon A_μ and the dual gauge field B_μ are defined on the Cartan subalgebra $\vec{H} = (T_3, T_8)$: $A^\mu \equiv A_3^\mu T_3 + A_8^\mu T_8$, $B^\mu \equiv B_3^\mu T_3 + B_8^\mu T_8$. The QCD-monopole field χ is defined on the nontrivial root vectors $E_a : \chi \equiv \sqrt{2}\sum_{a=1}^3 \chi_a E_a$. Here $\hat{K}_{\text{gauge}}(A_\mu,\ B_\mu)$ is the kinetic term of gauge fields $(A_\mu,\ B_\mu)$ in the Zwanziger form [16],

$$\hat{K}_{\text{gauge}}(A_\mu, B_\mu) \equiv -[n \cdot (\partial \wedge A)]^\nu [n \cdot^* (\partial \wedge B)]_\nu$$

$$- \tfrac{1}{2}[n \cdot (\partial \wedge A)]^2 - \tfrac{1}{2}[n \cdot (\partial \wedge B)]^2, \tag{2}$$

where the duality of the gauge theory is manifest. The parameters are chosen as $\lambda = 25$, $v = 0 \cdot 126$ GeV, $e = 5 \cdot 5$ so as to reproduce the inter-quark potential and the flux-tube radius $R \simeq 0 \cdot 4$ fm [2].

In the QCD-monopole condensed vacuum, the nonperturbative gluon propagator [2, 3] is derived by integrating out B_μ,

$$D_{\mu\nu}(p) = -\frac{1}{p^2}\left\{g_{\mu\nu} + (\alpha_e - 1)\frac{p_\mu p_\nu}{p^2}\right\}$$

$$+ \frac{1}{p^2}\frac{m_B^2}{p^2 - m_B^2}\frac{1}{(n \cdot p)^2 + a^2}\epsilon^\lambda{}_{\mu\alpha\beta}\epsilon_{\lambda\nu\gamma\delta}n^\alpha n^\gamma p^\beta p^\delta, \tag{3}$$

where the mass of B_μ, $m_B = \sqrt{3}gv$, is proportional to the QCD-monopole condensate v. As the polarization effect of light quarks, the infrared cutoff parameter a corresponding to the hadron size appears in relation with the scalar polarization function $\Pi(p^2)$ for the quark loop diagram: $\Pi(p^2 \simeq 0) = a^2$.

Our group showed the essential role of QCD-monopole condensation to dynamical chiral-symmetry breaking (DχSB) by solving the Schwinger–Dyson (SD) equation [2, 3, 17]. Taking a simple form for the full quark propagator

as $S(p) = 1/\{p - M(p^2) + i\epsilon\}$, one obtains the SD equation for the quark mass $M(p^2)$,

$$M(p^2) = \int \frac{d^4k}{(2\pi)^4} \vec{Q}^2 \frac{M(k^2)}{k^2 + M^2(k^2)} D_{\mu\mu}(k - p), \tag{4}$$

where $D_{\mu\mu}(p)$ has three parts,

$$D_{\mu\mu}(p) = \frac{2}{(n \cdot p)^2 + a^2} \cdot \frac{m_B^2}{p^2 + m_B^2} + \frac{2}{p^2 + m_B^2} + \frac{1 + \alpha_e}{p^2}$$

$$= D_{\mu\mu}^{\mathrm{conf.}}(p) + D_{\mu\mu}^Y(p) + D_{\mu\mu}^C(p). \tag{5}$$

The confinement part $D_{\mu\mu}^{\mathrm{conf.}}(p)$ relates to the linear confinement potential [2, 3] at the quenched level, $a = 0$. The Yukawa part $D_{\mu\mu}^Y(p)$ relates to the short-range Yukawa potential. The Coulomb part $D_{\mu\mu}^C(p)$ does not contribute to the quark static potential. However, it is difficult to separate each contribution in the nonlinear SD equation. Instead, we study DχSB in the DGL theory using the effective potential formalism [18] in order to separate each contribution to the confinement, Yukawa and Coulomb parts energetically. Within the ladder approximation, the effective potential $V_{\mathrm{eff}}[S]$ up to the two-loop diagram leads to the SD equation by imposing the extreme condition on the full quark propagator $S(p)$ [19]. Using the nonperturbative gluon propagator $D^{\mu\nu}(p)$ in the DGL theory, the effective potential, vacuum energy density as a function of the dynamical quark mass $M(p^2)$, is expressed as

$$V_{\mathrm{eff}}[S] = i\mathrm{Tr}\ln(SG^{-1}) + i\mathrm{Tr}(SG^{-1})$$

$$- \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{\vec{Q}^2 e^2}{2} \mathrm{tr}\left(\gamma_\mu S(p)\gamma_\nu S(q) D^{\mu\nu}(p - q)\right), \tag{6}$$

where $G(p)$ is the bare quark propagator, $G^{-1}(p) = \not{p} + i\epsilon$ in the chiral limit. The effective potential corresponding to the SD equation (4) is obtained by

$$V_{\mathrm{eff}}[M(p^2)] = -2N_c N_f \int \frac{d^4p}{(2\pi)^4} \left\{ \ln\left(\frac{p^2 + M^2(p^2)}{p^2}\right) - 2\frac{M^2(p^2)}{p^2 + M^2(p^2)} \right\}$$

$$+ N_f(N_c - 1) \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} e^2 \frac{M(p^2)}{p^2 + M^2(p^2)} \frac{M(q^2)}{q^2 + M^2(q^2)} D_{\mu\mu}(p - q)$$

$$= V_{\mathrm{quark}}(M(p^2)) + V_{\mathrm{conf.}}(M(p^2)) + V_Y(M(p^2)) + V_C(M(p^2)), \tag{7}$$

where the first term is the quark-loop contribution without gauge interaction. The second term with $D_{\mu\mu}$ is the two-loop contribution with the quark–gluon interaction, which is divided into the confinement, Yukawa and Coulomb parts $(V_{\mathrm{conf.}}, V_Y, V_C)$ corresponding to the decomposition of $D_{\mu\mu}$ in Eq. (5).

As for the Dirac-string direction n_μ, we take its average because of the light-quark movement [17], so that the effective potential V_{eff} does not depend on n_μ explicitly. From the renormalization group analysis of QCD [19], the approximate form of quark-mass function $M(p^2)$ is expected as

$$M(p^2) = M(0)\frac{p_c^2}{(p^2 + p_c^2)}\left\{\frac{\ln p_c^2}{\ln(p^2 + p_c^2)}\right\}^{1 - \frac{N_c^2 - 1}{2N_c} \cdot \frac{9}{11N_c - 2N_f}}. \tag{8}$$
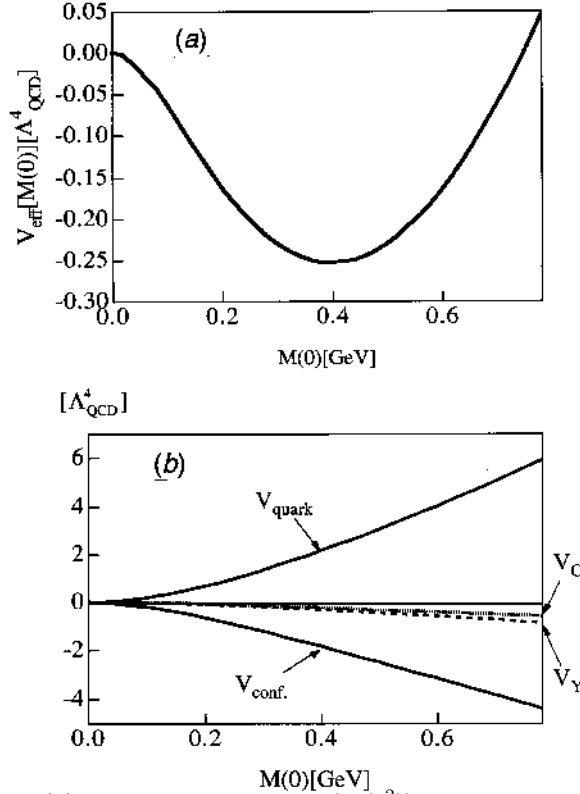

Fig. 2. (a) Effective potential $V_{\mathrm{eff}}(M(p^2))$ as a function of the infrared quark mass $M(0)$ using the quark-mass ansatz (8) with $p_c^2 \simeq 10\Lambda_{\mathrm{QCD}}^2$. (b) The separation of the effective potential: the quark-loop contribution $V_{\mathrm{quark}}(M(p^2))$, the confinement part $V_{\mathrm{conf.}}(M(p^2))$, the Yukawa part $V_Y(M(p^2))$ (dashed curve) and the Coulomb part $V_C(M(p^2))$ (dotted curve).

The exact solution $M_{SD}(p^2)$ of the SD equation (4) [17] is reproduced well by this ansatz (8) with $M(0) \simeq 0\cdot4$ GeV and $p_c^2 \simeq 10\Lambda_{\mathrm{QCD}}^2$. Hence, we use this form of $M(p^2)$ as a variational function in the effective potential formalism.

Fig. 2a shows $V_{\mathrm{eff}}(M(p^2))$ as a function of the infrared quark mass $M(0)$ using the quark-mass ansatz (8) with $p_c^2 \simeq 10\Lambda_{\mathrm{QCD}}^2$. One finds the clear double-well structure in $V_{\mathrm{eff}}(M(p^2))$, which has a nontrivial minimum at $M(0) \simeq 0\cdot4$ GeV. Fig. 2b shows the separated contribution V_{quark}, $V_{\mathrm{conf.}}$, V_Y and V_C: V_{quark} is the quark-loop contribution in Eq. (7); $V_{\mathrm{conf.}}$, V_Y and V_C are the confinement, Yukawa and Coulomb parts in the two-loop contribution in Eq. (7), which correspond to

the three terms in Eq.(5). There is a large cancellation between the quark part V_{quark} and the confinement part $V_{\text{conf.}}$ on DχSB. The effective potential is mainly lowered by $V_{\text{conf.}}$, although V_Y and V_C also contribute to lower it qualitatively. Since the lowering of the effective potential contributes to DχSB, the main driving force of DχSB is brought by the confinement part $D_{\mu\mu}^{\text{conf.}}(p)$ in the nonperturbative gluon propagator, which means monopole dominance for DχSB [7]–[9].
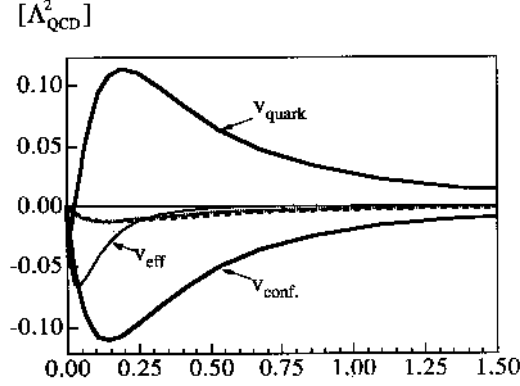

Fig. 3. Integrands $v_{\text{eff}}(p^2)$, $v_{\text{quark}}(p^2)$, $v_{\text{conf.}}(p^2)$, $v_Y(p^2)$ (dashed curve) and $v_C(p^2)$ (dotted curve) for the solution $M_{SD}(p^2)$ of the SD equation.

Finally, we investigate the integrand of the effective potential in the momentum space,

$$V_{\text{eff}}[M(p^2)] = \int_0^\infty dp^2 v_{\text{eff}}(p^2)$$

$$= \int_0^\infty dp^2 [v_{\text{quark}}(p^2) + v_{\text{conf.}}(p^2) + v_Y(p^2) + v_C(p^2)], \qquad (9)$$

where this separation corresponds to V_{quark}, $V_{\text{conf.}}$, V_Y and V_C. We show in Fig. 3 the integrands $v_{\text{eff}}(p^2)$, $v_{\text{quark}}(p^2)$, $v_{\text{conf.}}(p^2)$, $v_Y(p^2)$ and $v_C(p^2)$ for the solution $M_{SD}(p^2)$ of the SD equation. The confinement part $v_{\text{conf.}}(p^2)$ is dominant at any momentum p^2 in comparison with the Yukawa and Coulomb parts ($v_Y(p^2)$, $v_C(p^2)$). The low-energy component less than 1 GeV contributes to DχSB through the lowering of the effective potential.

3. Analytical Study on Instanton and QCD–Monopole

The instanton is another important topological object in the nonabelian gauge theory; $\pi_3(\text{SU}(N_c)) = Z_\infty$ [20]. Recent lattice studies [5]–[11] indicate abelian dominance for the nonperturbative quantities in the maximally abelian (MA) gauge and in the Polyakov gauge. If the system is completely described only by the abelian field, the instanton would lose the topological basis for its existence, and therefore it seems unable to survive in the abelian manifold. However, even in the abelian gauge, nonabelian components remain relatively large around the QCD-monopoles, which are nothing but the topological defects, so that instantons are expected to survive only around the QCD-monopole trajectories in the abelian-dominant system.

We examine such a close relation between instantons and QCD-monopoles in the continuum SU(2) gauge theory [3, 10, 11, 21]. We adopt the Polyakov-like gauge, where $A_4(x)$ is diagonalized, as an abelian gauge. Using the 't Hooft symbol $\overline{\eta}^{a\mu\nu}$, the multi-instanton solution is written as [20] $A^\mu(x) = i\overline{\eta}^{a\mu\nu}\frac{\tau^a}{2}\partial^\nu \ln(1+\sum_k a_k^2/|x-x_k|^2)$, where $x_k^\mu \equiv (\mathbf{x}_k, t_k)$ and a_k denote the centre coordinate and the size of kth instanton, respectively. Near the centre of the kth instanton, $A_4(x)$ takes a hedgehog configuration, $A_4(x) \simeq i\tau^a(\mathbf{x} - \mathbf{x}_k)^a/|x - x_k|^2$. In the Polyakov-like gauge, $A_4(x)$ is diagonalized by a singular gauge transformation [2, 11, 21], which leads to the QCD-monopole trajectory on $A_4(x) = 0$: $\mathbf{x} \simeq \mathbf{x}_k$. Hence, the QCD-monopole trajectory penetrates each instanton centre along the temporal direction. In other words, instantons only live along the QCD-monopole trajectory.
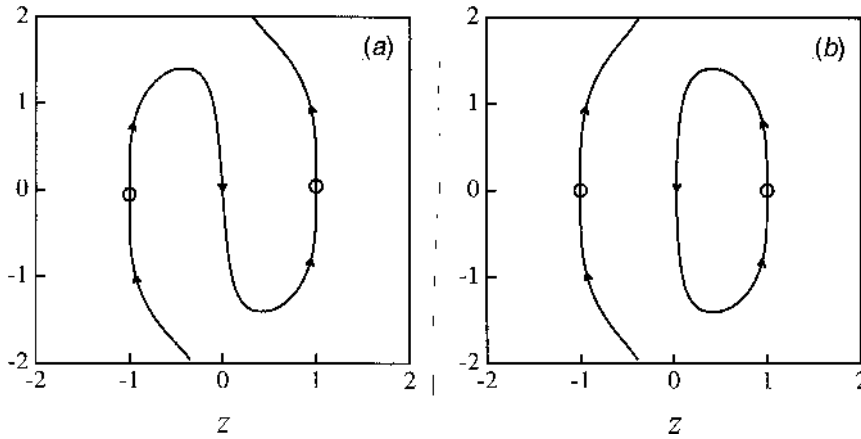

Fig. 4. Examples of the QCD-monopole trajectory in the two-instanton system with (a) $(z_1, t_1) = -(z_2, t_2) = (1, 0 \cdot 05)$, $a_1 = a_2$ and (b) $(z_1, t_1) = -(z_2, t_2) = (1, 0), a_2 = 1 \cdot 1 a_1$.

For the single-instanton system [3, 10, 11, 21, 22], the QCD-monopole trajectory $x^\mu \equiv (\mathbf{x}, t)$ is simply given by $\mathbf{x} = \mathbf{x}_1$ $(-\infty < t < \infty)$ at the classical level. For the two-instanton system, two instanton centres can be located on the zt plane without loss of generality: $x_1 = y_1 = x_2 = y_2 = 0$. Owing to the symmetry of the system, QCD monopoles only appear on the zt plane, and hence one has only to examine $A_4(x)$ on the zt plane $(x = y = 0)$. In this case, $A_4(x)$ is already diagonalized on the zt plane: $A_4(x) = A_4^3(z,t)\tau^3$, and therefore the QCD-monopole trajectory $x^\mu = (x, y, z, t)$ is simply given by $A_4^3(z,t) = 0$ and $x = y = 0$. Generally, the QCD-monopole trajectory is rather complicated even at the classical level in the two-instanton system [3, 10, 11, 21, 23]: it has a loop or a folded structure as shown in Figs 4a or 4b. It is remarkable that the QCD-monopole trajectory originating from instantons is very unstable against a small fluctuation of the location or the size of instantons [10, 11, 21].

The QCD-monopole trajectory tends to be highly complicated and unstable in the multi-instanton system even at the classical level, and the topology of the trajectory is often changed due to a small fluctuation of instantons [10, 11, 21, 24]. Hence, quantum fluctuations would make it more complicated and more unstable, which leads to the appearance of a long complicated trajectory as a result. Thus, instantons may contribute to promote monopole condensation, which is signaled by a long complicated monopole loop in the lattice QCD simulation [5, 6].

We study also the thermal instanton system in the Polyakov-like gauge [11, 21]. At high temperature, the QCD-monopole trajectory is reduced to simple straight lines in the temporal direction, which may correspond to the deconfinement phase transition through the vanishing of QCD-monopole condensation [25]. For the thermal two-instanton system, the topology of the QCD-monopole trajectory is drastically changed at $T_c \simeq 0 \cdot 6 d^{-1}$, where d is the distance between the two instantons [11, 21]. If one adopts $d \sim 1$ fm as a typical mean distance between instantons [26], such a topological change occurs at $T_c \sim 120$ MeV [11, 21].

4. Instanton and Monopole on a Lattice

We study the correlation between instantons and QCD-monopoles in the maximally abelian (MA) gauge and in the Polyakov gauge using the SU(2) lattice with 16^4 and $\beta = 2 \cdot 4$ [8, 10, 11, 27]. All measurements are done every 500 sweeps after a thermalization of 1000 sweeps using the heat-bath algorithm. After generating the gauge configurations, we examine monopole dominance [6]–[11] for the topological charge using the following procedure:

(1) We adopt the MA gauge and the Polyakov gauge as typical examples of the 't Hooft abelian gauge. The MA gauge is carried out by maximizing $R = \sum_{\mu,s} \text{tr}\{U_\mu(s)\tau_3 U_\mu^{-1}(s)\tau_3\}$. The Polyakov gauge is obtained by diagonalizing the Polyakov loop $P(s)$.

(2) The SU(2) link variable $U_\mu(s)$ is factorized as $U_\mu(s) = M_\mu(s)u_\mu(s)$ with the abelian link variable $u_\mu(s) = \exp\{i\tau_3\theta_\mu(s)\}$ and the 'off-diagonal' factor $M_\mu(s) \equiv \exp\{i\tau^1 C_\mu^1(s) + i\tau^2 C_\mu^2(s)\}$. Under the residual U(1)-gauge transformation, $u_\mu(s)$ behaves as a gauge field, while $M_\mu(s)$ behaves as a charged matter field.

(3) The abelian field strength $\theta_{\mu\nu} \equiv \partial_\mu\theta_\nu - \partial_\nu\theta_\mu$ is decomposed as $\theta_{\mu\nu}(s) = \bar{\theta}_{\mu\nu}(s) + 2\pi M_{\mu\nu}(s)$ with $-\pi \leq \bar{\theta}_{\mu\nu}(s) < \pi$ and $M_{\mu\nu}(s)\epsilon \mathbf{Z}$ [13]. Here, $\bar{\theta}_{\mu\nu}(s)$ and $2\pi M_{\mu\nu}(s)$ correspond to the regular part and the Dirac-string part, respectively

(4) Using the lattice Coulomb propagator in the Landau gauge [4, 13], the U(1) gauge variable $\theta_\mu(s)$ is decomposed as $\theta_\mu(s) = \theta_\mu^{Ds}(s) + \theta_\mu^{Ph}(s)$ with a singular part $\theta_\mu^{Ds}(s)$ and a regular part $\theta_\mu^{Ph}(s)$, which are obtained from $2\pi M_{\mu\nu}(s)$ and $\bar{\theta}_{\mu\nu}(s)$, respectively. The singular part carries almost the same amount of magnetic current as the original U(1) field, whereas it scarcely carries the electric current. The situation is just the opposite in the regular part. For this reason, we regard the singular part as 'monopole-dominating', and the regular part as 'photon-dominating' [8, 10, 11].

(5) The corresponding SU(2) variables are reconstructed from $\theta_\mu^{Ds}(s)$ and $\theta_\mu^{Ph}(s)$ by multiplying the off-diagonal factor $M_\mu(s)$: $U_\mu^{Ds}(s) = M_\mu(s)\exp\{i\tau_3\theta_\mu^{Ds}(s)\}$ and $U_\mu^{Ph}(s) = M_\mu(s)\exp\{i\tau_3\theta_\mu^{Ph}(s)\}$.

(6) The topological charge $Q = (1/16\pi^2)\int d^4x \text{Tr}(G_{\mu\nu}\tilde{G}_{\mu\nu})$, the integral of the absolute value of the topological density $I_Q \equiv (1/16\pi^2)\int d^4x|\text{Tr}(G_{\mu\nu}\tilde{G}_{\mu\nu})|$, and the action divided by 8π, $S \equiv (1/16\pi^2)\int d^4x \text{Tr}(G_{\mu\nu}G_{\mu\nu})$ are calculated by using $U_\mu(s)$, $U_\mu^{Ds}(s)$ and $U_\mu^{Ph}(s)$. Then, three sets of quantities are obtained; $\{Q(\text{SU}(2)), I_Q(\text{SU}(2)), S(\text{SU}(2))\}$ for the full SU(2) variable, $\{Q(\text{Ds}), I_Q(\text{Ds}), S(\text{Ds})\}$ for the singular part, and $\{Q(\text{Ph}), I_Q(\text{Ph}), S(\text{Ph})\}$ for the regular part. Here, I_Q has been introduced to get information on the instanton and anti-instanton pair.

(7) The correlations among these quantities are examined using the Cabibbo–Marinari cooling method (the heat-bath algorithm with $\beta \to \infty$).

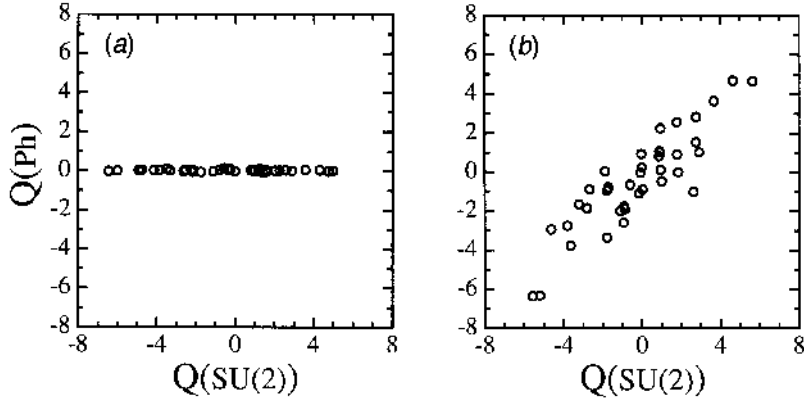**Fig. 5.** Correlations between (*a*) Q(Ds) and Q(SU(2)) at 80 cooling sweeps; (*b*) Q(Ph) and Q(SU(2)) at 10 cooling sweeps.
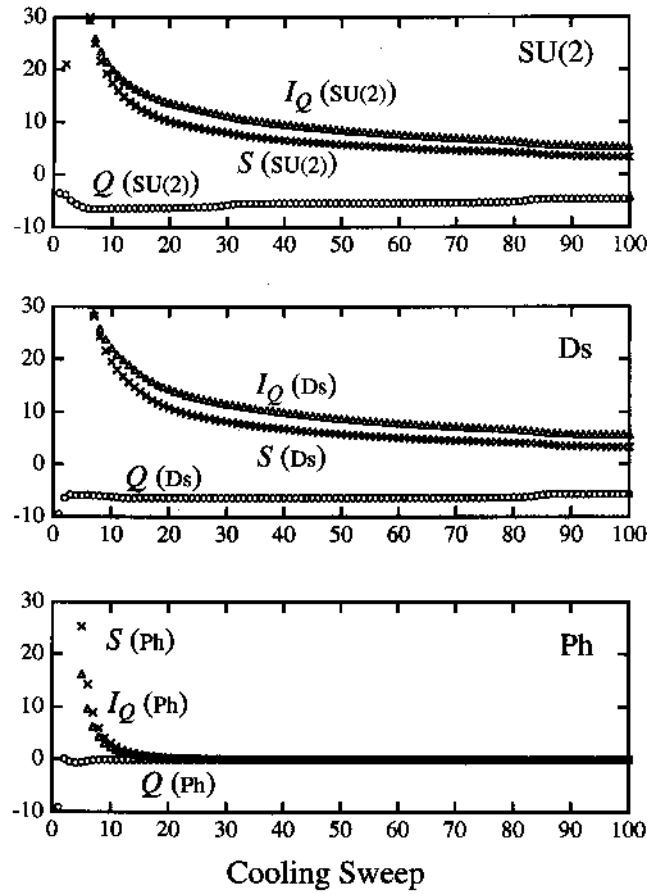
Fig. 6. Typical cooling curves for (*a*) Q(SU(2)), I_Q(SU(2)), S(SU(2)); (*b*) Q(Ds), I_Q(Ds), S(Ds); (*c*) Q(Ph), I_Q(Ph), S(Ph).

We prepared 40 samples for both the MA and the Polyakov gauge. Since quite similar results have been obtained in the MA gauge [8] and the Polyakov gauge, only the latter case is shown.

Fig. 5 shows the correlation among $Q(\mathrm{SU}(2))$, $Q(\mathrm{Ds})$ and $Q(\mathrm{Ph})$ after some cooling sweeps in the Polyakov gauge. A strong correlation is found between $Q(\mathrm{SU}(2))$ and $Q(\mathrm{Ds})$, which is defined in the singular (monopole) part. Such a strong correlation remains even at 80 cooling sweeps. On the other hand, $Q(\mathrm{Ph})$ quickly vanishes only by several cooling sweeps, and no correlation is seen between $Q(\mathrm{Ph})$ and $Q(\mathrm{SU}(2))$.

We show in Fig. 6 the cooling curves for Q, I_Q and S in a typical example in the Polyakov gauge. Similar to the full SU(2) case, $Q(\mathrm{Ds})$, $I_Q(\mathrm{Ds})$ and $S(\mathrm{Ds})$ in the singular (monopole) part remain finite during the cooling process. On the other hand, $Q(\mathrm{Ph})$, $I_Q(\mathrm{Ph})$ and $S(\mathrm{Ph})$ in the regular part quickly vanish by only less than 10 cooling sweeps. Therefore, instantons seem unable to live in the regular (photon) part, but only survive in the singular (monopole) part in the abelian gauges. In particular, finiteness of $I_Q(\mathrm{Ds})$ indicates the existence of the instanton and anti-instanton pair in the singular part, while vanishing of $I_Q(\mathrm{Ph})$ indicates the absence of such a topological pair excitation in the regular part.

Thus, monopole dominance for the topological charge is found in the 't Hooft abelian gauge. In particular, instantons would survive only in the singular (monopole-dominating) part in the abelian gauges [8, 10, 11], which agrees with the result in our previous analytical study. Monopole dominance for the $U_A(1)$ anomaly [28]–[31] is also expected.

Finally, we study also the finite-temperature system using the $16^3 \times 4$ lattice with various β around $\beta_c \simeq 2 \cdot 3$ [32]. Monopole dominance for the instanton is found also in the finite-temperature confinement phase. Near the critical temperature $\beta_c \simeq 2 \cdot 3$, Q and I_Q rapidly decrease, which means a reduction in the number of instantons and anti-instantons. Instantons vanish as well as QCD-monopole condensation in the deconfinement phase ($\beta > \beta_c$). Hence, the instanton configuration is expected to surive only around the condensed QCD-monopole trajectories [32].

Acknowledgments

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