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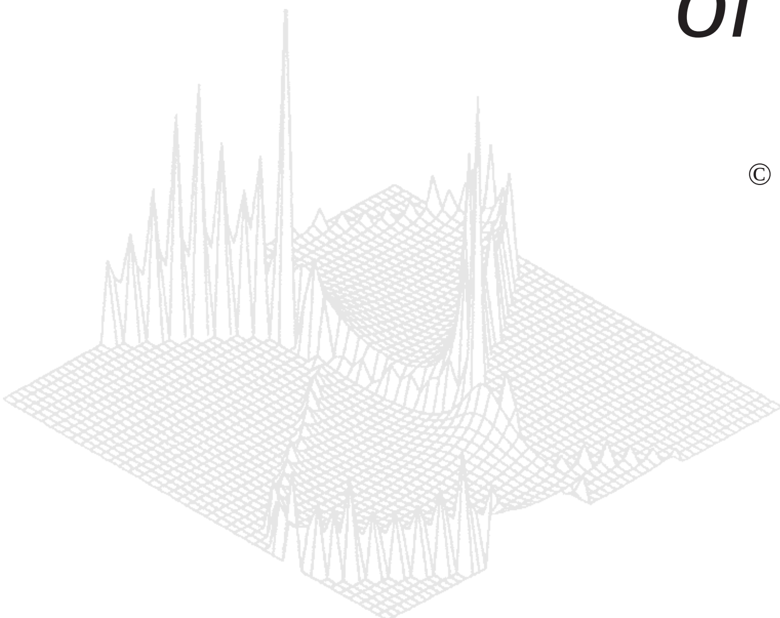

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Nuclear Correlation Effects in the Deep Inelastic Structure Function of ^{4}He*

Hiko Morita and Toru Suzuki[A]

Sapporo-Gakuin University, Ebetsu, Hokkaido 069, Japan.
[A] Tokyo Metropolitan University, Hachioji, Tokyo 192–03, Japan.

Abstract

The structure function of ^{4}He is calculated based on a realistic spectral function which is consistent with the momentum sum rule. The calculation reproduces the empirical suppression of the EMC ratio $R(x) = F_2(^4\mathrm{He})/F_2(\mathrm{D})$ at $x < 1$. At $x \sim 1$ the quasielastic contribution produces a sizable effect in the ratio, but it becomes negligible around $Q^2 \sim 20$ (GeV/c)^2.

1. Introduction

Deep inelastic lepton–nucleus scattering (DIS) provides information on quark distributions in nuclei. The deviation of nuclear structure functions measured in DIS from the nucleon structure function in the region $x < 1$ of the Bjorken variable (EMC effect [1]) has been studied in detail in a number of publications [2]. It is not clear yet, however, whether this is a genuine subhadronic effect which requires a drastic change in the nucleon structure inside nuclei or can be absorbed into the nuclear many-body scheme, e.g. the nuclear binding effect or the nucleon–nucleon correlation effect. The correlation effect at short distances, in particular, is a notoriously difficult but still important subject which has a long history of research. Nuclear structure functions in the region beyond $x = 1$ are expected to provide a clue to unravel the complicated correlation effect. A detailed study of this region will soon become available thanks to high-intensity continuous electron beams at several facilities.

A direct calculation of quark distributions in nuclei including nuclear correlations will be a desirable framework to study nuclear structure functions [3]. At the moment the quark correlations among nucleons inside nuclei seem highly model-dependent, and it would require detailed studies of nucleon correlations to give a constraint on the model building. It seems therefore worth while to investigate closely the nuclear structure functions based on the nuclear viewpoint as precisely as possible. The nucleus ^{4}He provides a suitable testing ground for this purpose, since a detailed wave function is available on the one hand, while on the other the nucleon density is high enough that the nuclear effect becomes apparent.

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It is the purpose of the present contribution to calculate the structure function of ^{4}He at low as well as the larger-x region using a spectral function based on the sophisticated nuclear wave function.

2. Spectral Function of ^{4}He

Lepton–nucleus scattering cross section at high energy is closely related to the nucleon spectral function in nuclei. For ^{4}He it is defined as follows:

$$S(\mathbf{k}, E) = \sum_f |\phi_{fi}(\mathbf{k})|^2 \delta(E - (E_f^{3N} - E_i^{4N}))\,, \tag{1}$$

where the overlap function ϕ_{fi} is defined by

$$\phi_{fi}(\mathbf{k}) = \sqrt{4}\langle \Psi_f^{3N} \Phi_{\mathbf{k}} | \Psi_i^{4N} \rangle, \quad \Phi_{\mathbf{k}}(\mathbf{R}) = (2\pi)^{-3/2} \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{R})\,. \tag{2}$$

Here Ψ_f^{3N} and Ψ_i^{4N} denote the final three-body and the initial ^{4}He wave functions, while E_f^{3N} and E_i^{4N} are the corresponding internal energies. According to the number of fragments, the final four-nucleon wave function given by the product $\Psi_f^{3N}\Phi_{\mathbf{k}}$ is classified into the two-body ($p+t$ or $n+^3$He), three-body ($p+n+d$), and four-body ($p+p+n+n$) breakup channels which are denoted as $2B$, $3B$ and $4B$, respectively. For the ground state wave functions of the three-nucleon system and of ^{4}He we applied the ATMS method [4] with the Reid soft-core V_8 model potential which is known to produce a realistic description of few-body systems. As for the wave function Ψ_f^{3N} in the $3B$ and $4B$ channels where the three-nucleons are in the continuum, we basically use the plane-wave (PW) approximation. This approximation is expected to become good at high momentum region where the NN-correlation effect is important. Fig. 1 shows the three-dimensional view of the spectral function $S(\mathbf{k}, E)$ calculated in the PW approximation. If one compares the result with that of a simple $(1s)^4$ oscillator wave function, it becomes clear that the nucleon correlation effect produces a large enhancement of the spectral function at high momentum ($k = |\mathbf{k}| > 2$ fm^{-1}) as well as at large removal energy ($E > 150$ MeV).

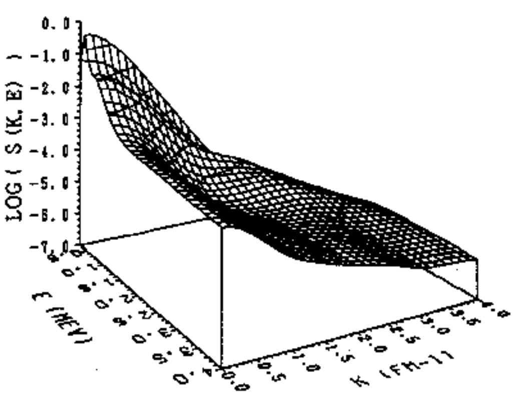

Fig. 1. Spectral function $S(\mathbf{k}, E)$ plotted in the $k(=|\mathbf{k}|) - E$ plane. The part $S_{2B}(\mathbf{k}, E)$ which gives a delta-function contribution at small E is not shown.

The simple PW approximation, however, does not satisfy the momentum sum rule for the spectral function, i.e.

$$\frac{1}{4}\int dES(\mathbf{k},E) = W_{2B}(\mathbf{k}) + W_{3B}(\mathbf{k}) + W_{4B}(\mathbf{k}) = W^{SN}(\mathbf{k})\,, \tag{3}$$

where W_{nB} denotes an n-body contribution to the momentum distribution function of the ^{4}He ground state calculated from the spectral function, while W^{SN} is the momentum distribution calculated directly from the ground state wave function. (The function W^{SN} is normalized in such a way that the momentum integral of W gives unity.) The discrepancy in the PW approximation comes from the non-orthogonality among three-nucleon final states Ψ_f^{3N}. To remedy this we make a projection on the three-nucleon continuum states so that they become orthogonal to the three-nucleon ground state [5]. We then are left with a discrepancy of $\sim$14%, which is used to uniformly renormalize the spectral function. The sum-rule consistent spectral function thus obtained reproduces nicely the k-dependence of the momentum distribution function $W^{SN}(\mathbf{k})$. Fig. 2 shows the n-body breakup contribution to the momentum distribution function. One may notice that the $2B$-contribution accounts for most of the momentum distribution at low k, while the $4B$ contribution becomes dominant at high momentum ($k > 2$ fm^{-1}) region.

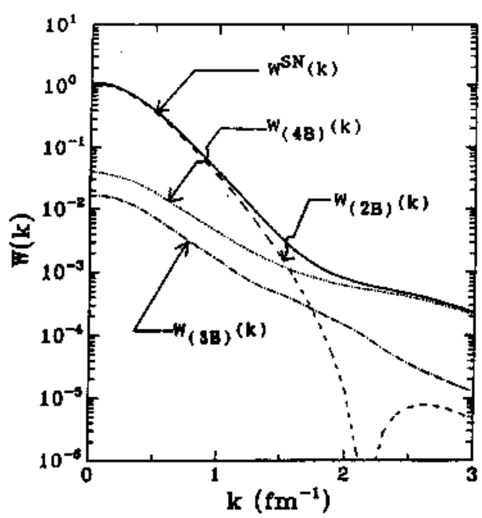

Fig. 2. The n-body breakup contributions to the momentum distribution functions $W_{nB}(\mathbf{k})$ shown together with the momentum distribution $W^{SN}(\mathbf{k})$ calculated directly from the ^{4}He ground state wave function. The sum $W_{2B} + W_{3B} + W_{4B}$ (not shown) is almost identical to W^{SN} up to $k \sim 3$ fm^{-1}.

3. Structure Function of ^{4}He

To construct a nuclear structure function we adopt the (single) nucleon convolution model, assuming that the nucleon cluster contribution is not important except close to the kinematical limit $x \simeq N$ for an N-body cluster. The effect of virtual mesons is not included, although it may be important in the small x region. Under this assumption the nuclear structure function W_2^A can be obtained (see e.g. [6]) as

$$W_2^A(x,Q^2) = \sum_{p,n}\int d\mathbf{k}\int dES(\mathbf{k},E)[C_1(\mathbf{k},\mathbf{q})W_1^N + C_2(\mathbf{k},\mathbf{q})W_2^N]\,, \tag{4}$$

where C_1, C_2 denote kinematical factors and W_1^N, W_2^N are nucleon structure functions. Empirical structure function data at x sufficiently smaller than 1 have been measured at large four-momentum transfer squared $Q^2 = -q^2$. In this region one may take the large Q^2 limit of the C and obtain a simpler form

$$F_2^A(x, Q^2) = \int_{z \le x} dz \left\{ \frac{Z}{A} f_A^p(z) F_2^p\left(\frac{x}{z}, Q^2\right) + \frac{N}{A} f_A^n(z) F_2^n\left(\frac{x}{z}, Q^2\right) \right\}, \qquad (5)$$

where Z (N) is the proton (neutron) number and $F_2^{p(n)}$ is the nucleon structure function. The light-cone momentum distribution functions are defined by

$$f_A^{p(n)}(z) = \int d^4k \;\; P_A(k) \;\; z\delta\left(z - \frac{M_A}{M} \cdot \frac{kq}{Pq}\right), \qquad (6)$$

where M_A and M are respectively the masses of the nucleus and of the nucleon, P is the nuclear four momentum, and $P_A(k)$ is the four momentum distribution of the nucleon in the nucleus. The delta-function implies that $z \sim (k_0 - k_{/\!/})/M$ in the Bjorken limit, where $k_{/\!/}$ is the component of $\mathbf{k}$ in the direction of $\mathbf{q}$. The flux factor z has been included in order to ensure relativistic normalization of f_A. In the nonrelativistic limit the four momentum distribution function is expressed in terms of the nuclear spectral function $S(\mathbf{k}, E)$ as

$$P_A(k) = S(\mathbf{k}, E)[1 + O(\mathbf{k}^2/M^2)]\,. \qquad (7)$$

In the calculation below we retained terms up to order $\mathbf{k}^2/M^2$ [7]. As shown in the previous section the spectral function consists of three components according to the number of fragments in the final states, i.e. $S = S_{2B} + S_{3B} + S_{4B}$.

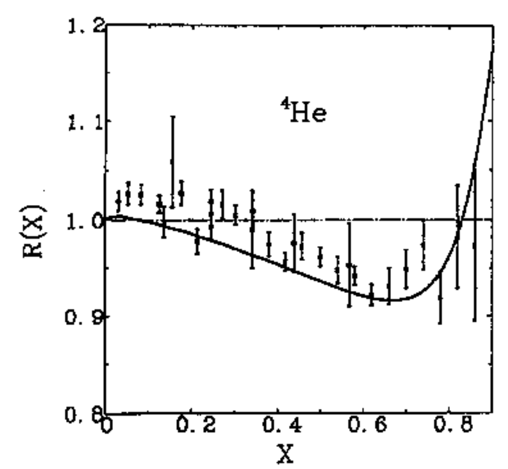

Fig. 3. Structure function ratio $R(x)$ of $F_2(x;\,^4\mathrm{He})$ to $F_2(x;\mathrm{D})$ at $x \le 1$ and $Q^2 = 4$ $(\mathrm{GeV}/c)^2$. Data are taken from refs [1, 8].

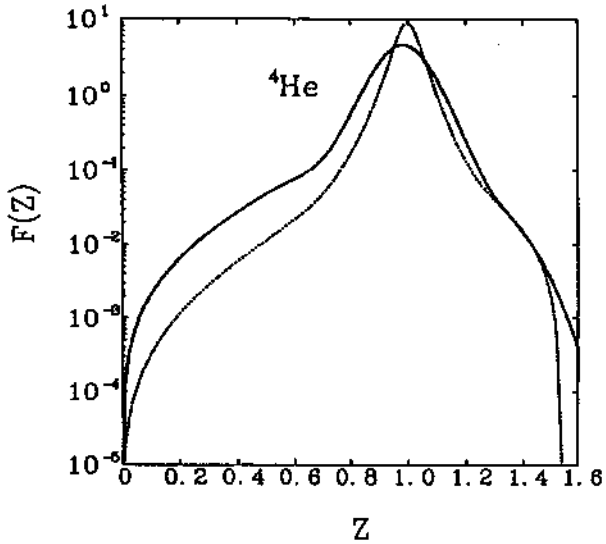

Fig. 4. Light-cone momentum distribution function $f(z)$ for ^{4}He (solid line) and for the deuteron (dotted curve).

Fig. 3 shows the calculated structure function ratio

$$R(x) = F_2(x; {}^4\mathrm{He})/F_2(x; \mathrm{D}) \tag{8}$$

for $x < 1$ at $Q^2 = 4$ $(\mathrm{GeV}/c)^2$. The structure function for the deuteron is obtained also from eq. (5) using a realistic wave function based on the same NN interaction. The experimental data have been taken from refs [1, 8]. The nucleon structure functions were obtained from the parametrization in ref. [9]. Calculations at $Q^2 = 20$ $(\mathrm{GeV}/c)^2$ give similar results.

Fig. 3 shows that the experimental ratio $R(x)$ is reproduced rather well in the region $0 \cdot 3 \leq x \leq 0 \cdot 8$. The difference in the structure functions of ^{4}He and of the deuteron comes from the difference in the momentum distribution function $f(z)$ shown in Fig. 4. One can see that the distribution function for ^{4}He is broader than that for the deuteron reflecting a more compact structure of the former. In particular, the shoulder at $|z - 1| \geq 0 \cdot 3$ reflects the effect of short-range correlations in nuclei. This effect is known to enhance the high-momentum and high removal energy components of the spectral function [5]. Thus the correlation effect (including binding effect) is sufficient to explain the degrading of the quark light-cone momentum distribution in ^{4}He.

We now consider the region around $x \simeq 1$. It is now important to take the quasielastic contribution into account. Here the term quasi-elastic means that the incident lepton scatters off nucleons elastically in the nucleus. Since the process involves elastic form factors of the nucleon, it is expected that this contribution would drop rapidly as Q^2 becomes larger. For this reason we use the expression (4) in order to retain all the Q^2 dependence in the convoluted structure function. It should be noted here that the integrand of the right-hand side of eq. (4) is actually dependent on the off-shell prescription of the nucleon contributions.

We take the extrapolation suggested by de Forest [10]. Fig. 5 shows the Q^2 dependence of the ^{4}He structure function. The quasielastic contribution shown by dashed curves is seen to drop rapidly with increasing Q^2, which is in contrast to the inelastic contribution. At $Q^2 = 15$ (GeV/c)^2 one can safely neglect the quasi-elastic contribution. We made a similar calculation for the deuteron structure function to obtain the ratio $R(x)$. Here the quasi-elastic contribution is still visible at $Q^2 = 15$ (GeV/c)^2 around $x = 1$. This is due to the rapid fall-off of the inelastic contribution of the structure function around $x = 1$, because of the small high momentum component in the deuteron. Our calculation suggests that the quasi-elastic contribution can be neglected at around $Q^2 = 20$ (GeV/c)^2 even for the deuteron.

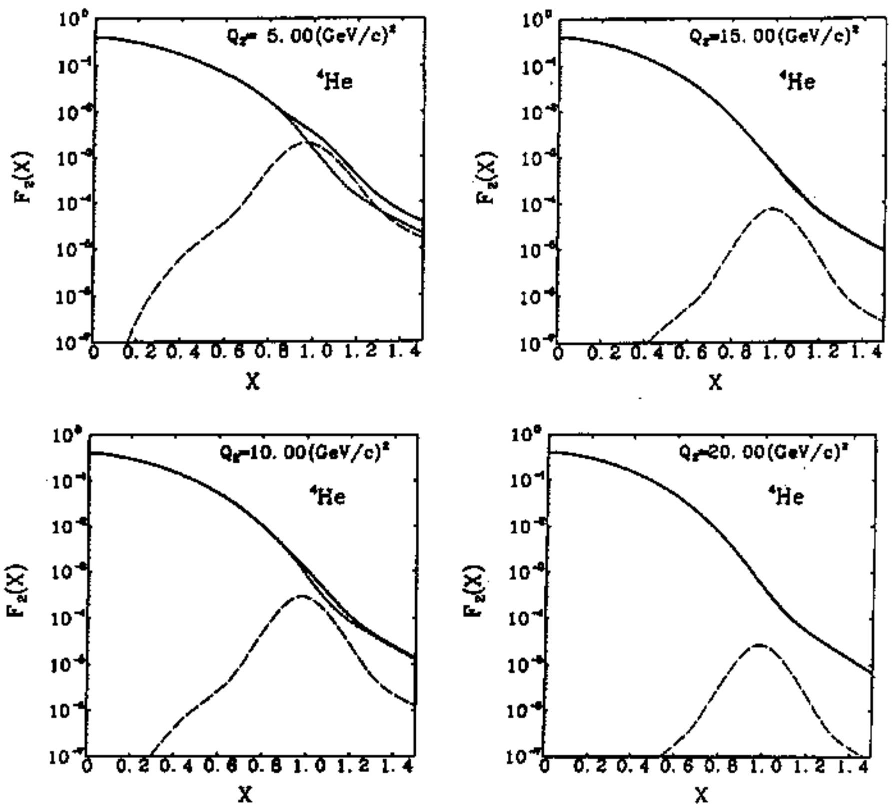

Fig. 5. Structure function F_2 of ^{4}He at four values of Q^2 calculated from the expression (4) where the full Q^2 dependence is included. The quasi-elastic contribution is shown by dashed curves, and the inelastic one by dotted curves. Solid curves show the sum of the two contributions.

Fig. 6 shows the ratio $R(x) = F_2(^4\mathrm{He})/F_2(\mathrm{D})$ up to $x = 1{\cdot}5$ at $Q^2 = 20$ (GeV/c)^2. The ratio shows a rapid rise around $x \simeq 1$. This behaviour originates from the strong x-dependence of different subprocesses of the spectral function as shown in the same figure. Here the denominator of the ratio is always fixed (total structure function F_2 of the deuteron) for these subprocess contributions.

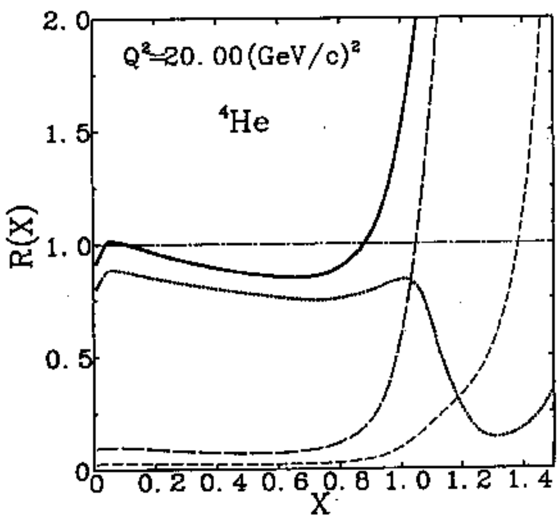

Fig. 6. Ratio $R(x)$ of the structure function up to $x = 1 \cdot 5$ (solid curve) at $Q^2 = 20$ $(\mathrm{GeV}/c)^2$. Contributions from different subprocesses for ^{4}He (S_{nB} for final n-body breakup) are also shown: $2B$ (dotted), $3B$ (short-dashed) and $4B$ (long-dashed). In each contribution, the denominator of the ratio is fixed to be $F_2(x; \mathrm{D})$.

The sharp drop of the two-body contributions (S_{2B}) at $x \geq 1$ is explained by the fact that there is only an S-wave between the final t and p of the $2B$ part of the spectral function of ^{4}He, while in the deuteron the D-wave contribution becomes dominant. The four-body contribution in ^{4}He however dominates the latter giving rise to the large value of the ratio. The transient pattern from $2B$ to $4B$ dominance in the ratio $R(x)$ is sensitively dependent on the spectral function, however. If one uses a model spectral function employed in ref. [6], for instance, one obtains a different behaviour of the ratio against x. Dependence on Q^2 around $x \sim 1$ is more complicated because of the quasielastic contribution which is important at lower Q^2. This in turn suggests that the measurement of the structure function in this region may provide a sensitive probe of the correlation effects in nuclei.

4. Summary

We performed a calculation of the structure function of ^{4}He using the nucleon convolution model together with realistic spectral function which is consistent with the momentum sum rule. The spectral function is dominated by the four-body breakup ($p + p + n + n$) contribution at high momentum ($k > 2$ fm^{-1}) as well as at large removal energy ($E > 150$ MeV). We studied the ratio $R(x)$ of the ^{4}He structure function to that of the deuteron. In the $x < 1$ region the nucleon correlation effect contained in the spectral function accounts for the suppression of $R(x)$ seen in the empirical data. At larger values of the Bjorken variable, i.e. around $x \sim 1$, the ratio may show a rather complicated behaviour as a function of x because the high momentum/energy contribution becomes dominant. The Q^2 dependence is not simple because of the quasielastic contributions in ^{4}He as well as in the deuteron. Our calculation suggests that at $Q^2 = 20$ $(\mathrm{GeV}/c)^2$ one may safely neglect the quasielastic contribution to the structure function.

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