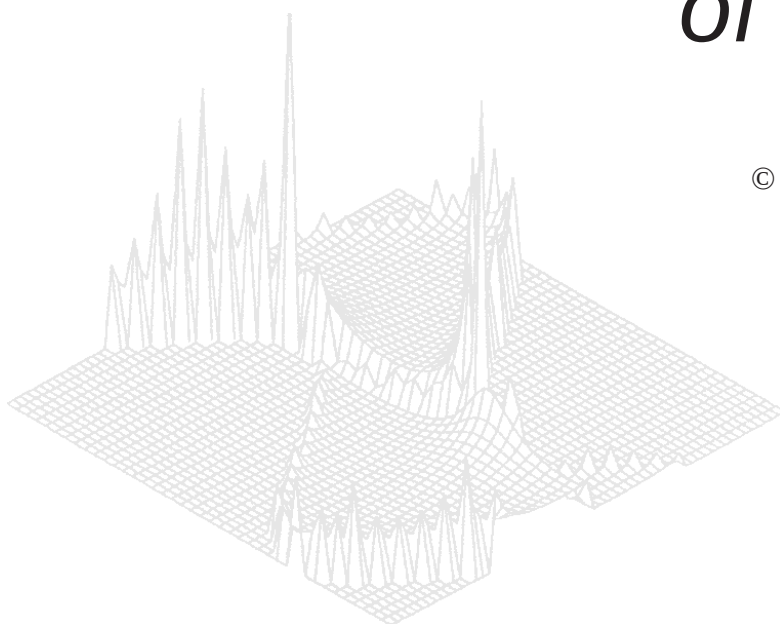

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η -Meson Decays and Strong $U_A(1)$ Breaking in the Three-flavour Nambu–Jona-Lasinio Model*

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Abstract

We study the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^0\gamma\gamma$ decays using an extended three-flavour Nambu–Jona-Lasinio model that includes the 't Hooft instanton induced interaction. We find that the η -meson mass, the $\eta \rightarrow \gamma\gamma$ decay width and the $\eta \rightarrow \pi^0\gamma\gamma$ decay width are in good agreement with the experimental values when the $U_A(1)$ breaking is strong and the flavour $SU(3)$ singlet-octet mixing angle θ is about zero. The effects of the $U_A(1)$ breaking on the baryon number one and two systems are also studied.

1. Introduction

It is well known that the QCD action has an approximate $U_L(3) \times U_R(3)$ chiral symmetry and its subsymmetry, $U_A(1)$ symmetry, is explicitly broken by the anomaly. The $U_A(1)$ symmetry breaking is manifested in the heavy mass of the η' meson.

The physics of the η and η' mesons have been extensively studied in the $1/N_C$ expansion approach [1]. In the $N_C \rightarrow \infty$ limit, the $U_A(1)$ anomaly is turned off and then the η meson becomes degenerate with the pion and the η' meson becomes a pure $\bar{s}s$ state with $m_{\eta'}^2(N_C \rightarrow \infty) = 2m_K^2 - m^2$ [2]. So the $U_A(1)$ anomaly pushes up m_η by about 400 MeV and $m_{\eta'}$ by about 300 MeV. It means that not only the η' meson but also the η meson is largely affected by the $U_A(1)$ anomaly.

In order to understand the role of the $U_A(1)$ anomaly in low-energy QCD, it may be important to study the η -meson decays as well as its mass and decay constant. Among the η -meson decays, $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^0\gamma\gamma$ decays are interesting. They have no final state interactions and involve only neutral mesons so that the electromagnetic transitions are induced only by the internal (quark) structure of the mesons. The $\eta \rightarrow \gamma\gamma$ decay is related to the Adler–Bell–Jackiw triangle anomaly [3] through partial conservation of the axial-vector current hypothesis. For the $\eta \rightarrow \pi^0\gamma\gamma$ decay, it is known that the chiral perturbation theory (ChPT)

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gives too small a prediction in the leading order and higher order terms are expected to be dominant.

The purpose of this paper is to study the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^0 \gamma\gamma$ decays in the framework of the three-flavour Nambu–Jona-Lasinio (NJL) model as a chiral effective quark Lagrangian of low-energy QCD. The three-flavour NJL model which involves the $U_L(3) \times U_R(3)$ symmetric four-quark interaction and the six-quark flavour-determinant interaction [4] incorporating effects of the $U_A(1)$ anomaly has been used widely in recent years to study such topics as the quark condensates in vacuum, the spectrum of low-lying mesons, the flavour-mixing properties of low-energy hadrons, etc. [5]. In this approach the effects of explicit breaking of the chiral symmetry by the current quark mass term and the $U_A(1)$ anomaly on the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^0 \gamma\gamma$ decay amplitudes can be calculated consistently with those on the η -meson mass, η decay constant and mixing angle within the model applicability.

2. Three-flavour Nambu–Jona-Lasinio model

We work with the following NJL model Lagrangian density:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_6, \quad (1)$$

$$\mathcal{L}_0 = \bar{\psi} (i\partial_\mu \gamma^\mu - \hat{m}) \psi, \quad (2)$$

$$\mathcal{L}_4 = \frac{G_S}{2} \sum_{a=0}^8 [(\bar{\psi} \lambda^a \psi)^2 + (\bar{\psi} \lambda^a i\gamma_5 \psi)^2], \quad (3)$$

$$\mathcal{L}_6 = G_D \{ \det[\bar{\psi}_i (1 - \gamma_5) \psi_j] + \det[\bar{\psi}_i (1 + \gamma_5) \psi_j] \}. \quad (4)$$

Here the quark field ψ is a column vector in colour, flavour and Dirac spaces and $\lambda^a (a = 0 \dots 8)$ is the $U(3)$ generator in flavour space. The free Dirac Lagrangian $\mathcal{L}_0$ incorporates the current quark mass matrix $\hat{m} = \text{diag}(m_u, m_d, m_s)$ which breaks the chiral $U_L(3) \times U_R(3)$ invariance explicitly. The $\mathcal{L}_4$ is a QCD motivated four-fermion interaction, which is chiral $U_L(3) \times U_R(3)$ invariant. The 't Hooft determinant $\mathcal{L}_6$ represents the $U_A(1)$ anomaly. It is a 3×3 determinant with respect to flavour with $i, j = u, d, s$.

Quark condensates and constituent quark masses are self-consistently determined by the gap equations in the mean field approximation. The covariant cutoff Λ is introduced to regularize the divergent integrals. The pseudoscalar channel quark–antiquark scattering amplitudes are then calculated in the ladder approximation. From the pole positions of the scattering amplitudes, the pseudoscalar meson masses are determined. We define the effective meson–quark coupling constants $g_{\eta qq}$ and $g_{\pi qq}$ by introducing additional vertex Lagrangians,

$$\mathcal{L}_{\eta qq} = g_{\eta qq} \bar{\psi} i\gamma_5 \lambda^8 \psi \phi_\eta, \quad (5)$$

$$\mathcal{L}_{\pi qq} = g_{\pi qq} \bar{\psi} i\gamma_5 \lambda^3 \psi \phi_{\pi^0}, \quad (6)$$

with $\lambda^\eta = \cos\theta\lambda^8 - \sin\theta\lambda^0$. Here ϕ is an auxiliary meson field introduced for convenience and the effective meson–quark coupling constants are calculated from the residues of the $q\bar{q}$ -scattering amplitudes at the corresponding meson poles. Because of the $SU(3)$ symmetry breaking, the flavour $\lambda^8 - \lambda^0$ components mix with each other. Thus we solve the coupled-channel $q\bar{q}$ scattering problem for the η meson. The mixing angle θ is obtained by diagonalization of the $q\bar{q}$ -scattering amplitude. It should be noted that θ depends on q^2 . At $q^2 = m_\eta^2$, θ represents the mixing angle of the λ^8 and λ^0 components in the η -meson state. In the usual effective pseudoscalar meson Lagrangian approaches, the η and η' mesons are analysed using the q^2 -independent η - η' mixing angle. Because of the q^2 -dependence, θ cannot be interpreted as the η - η' mixing angle. The origin of the q^2 -dependence is that the η and η' mesons have internal quark structures. The meson decay constant f_M ($M = \pi, K, \eta$) is determined by calculating the quark–antiquark one-loop graph. The explicit expressions are found in [6].

3. The $\eta \rightarrow \gamma\gamma$ Decay Amplitude

The $\pi^0, \eta \rightarrow \gamma\gamma$ decay amplitudes are given by

$$\langle \gamma(k_1)\gamma(k_2)|M(q) \rangle = i(2\pi)^4 \delta^4(k_1 + k_2 - q) \varepsilon_{\mu\nu\rho\sigma} \epsilon_1^\mu \epsilon_2^\nu k_1^\rho k_2^\sigma \tilde{T}_{M \rightarrow \gamma\gamma}(q^2), \quad (7)$$

where ϵ_1 and ϵ_2 are the polarization vectors of the photon. By calculating the pseudoscalar–vector–vector type quark triangle diagrams, we get the following results:

$$\tilde{T}_{\pi^0 \rightarrow \gamma\gamma} = \frac{\alpha}{\pi} g_{\pi qq} F(u, \pi^0), \quad (8)$$

$$\begin{aligned} \tilde{T}_{\eta \rightarrow \gamma\gamma} = \frac{\alpha}{\pi} g_{\eta qq} \frac{1}{3\sqrt{3}} & \left[\cos\theta \{5F(u, \eta) - 2F(s, \eta)\} \right. \\ & \left. - \sin\theta \sqrt{2} \{5F(u, \eta) + F(s, \eta)\} \right]. \end{aligned} \quad (9)$$

Here α is the fine structure constant of QED and $F(a, M)$ ($a = u, s$ and $M = \pi^0, \eta$) is defined as

$$F(a, M) = \int_0^1 dx \int_0^1 dy \frac{2(1-x)M_a}{M_a^2 - m_M^2 x(1-x)(1-y)}. \quad (10)$$

Then the $M \rightarrow \gamma\gamma$ decay width is given by $\Gamma(M \rightarrow \gamma\gamma) = |\tilde{T}_{M \rightarrow \gamma\gamma}|^2 m_M^3 / (64\pi)$.

In the chiral limit, the pion mass vanishes and $F(u, \pi^0)$ becomes $1/M_u$. In this limit, the Goldberger–Treiman (GT) relation at the quark level, $M_u = g_\pi f_\pi$, holds in the NJL model and this leads to $\tilde{T}_{\pi^0 \rightarrow \gamma\gamma} = \alpha/\pi f_\pi$ which is same as the tree-level results in the Wess–Zumino–Witten Lagrangian approach [7]. It should be mentioned that we have to integrate out the triangle diagrams without introducing a cutoff Λ in order to get the above result, though the cutoff is introduced in the gap equations in the NJL model. In the $U(3)_L \times U(3)_R$ version of the NJL model, the WZW term has been derived using the bosonization

method with the heat-kernel expansion [8, 9]. In their approach, the $O(1/\Lambda)$ term has been neglected and it is equivalent to taking the $\Lambda \rightarrow \infty$ limit.

4. The $\eta \rightarrow \pi^0 \gamma \gamma$ Decay Amplitude

The $\eta \rightarrow \pi^0 \gamma \gamma$ decay amplitude is given by

$$\langle \pi^0(p_\pi) \gamma(k_1, \epsilon_1) \gamma(k_2, \epsilon_2) | \eta(p) \rangle = i(2\pi)^4 \delta^4(p_\pi + k_1 + k_2 - p) \epsilon_1^\mu \epsilon_2^\nu T_{\mu\nu}. \quad (11)$$

The dominant contributions to this process in this model are the quark-box diagrams. Following the evaluation of the quark-box diagrams performed in [10], we obtain

$$T_{\mu\nu} = -i \frac{1}{\sqrt{3}} (\cos \theta - \sqrt{2} \sin \theta) e^2 g_{\eta qq} g_{\pi qq} \int \frac{d^4 q}{(2\pi)^4} \sum_{i=1}^6 U_{\mu\nu}^i, \quad (12)$$

$$U_{\mu\nu}^1 = \text{tr} \left\{ \gamma_5 \frac{1}{\not{q} - M + i\epsilon} \gamma_5 \frac{1}{\not{q} + \not{p} - \not{k}_1 - \not{k}_2 - M + i\epsilon} \right. \\ \left. \times \gamma_\nu \frac{1}{\not{q} + \not{p} - \not{k}_1 - M + i\epsilon} \gamma_\mu \frac{1}{\not{q} + \not{p} - M + i\epsilon} \right\}, \quad (13)$$

$$U_{\mu\nu}^2 = \text{tr} \left\{ \gamma_5 \frac{1}{\not{q} - M + i\epsilon} \gamma_5 \frac{1}{\not{q} + \not{k}_2 - M + i\epsilon} \right. \\ \left. \times \gamma_\nu \frac{1}{\not{q} + \not{p} - \not{k}_1 - M + i\epsilon} \gamma_\mu \frac{1}{\not{q} + \not{p} - M + i\epsilon} \right\}, \quad (14)$$

$$U_{\mu\nu}^3 = \text{tr} \left\{ \gamma_5 \frac{1}{\not{q} - M + i\epsilon} \gamma_\nu \frac{1}{\not{q} + \not{k}_2 - M + i\epsilon} \right. \\ \left. \times \gamma_\mu \frac{1}{\not{q} + \not{k}_1 + \not{k}_2 - M + i\epsilon} \gamma_5 \frac{1}{\not{q} + \not{p} - M + i\epsilon} \right\}, \quad (15)$$

$$U_{\mu\nu}^4 = U_{\nu\mu}^1(k_1 \leftrightarrow k_2), \quad (16)$$

$$U_{\mu\nu}^5 = U_{\nu\mu}^2(k_1 \leftrightarrow k_2), \quad (17)$$

$$U_{\mu\nu}^6 = U_{\nu\mu}^3(k_1 \leftrightarrow k_2). \quad (18)$$

Here M is the constituent u,d quark mass. Because the loop integration in (12) is not divergent, we again do not use the UV cutoff. Then the gauge invariance is preserved. The inclusion of the cutoff that is consistent with the gap equation will break the gauge invariance and make the present calculation too complicated. Note that the strange quark does not contribute to the loop.

On the other hand the amplitude $T_{\mu\nu}$ has a general form required by the gauge invariance [11]

$$T^{\mu\nu} = A(x_1, x_2)(k_1^\nu k_2^\mu - k_1 \cdot k_2 g^{\mu\nu}) \\ + B(x_1, x_2) \left[-m_\eta^2 x_1 x_2 g^{\mu\nu} - \frac{k_1 \cdot k_2}{m_\eta^2} p^\mu p^\nu + x_1 k_2^\mu p^\nu + x_2 p^\mu k_1^\nu \right], \quad (19)$$

with $x_i = p \cdot k_i / m_\eta^2$. With A and B , the differential decay rate with respect to the energies of the two photons is given by

$$\frac{d^2\Gamma}{dx_1 dx_2} = \frac{m_\eta^5}{256\pi^2} \left\{ |A + \frac{1}{2}B|^2 \left[2(x_1 + x_2) + \frac{m_\pi^2}{m_\eta^2} - 1 \right]^2 \right. \\ \left. + \frac{1}{4}|B|^2 \left[4x_1 x_2 - \left[2(x_1 + x_2) + \frac{m_\pi^2}{m_\eta^2} - 1 \right] \right]^2 \right\}. \quad (20)$$

Though the mass of η as a $\bar{q}q$ bound state depends on G_D^{eff} , we use the experimental value $m_\eta = 547$ MeV in evaluating (20). The Dalitz boundary is given by two conditions:

$$\frac{1}{2} \left(1 - \frac{m_\pi^2}{m_\eta^2} \right) \leq x_1 + x_2 \leq 1 - \frac{m_\pi}{m_\eta}, \quad (21)$$

$$x_1 + x_2 - 2x_1 x_2 \leq \frac{1}{2} \left(1 - \frac{m_\pi^2}{m_\eta^2} \right). \quad (22)$$

In evaluating (13)–(18), one only has to identify the coefficients of $p^\mu p^\nu$ and $g^{\mu\nu}$. Details of the calculation are given in [10]. Defining $\mathcal{A}$ and $\mathcal{B}$ by

$$\int \frac{d^4 q}{(2\pi)^4} \sum_{i=1}^6 U_i^{\mu\nu} = -i \left(\mathcal{A} g^{\mu\nu} + \mathcal{B} \frac{p^\mu p^\nu}{m_\eta^2} + \dots \right), \quad (23)$$

we find A and B as

$$A = \frac{1}{\sqrt{3}} (\cos \theta - \sqrt{2} \sin \theta) e^2 g_{\pi qq} g_{\eta qq} \frac{2}{m_\eta^2 \sigma} \left[\mathcal{A} - 2x_1 x_2 \frac{\mathcal{B}}{\sigma} \right], \quad (24)$$

$$B = \frac{1}{\sqrt{3}} (\cos \theta - \sqrt{2} \sin \theta) e^2 g_{\pi qq} g_{\eta qq} \frac{2}{m_\eta^2 \sigma} \frac{\mathcal{B}}{\sigma}, \quad (25)$$

with

$$\sigma = \frac{(k_1 + k_2)^2}{m_\eta^2} = 2(x_1 + x_2) + \frac{m_\pi^2}{m_\eta^2} - 1. \quad (26)$$

We evaluate $\mathcal{A}$ and $\mathcal{B}$ numerically and further integrate (20) to obtain the $\eta \rightarrow \pi^0 \gamma \gamma$ decay rate.

5. Numerical Results

The recent experimental results for the $\pi^0, \eta \rightarrow \gamma\gamma$ decay widths are $\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.7 \pm 0.6 \text{ eV}$ and [12] and the reduced amplitudes are

$$|\tilde{\mathcal{T}}_{\pi^0 \rightarrow \gamma\gamma}| = (2.5 \pm 0.1) \times 10^{-11} [\text{eV}]^{-1}, \quad (27)$$

$$|\tilde{\mathcal{T}}_{\eta \rightarrow \gamma\gamma}| = (2.5 \pm 0.06) \times 10^{-11} [\text{eV}]^{-1}. \quad (28)$$

From Eqs (8) and (9), we get $\tilde{\mathcal{T}}_{\eta \rightarrow \gamma\gamma} = \frac{5}{3} \tilde{\mathcal{T}}_{\pi^0 \rightarrow \gamma\gamma}$ in the $U_A(1)$ limit. Therefore in order to reproduce the experimental value of $\tilde{\mathcal{T}}_{\eta \rightarrow \gamma\gamma}$, the effect of the $U_A(1)$ anomaly should reduce $\tilde{\mathcal{T}}_{\eta \rightarrow \gamma\gamma}$ by a factor $\frac{3}{5}$. On the other hand, the experimental value of the $\eta \rightarrow \pi^0 \gamma\gamma$ decay width is [12]

$$\Gamma_{exp}(\eta \rightarrow \pi^0 \gamma\gamma) = 0.85 \pm 0.19 \text{ eV}. \quad (29)$$

In our theoretical calculations, the parameters of the NJL model are the current quark masses $m_u = m_d, m_s$, the four-quark coupling constant G_S , the six-quark determinant coupling constant G_D and the covariant cutoff Λ . We take G_D as a free parameter and study η meson properties as functions of G_D . We use the light current quark masses $m_u = m_d = 8.0 \text{ MeV}$ to reproduce $M_u = M_d \simeq 330 \text{ MeV}$ ($\simeq \frac{1}{3} M_N$) which is the value usually used in the nonrelativistic quark model. Other parameters, m_s , G_D , and Λ , are determined so as to reproduce the isospin averaged observed masses, m_π, m_K , and the pion decay constant f_π .

We obtain $m_s = 193 \text{ MeV}$, $\Lambda = 783 \text{ MeV}$, $M_{u,d} = 325 \text{ MeV}$ and $g_{\pi qq} = 3.44$, which are almost independent of G_D . The ratio of the current s-quark mass to the current u,d quark mass is $m_s/m_u = 24.1$, which agrees well with $m_s/\hat{m} = 25 \pm 2.5$ [$\hat{m} = \frac{1}{2}(m_u + m_d)$] derived from ChPT [13]. The kaon decay constant f_K is the prediction and is almost independent of G_D . We have obtained $f_K = 97 \text{ MeV}$ which is about 14% smaller than the observed value. We consider this is the typical predictive power of the NJL model in the strangeness sector.

Table 1 summarizes the fitted results of the model parameters and the quantities necessary for calculating the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^0 \gamma\gamma$ decay widths which depend on G_D . We define dimensionless parameters $G_D^{\text{eff}} \equiv -G_D(\Lambda/2\pi)^4 \Lambda N_c^2$ and $G_S^{\text{eff}} \equiv G_S(\Lambda/2\pi)^2 N_c$. When G_D^{eff} is zero, our Lagrangian does not cause the flavour mixing and therefore ideal mixing is achieved. The ‘ η ’ is purely $u\bar{u} + d\bar{d}$ and is degenerate to the pion in this limit.

We next discuss the $\pi^0 \rightarrow \gamma\gamma$ decay. The calculated result is $\tilde{\mathcal{T}}_{\pi^0 \rightarrow \gamma\gamma} = 2.50 \times 10^{-11} \text{ eV}^{-1}$ which agrees well with the observed value given in Eq. (27). The current algebra result is $\tilde{\mathcal{T}}_{\pi^0 \rightarrow \gamma\gamma} = \alpha/\pi f_\pi = 2.514 \times 10^{-11} \text{ eV}^{-1}$, and thus the soft pion limit is a good approximation for $\pi^0 \rightarrow \gamma\gamma$ decay. The chiral symmetry breaking affects $\tilde{\mathcal{T}}_{\pi^0 \rightarrow \gamma\gamma}$ in two ways. One is the deviation from the GT relation and another is the matrix element of the triangle diagram $F(u, \pi^0)$. Our numerical results are $g_\pi = 3.44$, $M_u/f_\pi = 3.52$ and $F(u, \pi^0)M_u = 1.015$, therefore the deviations from the soft pion limit are very small both in the GT relation and the matrix element of the triangle diagram.

Let us now turn to the discussion of the $\eta \rightarrow \gamma\gamma$ decay. The calculated results of the $\eta \rightarrow \gamma\gamma$ decay amplitude $\tilde{\mathcal{T}}_{\eta \rightarrow \gamma\gamma}$ are given in Table 1 and shown in Fig. 1.

The experimental value of the $\eta \rightarrow \gamma\gamma$ decay amplitude is reproduced at about $G_D^{\text{eff}} = 0.7$. The calculated η -meson mass at $G_D^{\text{eff}} = 0.7$ is $m_\eta = 510$ MeV which is 7% smaller than the observed mass. The value $G_D^{\text{eff}} = 0.7$ corresponds to $G_D\langle\bar{s}s\rangle/G_S = 0.44$, suggesting that the contribution from $\mathcal{L}_6$ to the dynamical mass of the up and down quarks is 44% of that from $\mathcal{L}_4$.

Table 1. Parameters of the model, the $\eta \rightarrow \gamma\gamma$ decay amplitude $\tilde{T}_{\eta \rightarrow \gamma\gamma}$ and $\eta \rightarrow \pi^0 \gamma\gamma$ decay width $\Gamma(\eta \rightarrow \pi^0 \gamma\gamma)$ for each G_D^{eff}

G_D^{eff}	G_S^{eff}	M_s [MeV]	m_η [MeV]	θ [deg]	$g_{\eta qq}$	$\tilde{T}_{\eta \rightarrow \gamma\gamma}$ [eV $^{-1}$]	$\Gamma(\eta \rightarrow \pi^0 \gamma\gamma)$ [eV]
0.00	0.73	556	138.1	-54.74	3.44	4.17×10^{-11}	2.88
0.10	0.70	552	285.3	-44.61	3.23	3.95×10^{-11}	2.46
0.20	0.66	549	366.1	-33.52	3.12	3.68×10^{-11}	2.06
0.30	0.63	545	419.1	-23.24	3.11	3.39×10^{-11}	1.71
0.40	0.60	541	455.0	-14.98	3.15	3.10×10^{-11}	1.42
0.50	0.57	537	479.7	-8.86	3.20	2.86×10^{-11}	1.20
0.60	0.54	533	497.3	-4.44	3.25	2.65×10^{-11}	1.04
0.70	0.51	529	510.0	-1.25	3.28	2.48×10^{-11}	0.92
0.80	0.47	525	519.6	1.09	3.30	2.35×10^{-11}	0.84
0.90	0.44	522	527.0	2.84	3.31	2.23×10^{-11}	0.77
1.00	0.41	518	532.8	4.17	3.32	2.14×10^{-11}	0.71
1.10	0.40	514	537.5	5.21	3.32	2.07×10^{-11}	0.67
1.20	0.35	511	541.3	6.02	3.31	2.00×10^{-11}	0.63
1.30	0.32	507	544.5	6.66	3.30	1.95×10^{-11}	0.61
1.40	0.29	504	547.2	7.17	3.29	1.91×10^{-11}	0.58
1.50	0.25	500	549.4	7.57	3.28	1.86×10^{-11}	0.56
1.60	0.22	497	551.4	7.90	3.26	1.83×10^{-11}	0.55

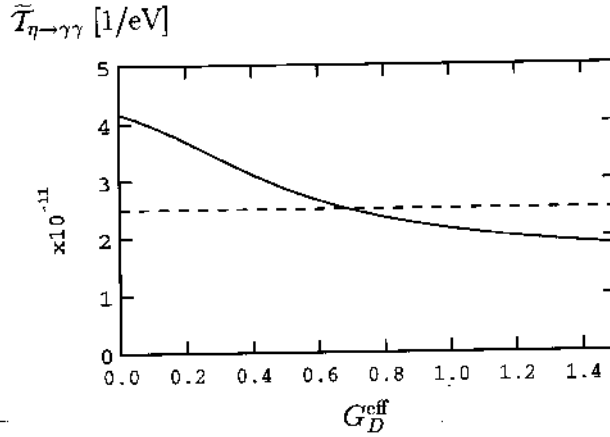


Fig. 1. Dependence of the $\eta \rightarrow \gamma\gamma$ decay amplitude on the dimensionless coupling constant G_D^{eff} . The horizontal dashed line indicates the experimental value.

The mixing angle at $G_D^{\text{eff}} = 0.7$ is $\theta = -1.3^\circ$ and that indicates a strong OZI violation and a large (u,d)-s mixing. This disagrees with the ‘standard’ value $\theta \simeq -20^\circ$ obtained in ChPT [14]. This is due to the stronger $U_A(1)$ breaking in the present calculation. The difference mainly comes from the fact that the mixing angle in the NJL model depends on q^2 of the $\bar{q}q$ state and thus reflects

the internal structure of the η meson. On the contrary the analyses of ChPT [14] assume an energy-independent mixing angle, i.e. $\theta(q^2=m_\eta^2) = \theta(q^2=m_{\eta'}^2)$.

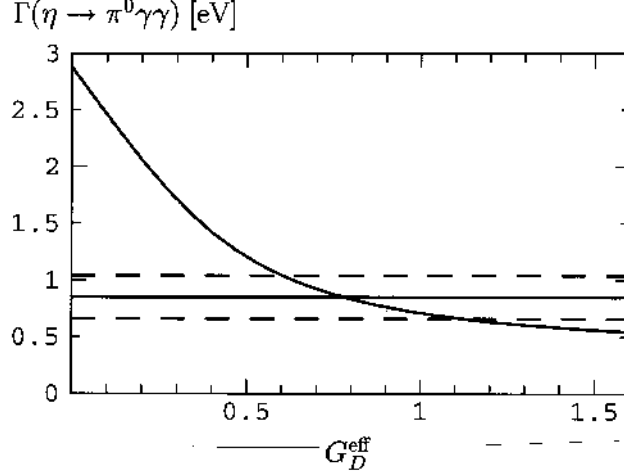


Fig. 2. Dependence of the $\eta \rightarrow \pi^0 \gamma \gamma$ decay width on the dimensionless coupling constant G_D^{eff} . The horizontal solid line indicates the experimental value and the dashed lines indicate its error widths.

The η decay constant is almost independent of G_D : $f_\eta = 91.2 \text{ MeV}$ ($\simeq f_\pi$) at $G_D^{\text{eff}} = 0.7$. Therefore it seems that the η meson does not lose the Nambu–Goldstone boson nature though its mass and mixing angle are strongly affected by the $U_A(1)$ breaking interaction.

The $\eta \rightarrow \pi^0 \gamma \gamma$ decay gives us independent information on the structure of the η meson. The calculated $\eta \rightarrow \pi^0 \gamma \gamma$ decay widths are also given in Table 1 and shown in Fig. 2. At $G_D^{\text{eff}} = 0.70$, we obtain $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = 0.92$, which is in good agreement with the experimental data shown in (29). This process was studied in ChPT [15] and in the extended NJL model [16]. The difference between these approaches and ours is discussed in [17].

6. Effects of the $U_A(1)$ Anomaly in Baryons

Since the effects of the $U_A(1)$ anomaly are rather large in the pseudoscalar sector, it is natural to ask if one can see some effects in the baryon sector. It was pointed out in [18] that the instanton can play an important role in the description of spin–spin forces, particularly for light baryons. The pattern of these effects can be very hard to disentangle from one-gluon exchange. The effects of the instanton induced interaction in baryon number $B = 2$ systems were studied in [19]. It was shown that an attraction between two nucleons is obtained by the two-body instanton induced interaction, while the three-body interaction is strongly repulsive in the H-dibaryon channel and makes the H-dibaryon almost unbound.

We estimate the effects of the $U_A(1)$ anomaly on the $B = 1$ and $B = 2$ systems by employing the six-quark determinant interaction given in Eq. (3) whose strength was determined so as to reproduce the observed η -meson mass, the $\eta \rightarrow \gamma \gamma$ decay width and the $\eta \rightarrow \pi^0 \gamma \gamma$ decay width, namely $G_D^{\text{eff}} = 0.7$. It is

done by calculating the matrix elements of the the $U_A(1)$ breaking interaction Hamiltonian with respect to unperturbed states of the MIT bag model and the nonrelativistic quark model (NRQM). For $B = 2$ systems, we only consider the $(0S)^6$ configuration of the six valence quark states. Therefore, the matrix element with respect to such a state gives a measure of the contribution of the $U_A(1)$ breaking interaction either to the dibaryon or to the short-range part of the interaction between two baryons. The determinant interaction induces not only three-body but also two-body interactions of valence quarks when the vacuum has a nonvanishing quark condensate. The details of the calculation are described in [20].

Table 2 shows the contribution of the two-body term for $B = 1$. The contribution to the decuplet baryons vanishes in the $SU(3)$ limit and therefore comes only from the $SU(3)$ asymmetry of the quark wave function. The three-body term does not contribute to the $B = 1$ states. Thus the $N\Delta$ mass difference due to the $U_A(1)$ breaking interaction is about 15% of the observed one.

Table 2. Contribution of the two-body term to octet and decuplet baryons

All the entries are in units of MeV

Wave function	N	Σ	Ξ	Λ	Δ	Σ^*	Ξ^*	Ω
MIT	-43.9	-41.2	-41.2	-42.9	0	0.12	0.12	0
NRQM	-40.88	-36.6	-36.6	-39.4	0	0.07	0.07	0

Table 3. Baryon component, $SU(3)$ multiplet, spin, isospin and strangeness of the eight channels of two octet baryons

Channel	Baryon component	$SU(3)$ multiplet	Spin	Isospin	Strangeness
I	NN	$\mathbf{10}^*$	1	0	0
II	NN	$\mathbf{27}$	0	1	0
III	$N\Sigma$	$\mathbf{27}$	0	3/2	-1
IV	$N\Sigma - N\Lambda$	$\mathbf{27}$	0	1/2	-1
V	$N\Sigma - N\Lambda$	$\mathbf{10}^*$	1	1/2	-1
VI	$N\Sigma$	$\mathbf{10}$	1	3/2	-1
VII	$N\Sigma - N\Lambda$	$\mathbf{8}$	1	1/2	-1
VIII	H	$\mathbf{1}$	0	0	-2

We next discuss the case of $B = 2$. We consider all the possible channels which are made of two octet baryons listed in Table 3. Table 4 shows the contribution of the two-body term. The channel VIII has the strongest attraction, about 170 MeV, and channel VII has the second strongest attraction. The contributions of the three-body term to the H-dibaryon and strangeness -1 channels are given in Table 5. It should be noted that the three-body term has no effect on the NN channels, and that the contributions to the channels III, IV and V reflect the $SU(3)$ breaking in the quark wave function. The contributions of the three-body term in channels VI, VII and VIII are remarkable and one will be able to observe some effects experimentally.

Table 4. Contributions of the two-body term to the eight channels of the two octet baryons listed in Table 3

All entries are in units of MeV

Wave function	I	II	III	IV	V	VI	VII	VIII
MIT	-89.9	-85.1	-88.9	-86.3	-93.9	-96.5	-121.5	-162.6
NRQM	-120.2	-105.3	-102.3	-103.9	-118.3	-116.7	-148.0	-182.6

Table 5. Contributions of the three-body term to the H-dibaryon (VIII) and strangeness -1 two octet baryon channels (III-VII)

All entries are in units of MeV

Wave function	III	IV	V	VI	VII	VIII
MIT	-6×10^{-2}	-6×10^{-2}	-7×10^{-2}	20.7	25.1	40.7
NRQM	-5×10^{-2}	-5×10^{-2}	-5×10^{-2}	28.3	34.9	56.1

We should comment on the difference between the determinant interaction used here and the instanton-induced interaction used in ref. [19]. The relative contributions of the $U_A(1)$ breaking interaction within the baryonic sector or within the mesonic sector are similar for the two interactions. However, the ratio of those in the baryonic sector to those in the mesonic sector is about $\frac{4}{7}$. Namely, if one fixes the strength of the interaction so as to give the same mass difference of η and η' , the effects of the instanton-induced interaction in the baryonic sector would be about $\frac{7}{4}$ stronger than those of the determinant interaction. After this correction the strength of the present $U_A(1)$ breaking interaction is consistent with that used in the calculation of the baryon-baryon interaction in ref. [19].

7. Summary

We have studied the $\eta \rightarrow \pi^0 \gamma \gamma$ decay in a three-flavour NJL model that includes the $U_A(1)$ breaking six-quark determinant interaction. The η -meson mass, the $\eta \rightarrow \gamma \gamma$ decay width and the $\eta \rightarrow \pi^0 \gamma \gamma$ decay width are reproduced well with a rather strong $U_A(1)$ breaking interaction that makes the $\eta_1 - \eta_8$ mixing angle $\theta \simeq 0^\circ$. The effects of the $U_A(1)$ breaking on the baryon number one and two systems have been also studied.

Finally, we should note that the NJL model does not confine quarks. Since the NG bosons π , K and η are strongly bound, the NJL model can describe their properties fairly well. However, the η' meson state in the NJL model has an unphysical decay of the $\eta' \rightarrow q\bar{q}$. Therefore we do not apply our model to the η' meson. Further study of the $U_A(1)$ breaking will require the construction of a framework which can be applied to the η' meson.

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