

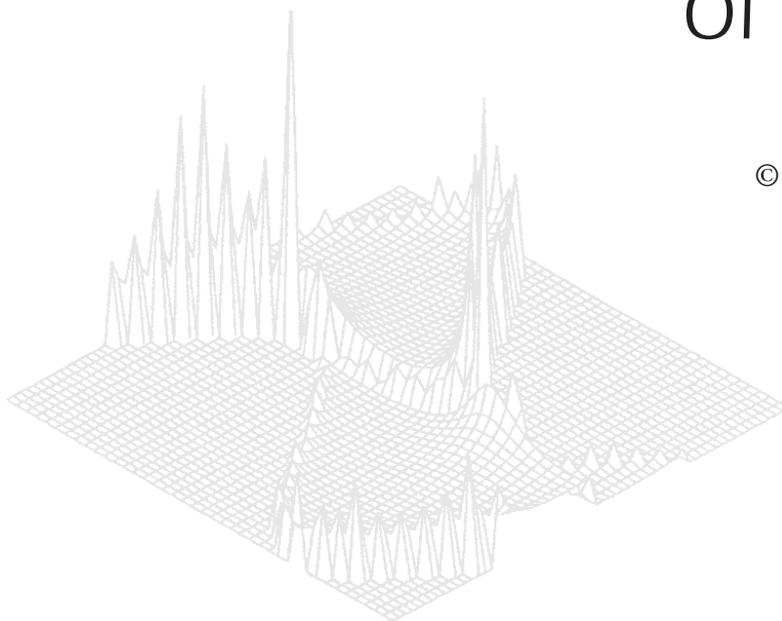
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## Chiral Symmetry Breaking in the Dual Ginzburg–Landau Theory\*

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### *Abstract*

Confinement and spontaneous chiral symmetry breaking are the most fundamental phenomena in quark nuclear physics, where hadrons and nuclei are described in terms of quarks and gluons. The dual Ginzburg–Landau (DGL) theory contains monopole fields as the most essential degrees of freedom. Their condensation in the vacuum is modelled to describe quark confinement in strong connection with QCD. We then demonstrate that the DGL theory is able to describe the spontaneous breakdown of chiral symmetry.

### **1. Introduction**

In quark nuclear physics (QNP) the most essential phenomena are the confinement of quarks and gluons and chiral symmetry breaking. Quarks are not found in free space, but they are seen in deep inelastic scattering. Quarks are present in hadrons and hence an understanding of confinement is essential for the sizes of hadrons.

Chiral symmetry is found in the QCD Lagrangian. In the u–d sector, the current mass is considered negligible ( $\sim 5$  MeV) as compared to the hadron mass ( $\sim 1$  GeV), and hence chiral symmetry is realized with high accuracy. We expect then the chiral partners (parity doublets) to be degenerate in the u–d meson spectrum. Nature, on the other hand, shows that the pion ( $0^-$ ) has a mass of 139 MeV and no  $0^+$  partner is found. The rho meson ( $1^-$ ) is 500 MeV apart from its chiral partner, the  $a_1$  meson ( $1^+$ ). The same features are also found for the baryon spectrum. These properties of the hadron spectrum tell us that chiral symmetry is broken spontaneously and the pion appears as its Goldstone particle. This chiral symmetry breaking should be understood in a full description of QNP. Chiral symmetry breaking would provide the constituent quark masses and even the small mass pions, which are responsible for the N–N interaction.

How do these phenomena happen? For this let us consider the QCD coupling strength  $\alpha_S$ , which runs with relevant momentum scale obeying the renormalization group of QCD [1]. At large momenta,  $\alpha_S$  decreases and the theory becomes asymptotically free, and hence the momentum dependence of deep inelastic

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scattering is calculable perturbatively. At small momenta, where confinement and chiral symmetry breaking are expected,  $\alpha_S$  blows up and hence we face highly non-perturbative processes. We need to find the essential degrees of freedom to gain insight into these phenomena.

## 2. Dual Ginzburg–Landau Theory

It was Nambu who first introduced an interesting view of colour confinement in 1974 [2]. Suppose we insert a superconductor into a magnetic field. The superconductor does not allow the magnetic field to pass through. If it were to allow the magnetic field, in a superconductor of the second kind, the magnetic field should be confined in a vortex-like configuration. This is known as the Meissner effect. Nambu took its dual version for quark confinement. If the vacuum is normal, the colour electric field should look like that for a Coulomb potential between a positive and a negative colour charge. If the vacuum is superconductor-like (dual superconductor), then the electric field is not allowed to pass through and hence the colour electric flux ought to be confined in a vortex-like configuration. This is then referred to as the dual Meissner effect. This picture, however, has not become popular, because it requires colour magnetic monopoles. In the superconductor, the charged object (i.e. the Cooper pair) condenses, while in the QCD vacuum the magnetically charged object (i.e. the colour magnetic monopole) condenses.

't Hooft was the first to demonstrate the natural appearance of colour magnetic monopoles in QCD [3]. In a non-abelian gauge theory like QCD, he introduced a particular gauge named the abelian gauge, to reduce it to an abelian gauge theory like QED. From a topological argument, colour magnetic monopoles appear naturally in the abelian space. This work then supports the idea of Nambu for confinement. Hence, QCD naturally reduces to QED with magnetic monopoles, which is the Maxwell equation with magnetic charges and currents studied by Dirac [4]. This Maxwell equation has duality symmetry, which naturally arises in QCD in this special gauge.

It took then about 10 years before the above idea was formulated in the form of a Lagrangian [5]. The dual Ginzburg–Landau (DGL) theory is expressed with

$$\mathcal{L}_{DGL} = \mathcal{L}_{\text{dual}} + \bar{q}(i\gamma_\mu\partial^\mu - m + e\gamma_\mu A^\mu)q + \text{tr}[\hat{D}_\mu, \chi]^\dagger[\hat{D}^\mu, \chi] - \lambda\text{tr}(\chi^\dagger\chi - v^2)^2. \quad (1)$$

Here  $\mathcal{L}_{\text{dual}}$  denotes the dual version of the gauge field tensor,  $q$  is the quark field and  $\chi$  is the monopole field. Also  $\hat{D}^\mu$  is the dual covariant derivative,  $\hat{D}^\mu = \partial^\mu + igB^\mu$ , where  $B^\mu$  is the dual gauge field and  $g$  is the dual coupling constant, which satisfies the Dirac condition,  $eg = 4\pi$ . The last term is the Higgs term which causes monopole condensation, where  $\lambda$  and  $v$  are the parameters of the DGL Lagrangian. It is important to note that this DGL Lagrangian is derived from the QCD Lagrangian by assuming the existence of the monopole field and abelian dominance [5]. It is at the same time supported by recent lattice QCD calculations [6]. This is the dream Lagrangian of Dirac, which ought to appear in some non-abelian gauge theory like QCD.

The first application is the  $q\bar{q}$  static potential by putting the  $q\bar{q}$  pair at distance  $r$  [7]. The potential comes out to have a Yukawa term and a linear confining term. We can fix the parameters of the DGL Lagrangian by fitting to the phenomenological potential. As for the glueball mass, which appears in the DGL theory and has a strong connection with the QCD vacuum and confinement, it turns out that  $M(0^+) \sim 1.5$  GeV. The appearance of the linear potential is not surprising, since it is modelled in the DGL theory. It is worth while to stress, however, that there are no other models which are able to realize confinement of colours and at the same time have a strong link with QCD. The real challenge is now to obtain chiral symmetry breaking, which is discussed next.

### 3. Chiral Symmetry Breaking

Chiral symmetry breaking is directly related to quark mass generation in the QCD vacuum. How do quarks then behave in a monopole condensed vacuum? It corresponds to solving the Schwinger–Dyson equation, where quarks get the self-energy corrections due to the non-perturbative interaction with gluons [7]. This seems, however, unphysical, because quarks are confined. It means that whenever a quark is present, there should be an anti-quark or a di-quark to make the system colour-singlet. In principle, therefore, we ought to solve the many-body system to talk about a single quark. Suppose we have the Schwinger–Dyson (SD) equation written schematically as

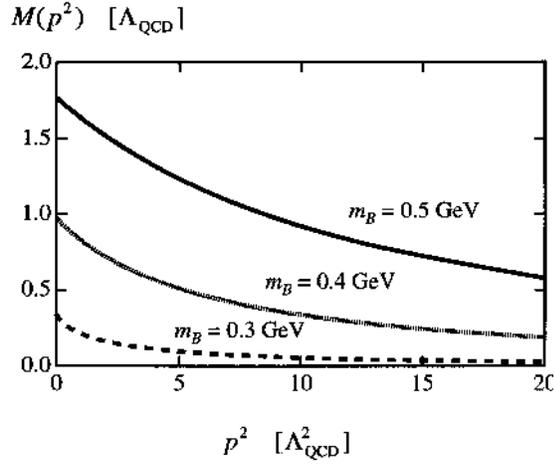
$$S^{-1}(p) = S_0^{-1}(p) + \int_0^\infty S(p-q)D(q)dq. \quad (2)$$

The quark, which is confined, should be seen from the position of the anti-quark. Then, the probability of finding the quark is finite only within the distance of the hadronic scale;  $\sim 1$  fm. Therefore, gluons cannot travel freely over any distance. Rather, they are confined also within hadronic distances. This indicates that there should be an infrared cut-off, which is of the order of the inverse of the confining distance  $R$  as  $q > q_c = 1/R$ . Hence, the SD equation is modified simply to

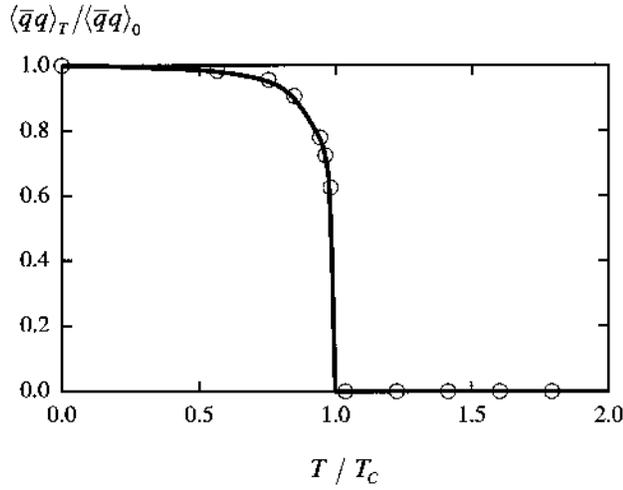
$$S^{-1}(p) = S_0^{-1}(p) + \int_{q_c}^\infty S(p-q)D(q)dq. \quad (3)$$

We show in Fig. 1 the result of chiral symmetry breaking, expressed in terms of the quark mass  $M(p)$ . It becomes finite by increasing the strength of monopole condensation. We find also that the pion decay constant and the quark condensate have values close to the semi-experimental values. This calculation demonstrates that monopole condensation is the source of both confinement and chiral symmetry breaking [8].

We discuss also the recovery of the chiral symmetry at finite temperature [9]. We can formulate this in the imaginary time formalism. We, in fact, find the recovery of chiral symmetry as indicated in Fig. 2 by the ratio of quark condensate at finite and zero temperatures. Here  $\langle \bar{q}q \rangle_T$  decreases with temperature and eventually drops to zero, indicating the recovery of chiral symmetry. We note that the temperature of the phase transition  $T_c \sim 0.11$  GeV seems smaller than the results of lattice QCD. This difference would be caused by the use of temperature



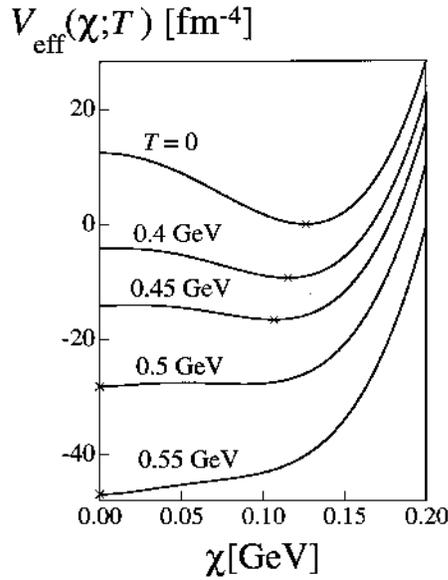
**Fig. 1.** Constituent quark mass calculated within the DGL theory with various values of the dual gauge mass, which indicates the strength of monopole condensation, as a function of the Euclidean momentum square. The unit  $\Lambda_{\text{QCD}}$  is 200 MeV.



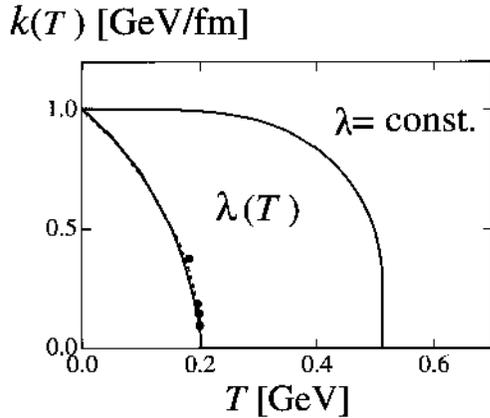
**Fig. 2.** Ratio of quark condensate at finite and zero temperatures within the DGL theory as a function of temperature. The critical temperature is about 0.11 GeV.

independent parameters in the Higgs term. Since this term is introduced at zero temperature, it is likely that they depend on temperature as the case of the superconductor. In addition, the hadronic scale should also depend on temperature. Here, the point of showing this result is merely to demonstrate that the DGL theory provides a phase transition to the normal phase at finite temperature.

We can even talk about the deconfinement phase transition at finite temperature [10]. In the quenched approximation, we can write the DGL Lagrangian in terms of the dual gauge fields by integrating out the gauge fields  $A_\mu$ . It amounts to



**Fig. 3.** Effective potential (thermodynamical potential) at various temperatures within the DGL theory as a function of the monopole condensate. The absolute minimum is indicated by  $\times$  for each curve, indicating the jump (phase transition of first order) around  $T = 0.5$  GeV.



**Fig. 4.** String tension between a quark and antiquark pair for constant and variable  $\lambda$  within the DGL theory as a function of temperature. The dots are the results of the pure-gauge lattice QCD [10].

calculating the partition function and we can derive the effective potential—the thermodynamical potential. The result is shown in Fig. 3, where the effective potential is plotted as a function of the monopole condensate  $\chi$ . The absolute minimum appears at a finite value of  $\chi$  on the lower temperature side and jumps to  $\chi = 0$ . This indicates a deconfinement phase transition of first order. We can calculate the string tension as a function of temperature, the result of which is shown in Fig. 4. Adjusting  $\lambda$  as a temperature dependent parameter so as to reproduce the critical temperature at 0.2 GeV, we find that the string tension reproduces the lattice QCD results [11].

It is useful to mention that lattice QCD is also undergoing very interesting developments. The  $q\bar{q}$  potential calculated with full lattice QCD agrees with the result of only the abelian part. This agreement indicates that confinement physics

can be described in terms of only the abelian gluons. Other studies in this direction are being carried out by several groups [12, 13]. All these results indicate that the low momentum phenomena could be described by the abelian gauge gluons (abelian dominance), when the abelian gauge is suitably chosen. At RCNP, we have started a numerical experimental program with the use of lattice QCD [14]. The largest task is to predict the properties of the glueballs [15] associated with confinement, particularly the decay properties for experimental identification.

#### 4. Conclusion

Quark nuclear physics (QNP) is the field which describes nucleons, mesons and nuclei in terms of quarks and gluons. The most essential phenomena in QNP are confinement of quarks and gluons and chiral symmetry breaking. Confinement is modelled as arising from the dual Meissner effect and is expressed in terms of the dual Ginzburg–Landau theory, where the QCD monopoles and their condensation in the QCD vacuum are the essential ingredients. We have demonstrated in this paper that the DGL theory is also able to describe chiral symmetry breaking. We have then discussed the restoration of these symmetries at finite temperature. Now the DGL theory is ready to be applied to exciting experimental phenomena in QNP.

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