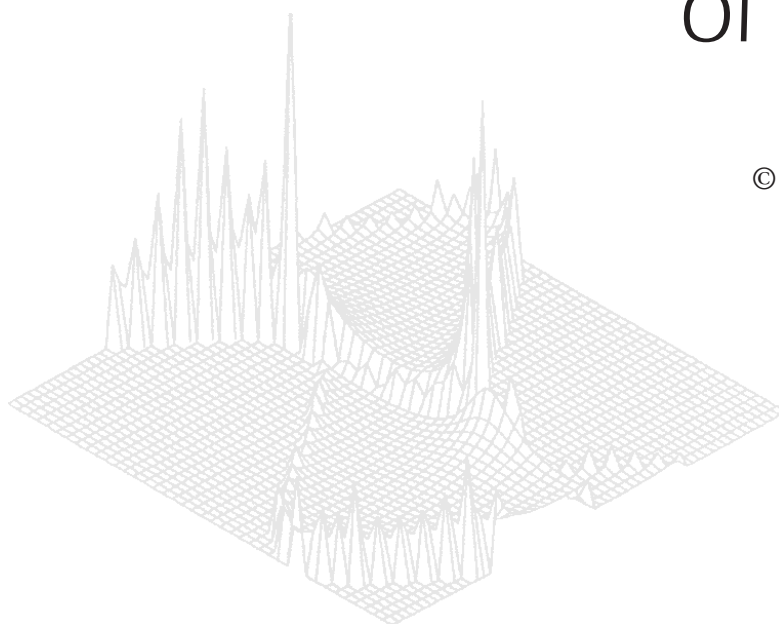

CSIRO PUBLISHING

Australian Journal of Physics

Volume 50, 1997
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A Study of the $\pi B \rightarrow YK$ Reactions for Kaon Production in Heavy Ion Collisions*

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Abstract

Parametrizations of total cross sections sufficient for all channels of the $\pi B \rightarrow YK$ reactions are completed using a resonance model. As well as discussing the $\pi N \rightarrow \Lambda K$ reactions, which were not presented in our previous publications, we present the differential cross section for $\pi N \rightarrow \Lambda K$. This report also aims at presenting supplementary discussions to our previous work.

One of the main goals of studying heavy ion collisions is to determine the equation of state (EOS) of nuclear matter. Because positive kaons (K^+) have a long mean free path inside the nucleus they are suggested as a good probe for the reactions occurring in the central region of the collisions [1]. Indeed, theoretical studies show that the kaons produced in heavy ion collisions are sensitive to the EOS [2, 3]. Although one can point out many important ingredients for the theoretical investigations of kaon production in heavy ion collisions, the discussions presented here are concerned with the elementary kaon production cross sections necessary for microscopic calculations.

One of the purposes of this report is to complete the parametrizations of total cross sections sufficient for all channels of the $\pi B \rightarrow YK$ reactions by a resonance model [7, 8] ($B = N, \Delta$ and $Y = \Lambda, \Sigma$). The results for the $\pi N \rightarrow \Lambda K$ reactions which were not given in our previous publications [7, 8] are presented. Furthermore, supplementary discussions to our previous work are presented.

The microscopic transport models [4] used for the calculations of kaon production in heavy ion collisions contain the following processes as the main collision terms: $B_1 B_2 \rightarrow B_1 B_2$, $NN \rightarrow NN\pi$, $NN \leftrightarrow N\Delta$, $\pi N \leftrightarrow \Delta$. Kaons are produced through the $B_1 B_2 \rightarrow B_3 YK$ and $\pi B \rightarrow YK$ reactions.

For a given impact parameter b , the Lorentz-invariant differential kaon multiplicity in the microscopic calculations is given by

* Refereed paper based on a contribution to the Japan–Australia Workshop on Quarks, Hadrons and Nuclei held at the Institute for Theoretical Physics, University of Adelaide, in November 1995.

$$\begin{aligned}
E \frac{d^3 N(b)}{d^3 p} = & \sum_{B_1 B_2} \int \left(E' \frac{d^3 \sigma_{B_1 B_2 \rightarrow B_3 Y K}(\sqrt{s_{B_1 B_2}})}{d^3 p'} \right) / \sigma_{B_1 B_2}^{tot}(\sqrt{s_{B_1 B_2}}) \\
& \times [1 - f(\mathbf{r}, \mathbf{p}, t)] \frac{d\Omega_{3Y}}{4\pi} \\
& + \sum_{\pi B} E'' \frac{d^3 \sigma_{\pi B \rightarrow Y K}(\sqrt{s_{\pi B}})}{d^3 p''} / \sigma_{\pi B}^{tot}(\sqrt{s_{\pi B}}). \quad (1)
\end{aligned}$$

Here the primed and the double-primed quantities are in the centre-of-momentum (c.m.) frames of the two colliding baryons ($B_1 B_2$) and pion-baryon (πB), respectively, while the unprimed quantities are those in the c.m. of the two nuclei. Here also $\sigma_{B_1 B_2}^{tot}(\sqrt{s_{B_1 B_2}})$ and $\sigma_{\pi B}^{tot}(\sqrt{s_{\pi B}})$ are the total cross sections as functions of the respective c.m. energies $\sqrt{s_{B_1 B_2}}$ and $\sqrt{s_{\pi B}}$. The factor $[1 - f(\mathbf{r}, \mathbf{p}, t)]$ stands for the Pauli blocking effects for the final baryon B_3 , and Ω_{3Y} is the solid angle of the relative momentum between the final baryon B_3 and hyperon Y . The Lorentz-invariant double differential kaon production cross section is obtained by integrating the kaon multiplicity Eq. (1) over the impact parameter b multiplied by the factor $2\pi b$. Eq. (1) shows that the elementary kaon production cross sections are directly related to the differential kaon yields. Thus, it is important to use correct elementary kaon production cross sections for the microscopic investigations of kaon production in heavy ion collisions.

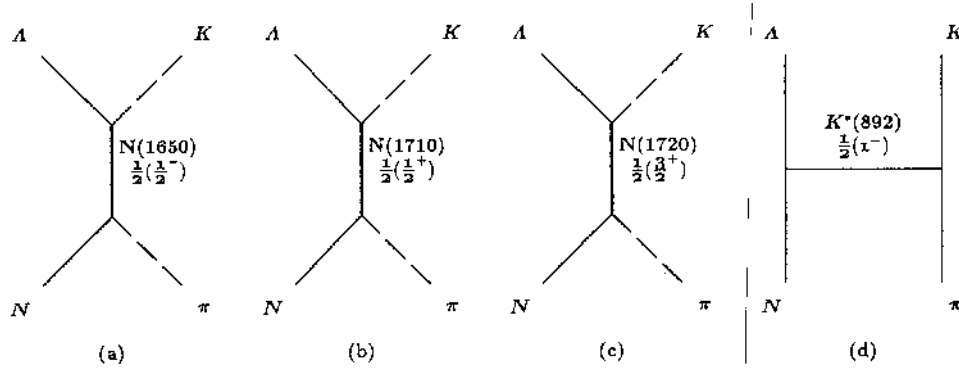


Fig. 1. Processes contributing to the $\pi N \rightarrow \Lambda K$ reactions. The diagrams correspond to (a) $N(1650)$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, (b) $N(1710)$ $\frac{1}{2}(\frac{1}{2}^+)$, (c) $N(1720)$ $\frac{1}{2}(\frac{3}{2}^+)$ s-channels and (d) $K^*(892)$ -exchange, respectively.

The elementary kaon production cross sections used for the microscopic calculations are usually taken from Randrup and Ko [5] in baryon-baryon collisions. In pion-nucleon collisions they are usually taken from Cugnon and Lombard [6]. Our interest at the moment is with the latter case. In the work of Cugnon and Lombard [6] total cross sections are parametrized by introducing an isospin-averaging procedure, and by using the limited experimental data assuming proton and neutron number ($Z = N$) symmetry. This means that the differential cross sections must be investigated using a more general procedure. Furthermore, the reactions $\pi \Delta \rightarrow Y K$ cannot be investigated because no experimental data

are available. One of the motivations for this work is to study both the total and differential cross sections $\pi B \rightarrow YK$ based on a microscopic model. Below, we give a theoretical description of the $\pi N \rightarrow \Lambda K$ reactions which will be our main concern later.

The experimental data [9] show that three resonances $N(1650)(\frac{1}{2}^-)$, $N(1710)(\frac{1}{2}^+)$ and $N(1720)(\frac{3}{2}^+)$ make contributions to the $\pi N \rightarrow \Lambda K$ reactions. In addition, $K^*(892)$ exchange is included as an effective process which also simulates the other heavier K^* meson effects. The relevant processes are depicted in Fig. 1. The interaction Lagrangians used are:

$$\mathcal{L}_{\pi NN(1650)} = -g_{\pi NN(1650)}(\bar{N}(1650)\vec{\tau}N \cdot \vec{\phi} + \bar{N}\vec{\tau}N(1650) \cdot \vec{\phi}), \quad (2)$$

$$\mathcal{L}_{\pi NN(1710)} = -ig_{\pi NN(1710)}(\bar{N}(1710)\gamma_5\vec{\tau}N \cdot \vec{\phi} + \bar{N}\vec{\tau}\gamma_5N(1710) \cdot \vec{\phi}), \quad (3)$$

$$\mathcal{L}_{\pi NN(1720)} = \frac{g_{\pi NN(1720)}}{m_\pi}(\bar{N}^\mu(1720)\vec{\tau}N \cdot \partial_\mu\vec{\phi} + \bar{N}\vec{\tau}N^\mu(1720) \cdot \partial_\mu\vec{\phi}), \quad (4)$$

$$\mathcal{L}_{K\Lambda N(1650)} = -g_{K\Lambda N(1650)}(\bar{N}(1650)\Lambda K + \bar{K}\bar{\Lambda}N(1650)), \quad (5)$$

$$\mathcal{L}_{K\Lambda N(1710)} = -ig_{K\Lambda N(1710)}(\bar{N}(1710)\gamma_5\Lambda K + \bar{K}\bar{\Lambda}\gamma_5N(1710)), \quad (6)$$

$$\mathcal{L}_{K\Lambda N(1720)} = \frac{g_{K\Lambda N(1720)}}{m_K}(\bar{N}^\mu(1720)\Lambda\partial_\mu K + (\partial_\mu\bar{K})\bar{\Lambda}N^\mu(1720)), \quad (7)$$

$$\mathcal{L}_{K^*(892)\Lambda N} = -g_{K^*(892)\Lambda N}(\bar{N}\gamma^\mu\Lambda K_\mu^*(892) + \bar{K}^*_\mu(892)\bar{\Lambda}\gamma^\mu N), \quad (8)$$

$$\mathcal{L}_{K^*(892)K\pi} = if_{K^*(892)K\pi}(\bar{K}\vec{\tau}K_\mu^*(892) \cdot \partial^\mu\vec{\phi} - (\partial^\mu\bar{K})\vec{\tau}K_\mu^*(892) \cdot \vec{\phi}) + \text{h.c.} \quad (9)$$

The amplitudes are given by: $\mathcal{M}_{\pi^0 p \rightarrow \Lambda K^+} = -\mathcal{M}_{\pi^0 n \rightarrow \Lambda K^0} = \sqrt{\frac{1}{2}}\mathcal{M}_{\pi^+ n \rightarrow \Lambda K^+} = \sqrt{\frac{1}{2}}\mathcal{M}_{\pi^- p \rightarrow \Lambda K^0} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d$, where the amplitudes \mathcal{M}_a , \mathcal{M}_b , \mathcal{M}_c and \mathcal{M}_d correspond to the diagrams (a), (b), (c) and (d) respectively in Fig. 1.

To carry the calculations further, form factors are introduced which reflect the finite size of the hadrons. Those form factors are carried by each vertex. For the meson-baryon-(baryon resonance) vertex, the following form factor is used:

$$F(\vec{q}) = \frac{\Lambda_C^2}{\Lambda_C^2 + \vec{q}^2}, \quad (10)$$

where \vec{q} is the meson momentum, and Λ_C is the cut-off parameter. On the other hand, for the $K^*(892)-K-\pi$ vertex, the form factor studied in Ref. [10] is used:

$$F_{K^*(892)K\pi}(|\frac{1}{2}(p_K - p_\pi)|) = C |\frac{1}{2}(p_K - p_\pi)| \exp(-\beta |\frac{1}{2}(p_K - p_\pi)|^2). \quad (11)$$

Before discussing the results, the model parameters need to be specified. The cut-off parameter Λ_C appearing in Eq. (10) is $\Lambda_C = 0.8 \text{ GeV}$ for all meson-baryon-(baryon resonance) vertices. The values obtained for the coupling

constants with this cut-off value are given in Table 1. The fitted value for $g_{K^*(892)\Lambda N}$ is $g_{K^*(892)\Lambda N} = 0.45$. The other parameters C and β appearing in Eq. (11) are $C = 2.72$ fm and [10].

Table 1. Calculated coupling constants and the experimental branching ratios

$B^*(\text{resonance})$	$\Gamma^{full}(\text{MeV})$	$\Gamma_{N\pi}(\%)$	$g_{\pi NB^*}^2$	$\Gamma_{\Lambda K}(\%)$	$g_{K\Lambda B^*}^2$
$N(1650)$	150	70.0	1.41	7.0	6.40×10^{-1}
$N(1710)$	100	15.0	2.57	15.0	4.74×10^{-1}
$N(1720)$	150	15.0	5.27×10^{-2}	6.5	3.91
$f_{K^*(892)K\pi}^2$			$g_{K^*(892)\Lambda N}^2$		
6.89×10^{-1}			2.03×10^{-1}		
$(\Gamma = 50 \text{ MeV}, \quad \Gamma_{K\pi} = 100\%)$					

Here, it is appropriate to discuss $K^*(892)$ exchange. Our calculations were also performed with the inclusion of the tensor coupling interaction. However, it was found that the results show similar dependence on both c.m. energy and angle (or $\cos\theta_{c.m.}$) to those results calculated with the inclusion of the vector coupling interaction alone. Thus, for the present purposes it is enough to include only the vector coupling interaction.

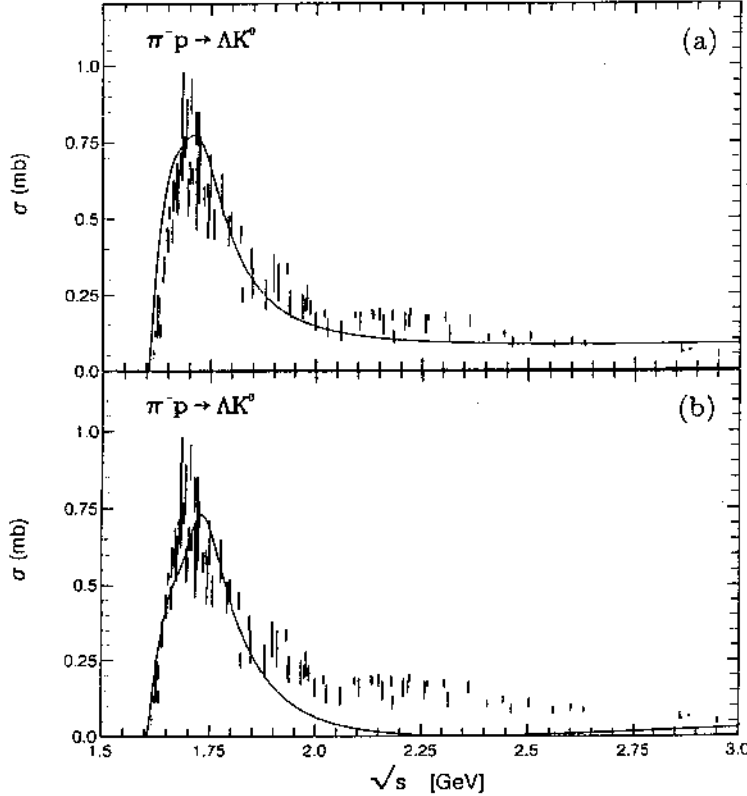


Fig. 2. Total cross sections $\pi^- p \rightarrow \Lambda K^0$. The experimental data are taken from Ref. [11]. The results are (a) without and (b) with the inclusion of interference terms.

The energy dependence of the total cross sections $\pi^- p \rightarrow \Lambda K^0$ is shown in Fig. 2, where (a) and (b) correspond to cases without and with the inclusion of interference terms. The sign combination among the interference terms in calculation (b) is selected in such a way that *both* total and differential cross sections are reproduced simultaneously. Mere inclusion of the single resonance or the K^* exchange alone cannot reproduce the energy dependence of the total cross section data.

Next, the differential cross sections $\pi^- p \rightarrow \Lambda K^0$ are shown in Fig. 3, where (a), (b) and (c) correspond to the pion beam momenta 0.980, 1.13 and 1.455 GeV/c, respectively. The general trends are reproduced, but the details are not yet satisfactory.

In Fig. 4 we give the energy dependence of the total cross sections $\pi N \rightarrow \Sigma K$. In Ref. [7], the $\Delta(1920)$ resonance was treated as an effective resonance which simulates other Δ resonance effects around the mass region 1.9 GeV. The two coupling constants $g_{K\Sigma\Delta(1920)}$ and $g_{\pi N\Delta(1920)}$ were scaled in Ref. [7]. The results obtained by using these scaled coupling constants are denoted by *set 1*. A more quantitative discussion of this scaling will be made below.

The experimental data [14] show that there are six Δ resonances which make contributions to the $\pi N \rightarrow \Sigma K$ reactions around the mass region 1.9 GeV (see Table 2). In order to understand the scaling factor quantitatively, we compare:

$$\frac{\text{the contribution of } \Delta(1920) \text{ to } \pi N \rightarrow \Sigma K}{\text{all } \Delta'(\text{masses around 1.9 GeV}) \text{ contribution to } \pi N \rightarrow \Sigma K} = \frac{10.4}{37.37} = 0.278,$$

$$\sqrt{\frac{g_{K\Sigma\Delta(1920)}^2 g_{\pi N\Delta(1920)}^2 (\text{from branching ratio})}{g_{K\Sigma\Delta(1920)}^2 g_{\pi N\Delta(1920)}^2 (\text{scaled})}} = \sqrt{\frac{1.11 \times 0.417}{3.83 \times 1.44}} = 0.289.$$

This comparison shows that the scaling factor ($=1.861$) for each coupling constant $g_{K\Sigma\Delta(1920)}$ and $g_{\pi N\Delta(1920)}$ is consistent with the total branching ratio obtained by summing these Δ resonance contributions. However, because of this effective description, the differential cross sections cannot be reproduced well. A more accurate determination of the branching ratios for these Δ resonances is necessary.

Here the $\pi\Delta \rightarrow YK$ reactions should also be mentioned. The parametrizations and figures given in Ref. [8] were obtained by using the lower values of the branching ratios for the resonances to decay to $\pi\Delta$. The parametrizations obtained by using the averaged values as given in Tables 1 and 2 of Ref. [8] will be given later. The difference between the previous parametrizations and those to be given later is that in the latter the change in the multiplication factor is large. However, the second term of the parametrization for the $\pi^+\Delta^0 \rightarrow \Sigma^0 K^+$ total cross section remains the same as before. As for the differential cross sections $\pi\Delta \rightarrow YK$, they are almost constant as a function of $\cos\theta$ in the c.m. frame for the beam energies for which calculations were made.

Finally, the parametrizations of the total cross sections sufficient for all channels of the $\pi B \rightarrow YK$ reactions in units of mb are given by:

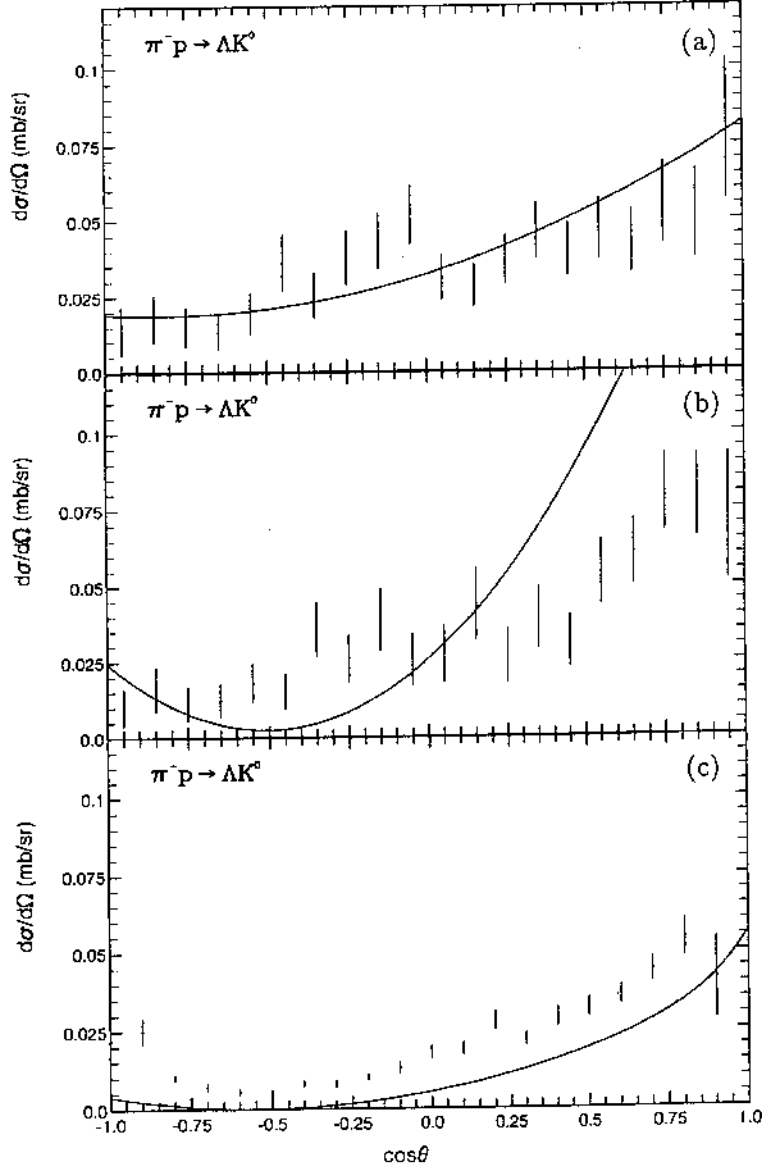


Fig. 3. Differential cross sections $\pi^- p \rightarrow \Lambda K^0$ in the c.m. frame: (a), (b) and (c) correspond to the pion beam momenta 0.980 GeV/c ($\sqrt{s} = 1.66$ GeV), 1.13 GeV/c ($\sqrt{s} = 1.742$ GeV) and 1.455 GeV/c ($\sqrt{s} = 1.908$ GeV), respectively. For (a) and (b), the experimental data are taken from Ref. [12], while for (c) they are taken from Ref. [13].

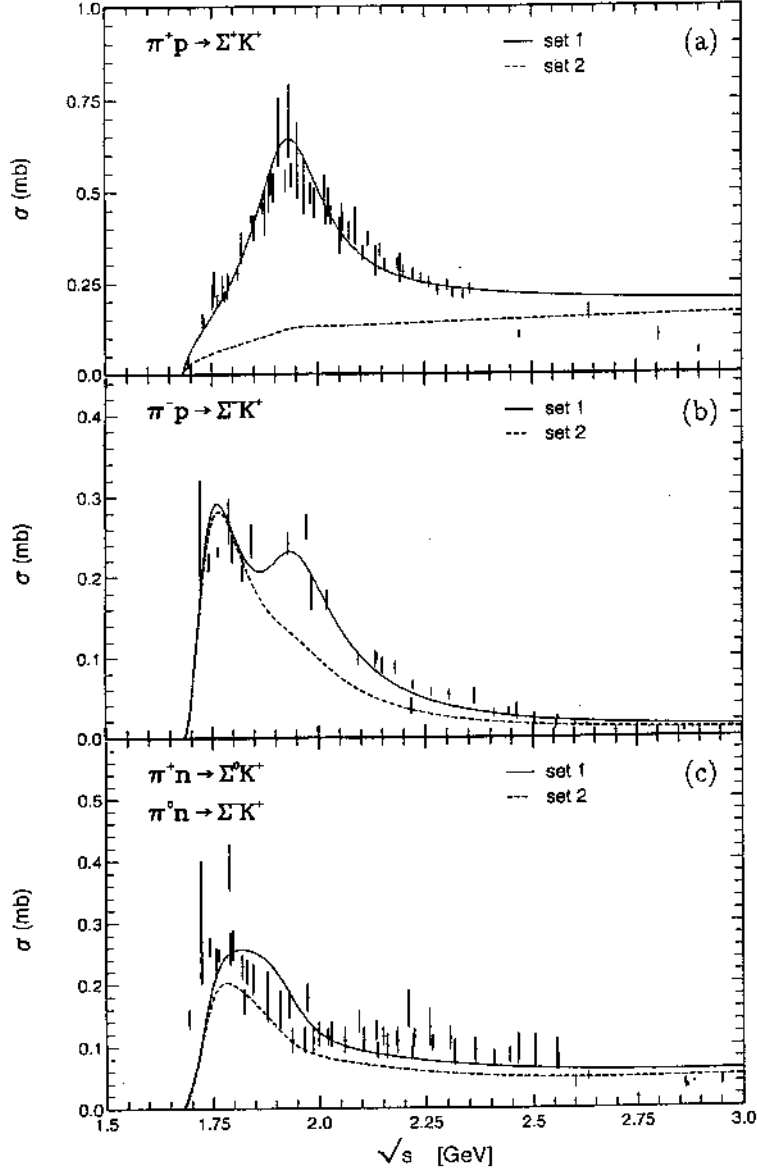


Fig. 4. Total sections for the reactions (a) $\pi^+ p \rightarrow \Sigma^+ K^+$, (b) $\pi^- p \rightarrow \Sigma^- K^+$ and (c) $\pi^+ n \rightarrow \Sigma^0 K^+$ and $\pi^0 n \rightarrow \Sigma^- K^+$, respectively. The notation set 1 in these figures indicates the results obtained by using the scaled values for $g_{K\Sigma\Delta(1920)}$ and $g_{\pi N\Delta(1920)}$. For further explanation, see Ref. [7].

Table 2. Contribution of $\Delta(1920)$ to the $\pi N \rightarrow \Sigma K$ reactions compared with other possible Δ resonances between 1900 and 1940 MeV [14]

The values in column four are all upper values

Δ^* (resonance)	Γ_{total} (MeV)	$(\Gamma_{\pi N \Delta^*} \Gamma_{K \Sigma \Delta^*})^{\frac{1}{2}} / \Gamma_{total}$	$(\Gamma_{\pi N \Delta^*} \Gamma_{K \Sigma \Delta^*})^{\frac{1}{2}}$ (MeV)
$\Delta(1900)$	200	<0.03	6.00
$\Delta(1905)$	350	$ 0.015 $	5.25
$\Delta(1910)$	250	<0.03	7.50
$\Delta(1920)$	200	$ 0.052 $	10.4
$\Delta(1930)$	350	<0.015	5.25
$\Delta(1940)$	198.4	<0.015	2.98

$$\begin{aligned}
\sigma(\pi^- p \rightarrow \Lambda K^0) &= \frac{0.007665(\sqrt{s} - 1.613)^{0.1341}}{(\sqrt{s} - 1.720)^2 + 0.007826}, \\
\sigma(\pi^+ p \rightarrow \Sigma^+ K^+) &= \frac{0.03591(\sqrt{s} - 1.688)^{0.9541}}{(\sqrt{s} - 1.890)^2 + 0.01548} \\
&\quad + \frac{0.1594(\sqrt{s} - 1.688)^{0.01056}}{(\sqrt{s} - 3.000)^2 + 0.9412}, \\
\sigma(\pi^- p \rightarrow \Sigma^- K^+) &= \frac{0.009803(\sqrt{s} - 1.688)^{0.6021}}{(\sqrt{s} - 1.742)^2 + 0.006583} \\
&\quad + \frac{0.006521(\sqrt{s} - 1.688)^{1.4728}}{(\sqrt{s} - 1.940)^2 + 0.006248}, \\
\sigma(\pi^+ n \rightarrow \Sigma^0 K^+) &= \sigma(\pi^0 n \rightarrow \Sigma^- K^+) \\
&\quad + \frac{0.05014(\sqrt{s} - 1.688)^{1.2878}}{(\sqrt{s} - 1.730)^2 + 0.006455}, \\
\sigma(\pi^0 p \rightarrow \Sigma^0 K^+) &= \frac{0.003978(\sqrt{s} - 1.688)^{0.5848}}{(\sqrt{s} - 1.740)^2 + 0.006670} \\
&\quad + \frac{0.04709(\sqrt{s} - 1.688)^{2.1650}}{(\sqrt{s} - 1.905)^2 + 0.006358}, \\
\sigma(\pi^- \Delta^{++} \rightarrow \Lambda K^+) &= \frac{0.009883(\sqrt{s} - 1.613)^{0.7866}}{(\sqrt{s} - 1.720)^2 + 0.004852}, \\
\sigma(\pi^- \Delta^{++} \rightarrow \Sigma^0 K^+) &= \frac{0.007448(\sqrt{s} - 1.688)^{0.7785}}{(\sqrt{s} - 1.725)^2 + 0.008147}, \\
\sigma(\pi^0 \Delta^0 \rightarrow \Sigma^- K^+) &= \frac{0.01052(\sqrt{s} - 1.688)^{0.8140}}{(\sqrt{s} - 1.725)^2 + 0.007713}, \\
\sigma(\pi^+ \Delta^0 \rightarrow \Sigma^0 K^+) &= \frac{0.003100(\sqrt{s} - 1.688)^{0.9853}}{(\sqrt{s} - 1.725)^2 + 0.005414} \\
&\quad + \frac{0.3179(\sqrt{s} - 1.688)^{0.9025}}{(\sqrt{s} - 2.675)^2 + 44.88}, \\
\sigma(\pi^+ \Delta^- \rightarrow \Sigma^- K^+) &= \frac{0.02629(\sqrt{s} - 1.688)^{1.2078}}{(\sqrt{s} - 1.725)^2 + 0.003777},
\end{aligned}$$

where all the parametrizations given above should be understood to be zero below threshold. Furthermore, the parametrizations for the $\pi\Delta \rightarrow YK$ reactions are obtained without the inclusion of interference terms. The above are the completion of the parametrizations for the total cross sections $\pi B \rightarrow YK$. Parametrizations for the other channels can be obtained by multiplying the relevant constant factors arising from isospin space [7, 8].

The next task is to investigate the $B_1 B_2 \rightarrow B_3 YK$ reactions by using the same resonance model, where most of the model parameters have already been fixed by this investigation. This program is now in progress.

Acknowledgments

One of the authors (K.T.) expresses his thanks to Prof. A. W. Thomas and to Drs A. G. Williams and K. Saito who organized ‘Quarks, Hadron and Nuclei Workshop’ held in Adelaide for the opportunity to report the results mentioned here. KT acknowledges the support of the Australian Research Council and SWH acknowledges support by GSI under contract No. T  F s T.

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