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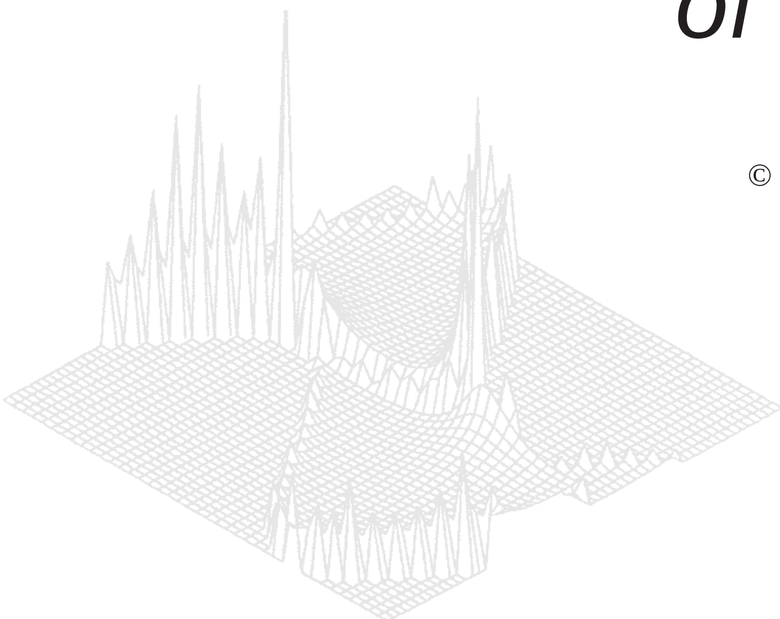

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Effects of Positron Density and Temperature on Large Amplitude Ion-acoustic Waves in an Electron–Positron–Ion Plasma

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Abstract

The nonlinear wave structures of large amplitude ion-acoustic waves are studied in a plasma with positrons. We have presented the region of existence of the ion-acoustic waves by analysing the structure of the pseudopotential. The region of existence sensitively depends on the positron to electron density ratio, the ion to electron mass ratio and the positron to electron temperature ratio. It is shown that the maximum Mach number increases as the positron temperature increases and the region of existence of the ion-acoustic waves spreads as the positron temperature increases. The present theory is applicable to analyse large amplitude ion-acoustic waves associated with positrons which may occur in space plasmas.

1. Introduction

In contrast to the usual plasma with electrons and positive ions, it has been known that the nonlinear waves in plasmas having positrons behave differently (Rizzato 1988). In fact, electron–positron–ion plasmas appear in the early universe (Rees 1983), active galactic nuclei (Miller and Witta 1987) and in pulsar magnetospheres (Michel 1982). When positrons are introduced in the plasma, the response of the plasma to disturbances is found to be drastically modified. There are several reports on solitons with small amplitudes in plasma, with a significant percentage of positrons (Tandberg Hansen and Emslie 1988). An electron–positron–ion plasma is usually characterised as a fully ionised gas consisting of electrons and positrons, the masses of which are equal (Tandberg Hansen and Emslie 1988). Nonlinear waves propagating in such plasmas have received a great deal of attention in investigating the nonlinear structures. The studies of nonlinear waves have been focused on the wave structures, such as solitons, double layers, vortices and so on. We have also suggested that the high-speed streaming particles excite various kind of nonlinear waves in space plasmas (Nejoh 1988, 1992, 1994*a*, 1994*b*, 1996*a*, 1996*b*; Nejoh and Sanuki 1994, 1995).

On the other hand, large amplitude nonlinear waves and, in particular, the effect of the ratio of the ion to electron mass of large amplitude ion-acoustic waves have not yet been studied in electron–positron–ion plasmas. Hence, in this

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paper, we investigate the region of existence of the large amplitude ion-acoustic waves in the presence of ions with finite temperature and hot electrons and positrons. We take a model where the dynamics of the nonlinear wave motion is governed by the hydrodynamic equations, whereas the temperature of the positron fluid is finite and the fluid follows the Boltzmann distribution.

The purpose of this paper is to derive the pseudopotential for ion-acoustic waves in an electron–positron–ion plasma and to show the dependence of the existence of ion-acoustic waves on the positron density and temperature. Stationary nonlinear potential structures can be formulated in terms of an integral equation of the same form as that governing the motion of particles in a potential well. The conditions for existence of large amplitude nonlinear ion-acoustic waves will be confirmed by considering the ratio of the positron to electron density, the ratio of the positron to electron temperature and normalised potential in an electron–positron–ion plasma. If there are no positrons, our results reduce to those obtained from ordinary electron–ion plasma theory. However, we are interested in the case where positrons exist in addition to electrons and positive ions in space.

The structure of this paper is as follows. In Section 2 we present the basic hydrodynamic equations for an electron–positron–ion plasma and derive an energy equation with the pseudopotential. In Section 3 we discuss the condition for existence of large amplitude ion-acoustic waves on the basis of an energy equation. The dependence of the pseudopotential on the normalised potential, the ratio of the positron to electron density, the positron to electron temperature ratio and the ratio of the ion to electron mass is presented. Section 4 is devoted to our concluding discussion.

2. Theory

We consider a plasma consisting of electrons, positrons and positive ions. In order to study one-dimensional propagation of large amplitude ion-acoustic waves, we describe a set of the fluid equations. The nonlinear wave propagation of low phase velocity is governed by the hydrodynamic equations of the species. We assume that the phase velocity is much smaller than the electron (positron) thermal velocity and is larger than the ion thermal velocity. The continuity equation and the equation of motion for electrons are described by

$$\frac{\partial}{\partial t} n_{\mathrm{e}} + \frac{\partial}{\partial x}(n_{\mathrm{e}}\, v_{\mathrm{e}}) = 0\,, \tag{1a}$$

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{e}}\frac{\partial}{\partial x}\right)v_{\mathrm{e}} - \frac{\partial \phi}{\partial x} = 0\,. \tag{1b}$$

We have the following equation for positrons:

$$n_{\mathrm{p}} = a\exp(-\phi/\beta)\,. \tag{2}$$

The continuity equation and the equation of motion for ions are written as

$$\frac{\partial}{\partial t} n_{\mathrm{i}} + \frac{\partial}{\partial x}(n_{\mathrm{i}} v_{\mathrm{i}}) = 0 \,, \tag{3a}$$

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{i}} \frac{\partial}{\partial x}\right) v_{\mathrm{i}} + \frac{1}{Q} \frac{\partial \phi}{\partial x} = 0 \,. \tag{3b}$$

The Poisson equation is given by

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\mathrm{e}} - n_{\mathrm{p}} - n_{\mathrm{i}} \,. \tag{4}$$

Here the parameters $\alpha = n_{\mathrm{p0}}/n_0$, $\beta = T_{\mathrm{p}}/T_{\mathrm{e}}$ and $Q = m_{\mathrm{i}}/m_{\mathrm{e}}$ are the ratio between the unperturbed positron and electron densities, the ratio between the positron temperature and electron temperature and the ion mass to electron mass ratio, respectively. The variable n_{i} stands for the ion density. The densities are normalised by the unperturbed background electron density n_0. The subscripts e, p and i denote electrons, positrons and ions, respectively. The space coordinate x, time t, velocities and electrostatic potential ϕ are normalised by the electron Debye length $\lambda_{\mathrm{D}} = (\epsilon_0 \, \kappa \, T_{\mathrm{e}}/n_0 \, e^2)^{1/2}$, the ion plasma period $\omega_{\mathrm{i}}^{-1} = (\epsilon_0 \, m_{\mathrm{i}}/n_0 e^2)^{1/2}$, the sound velocity $C_{\mathrm{s}} = (\kappa T_{\mathrm{e}}/m_i)^{1/2}$, and $\kappa T_{\mathrm{e}}/e$, respectively, where m_{i}, ϵ_0 and e are the ion mass, the permittivity of the vacuum and the electric charge, respectively. In equilibrium, we have $n_{\mathrm{i0}} + n_{\mathrm{p0}} = n_0$.

In the linear limit, equations (1a)–(4) give rise to the dispersion relation of the ion-acoustic waves in an electron–positron–ion plasma. We derive the dispersion relation as

$$\omega^2 = \frac{1 + (1-\alpha)/Q}{\alpha/\beta} \, \frac{k^2}{1 + (\alpha/\beta)^{-1} k^2} \,, \tag{5}$$

where ω and k are the frequency and the wave number.

In order to solve equations (1a)–(4), we consider the physical quantities derived in the stationary state. We introduce a variable $\xi = x - Mt$ and assume the stationary state in the moving frame, where M denotes the speed of the nonlinear structure. Integrating equations (1a) and (1b), we obtain the electron number density as

$$n_{\mathrm{e}} = \frac{1}{\sqrt{1 + 2\phi/M^2}} \,. \tag{6}$$

Integrating equations (3a) and (3b), we get the ion density

$$n_{\mathrm{i}} = (1-\alpha) \frac{1}{\sqrt{1 - 2\phi/QM^2}} \,, \tag{7}$$

where we used the boundary conditions $n_{\mathrm{e}} \to 1$, $n_{\mathrm{p}} \to \alpha$, $n_{\mathrm{i}} \to 1-\alpha$, v_{e}, v_{p}, $v_{\mathrm{i}} \to 0$, $\phi \to 0$ at $\xi \to \pm\infty$. From (6) and (7), equation (4) reduces to the nonlinear Poisson equation

$$\frac{\partial\phi}{\partial x^2} = \frac{1}{\sqrt{1+2\phi/M^2}} - \alpha\exp(-\phi/\beta) - \frac{1-\alpha}{\sqrt{1-2\phi/QM^2}} \equiv -\frac{\partial V(\phi)}{\partial\phi}\,, \tag{8}$$

where $V(\phi)$ is the pseudopotential.

From equation (8), we obtain *the energy integral*

$$\tfrac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + V(\phi) = 0\,. \tag{9}$$

The pseudopotential reads as

$$V(\phi) = M^2(1-\sqrt{1+2\phi/M^2}) + \alpha\beta[1-\exp(-\phi/\beta)]$$

$$+\,QM^2(1-\alpha)(1-\sqrt{1-2\phi/QM^2})\,. \tag{10}$$

It should be noted, from equation (10), that $0 < \phi < QM^2/2$.

In order for the large amplitude ion-acoustic wave to exist, the following two conditions must be satisfied:

(i) The pseudopotential must have a local maximum at the point $\phi = 0$, and the equation $V(\phi) = 0$ should have at least one real solution. This condition derives the inequality

$$\frac{1}{M^2} - \frac{\alpha}{\beta} + \frac{1-\alpha}{QM^2} < 0\,. \tag{11}$$

We note that the condition (11) is a consequence of the inequality

$$\frac{\mathrm{d}^2V(\phi)}{\mathrm{d}\phi^2} < 0 \quad \text{at} \quad \phi = 0\,. \tag{12}$$

Moreover, it follows from (11) that supersonic ion-acoustic waves can exist in electron–positron–ion plasmas.

(ii) Nonlinear ion-acoustic waves exist only when $V(\phi_{\mathrm{M}}) \geq 0$, where the maximum potential ϕ_{M} is determined by $\phi_{\mathrm{M}} = QM^2/2$. The equation $V(\phi) = 0$ can have only one real nonzero solution; this solution being positive. This condition can be described as

$$\alpha\beta\left[1-\exp\left(-\frac{QM^2}{2\beta}\right)\right] + M^2[1-\sqrt{1+Q}+Q(1-\alpha)] \geq 0\,. \tag{13}$$

Fig. 1 illustrates the dependence of the maximum Mach number on the positron temperature β, for which large amplitude ion-acoustic waves can exist, where $\alpha = 0{\cdot}97745$ and $Q = 1836$, that is, the ions being protons. It is shown that only supersonic ion-acoustic waves can propagate and the positron temperature increases the maximum Mach number in the plasma under consideration. The maximum Mach number and, correspondingly, the maximum amplitude of the ion-acoustic wave depend significantly on the parameters α and β. The region for existence of the ion-acoustic wave is characterised by these conditions.

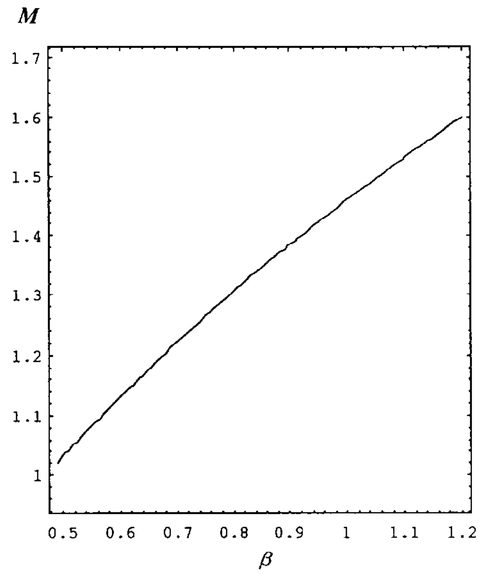

Fig. 1. Dependence of the maximum Mach number M on the positron temperature β for $\alpha = 0\cdot 97745$ and $Q = 1836$.

A complete analytical investigation of the ion-acoustic solitons in this system is possible for the small amplitude wave limit ($\phi \ll 1$). The specific results can be obtained by expanding $V(\phi)$ in powers of ϕ and keeping up to the third-order terms ϕ^3. Accordingly, equation (10) takes the form

$$V(\phi) \approx \frac{1}{2M^2}\left(1 - \frac{\alpha}{\beta}M^2 + \frac{1-\alpha}{Q}\right)\phi^2 + \frac{1}{6M^4}\left(-3 + \frac{\alpha}{\beta^2}M^4 + 3\,\frac{1-\alpha}{Q^2}\right)\phi^3\,. \tag{14}$$

Then, integrating (9) with (14), we obtain a soliton solution

$$\phi = \frac{M^2\left(1 - \frac{\alpha}{\beta}M^2 + \frac{1-\alpha}{Q}\right)}{-1 + \frac{1}{3}\frac{\alpha}{\beta^2}M^4 + \frac{1-\alpha}{Q^2}}\,\mathrm{sech}^2\left(\frac{1}{2M}\sqrt{1 - \frac{\alpha}{\beta}M^2 + \frac{1-\alpha}{Q}}\,(\xi - \xi_0)\right).$$

It should be noted that the ion-acoustic soliton exists in the limiting case with $\phi \ll 1$. If there are no positrons, $\alpha \to 0$, it is obvious that equations (10), (11) and (14) reduce to the results obtained from ordinary electron–ion plasma theory.

We study the nonlinear potential structures for ion-acoustic waves on the basis of equations (9)–(13) in the following section.

3. Pseudopotential Structure and the Region of Large Amplitude Ion-acoustic Waves

We consider the nonlinear wave structures of large amplitude ion-acoustic waves in the case where the positron temperature and number density are important. We show a bird's eye view of the pseudopotential $V(\phi)$ in Fig. 2, in the case of $M = 1 \cdot 1$, $\beta = 5$, by numerical calculation. Fig. 3 illustrates the dependence of $V(\phi)$ on the potential ϕ when $M = 1 \cdot 1$, $\beta = 5$ and $\alpha = 0 \cdot 95$. In the case where the positron temperature increases, we show a bird's eye view of the pseudopotential in Fig. 4 when $M = 1 \cdot 4$, $\beta = 20$. Here we assume that $Q = 1836$. The pseudopotential $V(\phi)$ versus ϕ in this case is also illustrated in Fig. 5 for $M = 1 \cdot 4$, $\beta = 20$ and $\alpha = 0 \cdot 97$.

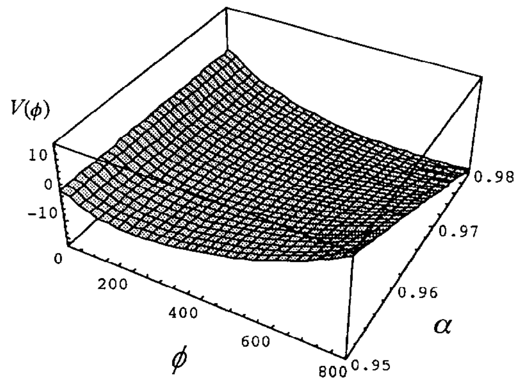

Fig. 2. Bird's eye view of the pseudopotential for the large amplitude ion-acoustic waves in the case of $M = 1 \cdot 1$ and $\beta = 5$.

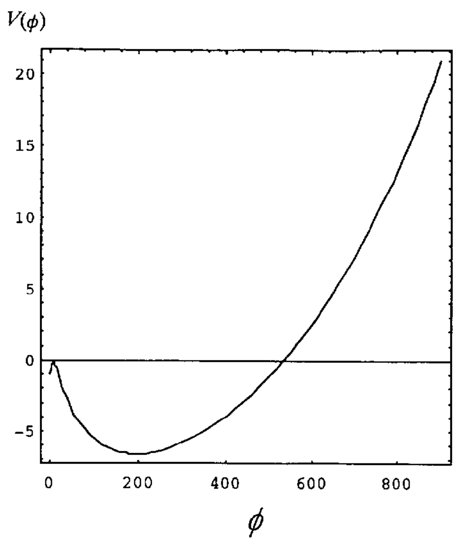

Fig. 3. Pseudopotential $V(\phi)$ against the electrostatic potential ϕ for $M = 1 \cdot 1$, $\beta = 5$ and $\alpha = 0 \cdot 95$.

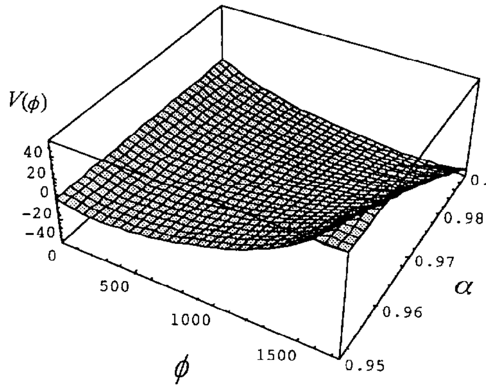

Fig. 4. Bird's eye view of the pseudopotential for the large amplitude ion-acoustic waves in the case of $M = 1 \cdot 4$ and $\beta = 20$.

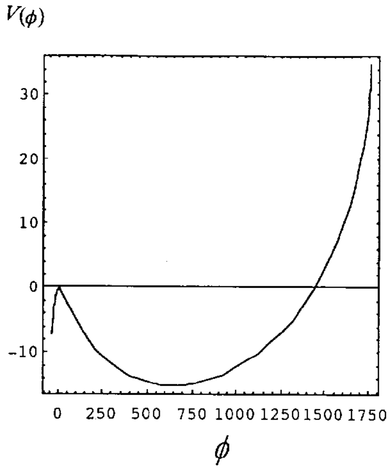

Fig. 5. Pseudopotential $V(\phi)$ against the electrostatic potential ϕ for $M = 1 \cdot 4$, $\beta = 20$ and $\alpha = 0 \cdot 97$.

In Fig. 6 we illustrate the region for existence of large amplitude ion-acoustic waves, depending on the ratio α of the positron density to the background electron density in the case of $M = 1 \cdot 1$ and $\beta = 5$ (Fig. 6*a*). Large amplitude ion-acoustic waves propagate in the lower region bounded by the curve but do not exist in other regions. We show the region for existence in the ϕ–α plane in Fig. 6*b* for $M = 1 \cdot 4$ and $\beta = 20$. Ion waves exist in the lower region bounded by the curve. It turns out that large amplitude ion-acoustic waves can propagate under the proper conditions mentioned above.

4. Concluding Discussion

The nonlinear wave structures of large amplitude ion-acoustic waves were studied in a plasma with positrons. We have presented the region for existence of the ion-acoustic waves on the basis of the fluid equations. We investigated the conditions for existence of the stationary supersonic ion-acoustic waves by analysing the structures of the pseudopotential. Typical results are shown in

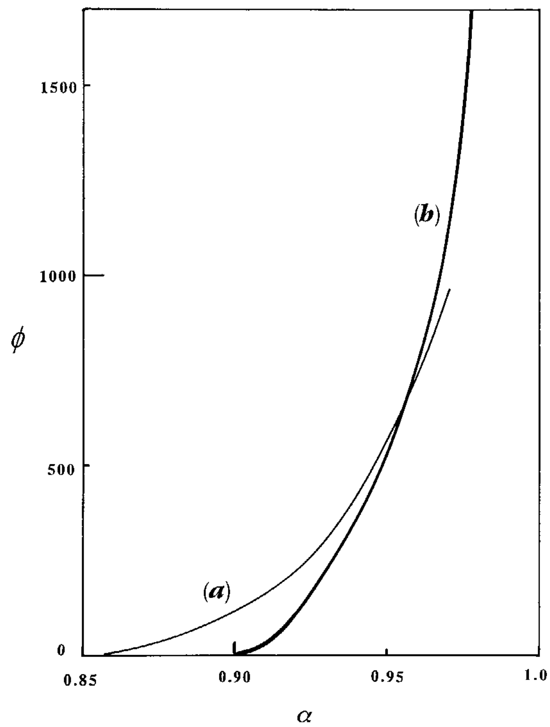

Fig. 6. The ϕ–α plane where the ion-acoustic wave exists in the case of (a) $M = 1{\cdot}1$ and $\beta = 5$; (b) $M = 1{\cdot}4$ and $\beta = 20$, where $Q = 1836$. The region for existence of the large amplitude ion-acoustic waves lies in the lower region bounded by the two curves.

Figs 1–6. The properties of nonlinear ion-acoustic waves change drastically due to the contribution of positrons. Thus we can show the characteristic features of the waves presented here as follows:

(1) The supersonic ion-acoustic waves can propagate in the electron–positron–ion plasma. The maximum Mach number increases as the positron temperature increases.
(2) The conditions for existence of the large amplitude ion-acoustic waves sensitively depend on the positron density, positron temperature and the electrostatic potential.
(3) Unlike the case of the electron beam–plasma system, the ion-acoustic wave exists when the positron density is nearly equal to the electron density. The region for existence of the large amplitude ion-acoustic waves spreads as the positron temperature increases.

The present investigation predicts new findings on large amplitude ion-acoustic waves in an electron–positron–ion plasma. In actual situations, the large amplitude ion-acoustic wave events associated with positrons are frequently observed in the solar atmosphere. Hence, referring to the present studies, we can understand the properties of large amplitude ion-acoustic waves in space where positrons exist.

Although we have not referred to any specific observations, the present theory is applicable to analyse large amplitude ion-acoustic shock and solitary waves associated with positrons which may occur in space plasmas.

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References

Michel, F. C. (1982). *Rev. Mod. Phys*. **54**, 1.
Miller, H. R., and Witta, P. J. (1987). 'Active Galactic Nuclei', p. 202 (Springer: Berlin).
Nejoh, Y. N. (1988). *Phys. Fluids* **31**, 2914.
Nejoh, Y. N. (1992). *Phys. Fluids* B **4**, 2830.
Nejoh, Y. N. (1994*a*). *J. Plasma Phys*. **51**, 441.
Nejoh, Y. N. (1994*b*). *IEEE Trans. Plasma Sci.* **22**, 205.
Nejoh, Y. N. (1996*a*). *Phys. Plasmas* **3**, 1447.
Nejoh, Y. N. (1996*b*). *Aust. J. Phys*. **49,** 967.
Nejoh, Y. N., and Sanuki, H. (1994). *Phys. Plasmas* **1**, 2154.
Nejoh, Y. N., and Sanuki, H. (1995). *Phys. Plasmas* **2**, 4122.
Rees, M. J. (1983). 'The Very Early Universe' (Cambridge Univ. Press).
Rizzato, F. B. (1988). *J. Plasma Phys*. **40,** 289.
Tandberg-Hansen, E., and Emslie, A. G. (1988). 'The Physics of Solar Flares', p. 124 (Cambridge Univ. Press).

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