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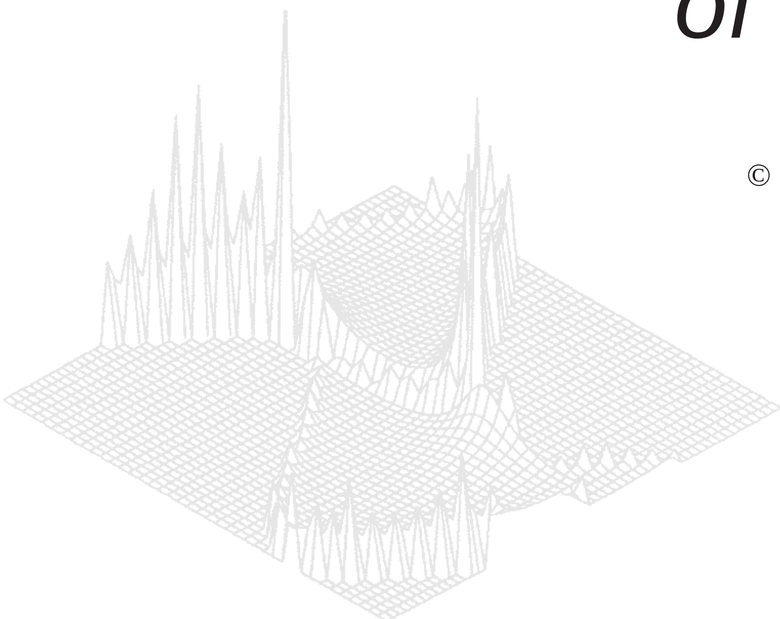

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Phase Transitions and Topological Defects in the Early Universe*

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Abstract

Our present theories of particle physics and cosmology, taken together, suggest that very early in its history, the universe underwent a series of phase transitions, at which topological defects, similar to those formed in some condensed matter transitions, may have been created. Such defects, in particular cosmic strings, may survive long enough to have important observable effects in the universe today. Predicting these effects requires us to estimate the initial defect density and the way that defects subsequently evolve. Very similar problems arise in condensed matter systems, and recently it has been possible to test some of our ideas about the formation of defects using experiments with liquid helium-3 (in collaboration with the Low Temperature Laboratory in Helsinki). I shall review the present status of this theory.

1. Introduction

Over the last twenty years there has been a remarkable interaction between the physics of the very small — elementary particles — and the very large — cosmology. Increasingly, particle physicists are looking to the early universe for tests of their theories, while cosmologists turn to ideas from particle physics to help to understand the large-scale structure of the universe. In interesting ways this interaction is now being extended to include also intermediate scales — condensed matter physics. One intriguing idea is that topological defects might have been formed at phase transitions in the early universe, similar to those formed in some phase transitions in condensed matter, and that such defects might have left traces still visible today (Kibble 1976). In trying to predict the observable consequences, we have to know how defects are formed and evolve. Direct tests of these mechanisms are impossible, for the obvious reason that the early universe is not accessible to experimentation. But we can test some of our ideas in the very different world of condensed matter.

(1a) Phase Transitions in the Early Universe

Let me start by explaining why we think there were phase transitions in the early universe. A key idea in modern particle physics is that of unification. One of the great successes was the bringing together of weak and electromagnetic interactions in the unified *electroweak* gauge theory, now part of the accepted

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'standard model' of particle physics. At ordinary laboratory energies, the weak and electromagnetic interactions seem very different. But we now understand their differences as low-energy phenomena, stemming from the fact that the carriers of weak interactions, the W and Z particles, have large masses, of order 100 GeV/c^2, while the photon is massless. The W and Z masses — and indeed the masses of other particles too — arise from a spontaneous breaking of the underlying symmetry. There is in this theory a phase transition, at a temperature $T_c \sim 100$ GeV. (I shall generally use units with $c = \hbar = k_B = 1$; this really means $k_B T_c \sim 100$ GeV.) The basic symmetry only becomes apparent above this phase transition, at energy scales well above 100 GeV.

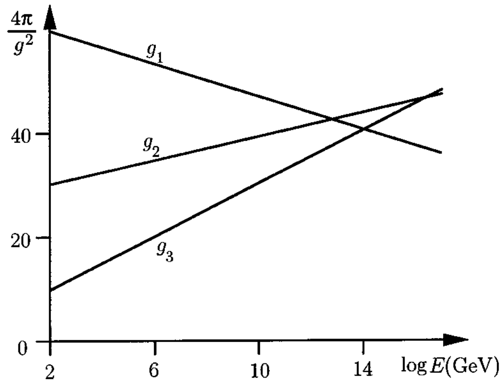

Fig. 1. Coupling constants in the standard model.

The standard model of particle physics, which has stood up astonishingly well to all experimental tests, comprises the electroweak theory, based on the symmetry group SU(2)×U(1), and the gauge theory of strong interactions, *quantum chromodynamics* (QCD), based on SU(3). We would like to believe that the separate parts of this model would themselves become unified at an even higher energy scale, forming a 'grand unified theory' (GUT). There is indeed some evidence to support this idea. The coupling constants that determine the strengths of the various interactions have a weak, logarithmic energy dependence, and would all become roughly equal at an energy scale of around 10^{15} or 10^{16} GeV (see Fig. 1). This strongly suggests that something interesting should happen in that energy region, most probably another symmetry-breaking phase transition.

Indeed we would like to go further, and combine all the interactions including gravity in what is often rather grandly, and perhaps misleadingly, called a 'theory of everything' (TOE). The best candidate at present is *superstring theory*, in which the fundamental objects are no longer point particles but tiny one-dimensional loops of string. (It must be emphasised that these superstrings are quite different from the 'cosmic strings' about which I will have much to say later.) If such unification is indeed possible, we would expect it to happen at the *Planck energy*. This is the unique energy one can construct from the three fundamental constants of physics — the speed of light c, Planck's constant $\hbar$ and Newton's constant G — and is about $M_{Pl} \sim 10^{19}$ GeV. It is the energy scale at which gravity becomes as strong as the other interactions. It is very suggestive that the Planck energy is only a few orders of magnitude above the GUT energy scale.

According to currently accepted theories, the early universe began with a 'hot big bang', and has been cooling more or less adiabatically ever since. This process may have been interrupted or preceded by a period of *inflation*, very rapid, possibly exponential, expansion, during which the universe was dominated by 'vacuum energy', followed by reheating. So at least from the end of inflation, the temperature has been steadily falling, and if ideas of unification are correct, the universe must have gone through a sequence of phase transitions at successively lower temperatures. One of these is the electroweak transition at about 100 GeV, a temperature that would have been reached in the standard picture about 10^{-10} s after the Big Bang. There may well be another transition even after that — the *quark–hadron* transition at around 100 MeV or 10^{-4} s, at which the dense soup of quarks and gluons condenses into individual hadrons, such as protons, neutrons or pions. However, no symmetry-breaking is involved in that case.

More interesting, though, from a cosmological point of view, are the earlier transitions, at scales between the Planck and electroweak energies. Whether the Planck energy itself corresponds to a phase transition is unclear. We can hardly do more than speculate about conditions at or above the Planck scale. Our present theories break down there, because we lack a consistent quantum theory of gravity. It does seem likely, however, that a transition of some sort occurred at the GUT scale, say just 10^{-37} s after the Big Bang, though any details are at present quite speculative. It is also quite probable that there was a later transition associated with the breaking of *supersymmetry*. This is an essential ingredient of superstring theory and plays a very important role in eliminating divergences. There is as yet no direct experimental support for supersymmetry, in particular no sign of the 'supersymmetric partners' that all particles should have. So if it is present this symmetry must be broken at a fairly high energy scale, probably somewhere between 10^3 and 10^{11} GeV. However, there is some indirect support, in the fact that the lines in Fig. 1 do not quite interesect at a point: in the minimal supersymmetric extension of the standard model, they can be brought together.

The picture that emerges then is of a sequence of symmetry-breaking phase transitions very early in the history of the universe. We have of course no direct way of testing such ideas. The question is: what evidence could we find in the universe today for these early, violent events? Surprisingly perhaps, there *could* be some very clear evidence. Many symmetry-breaking phase transitions, in particle physics models as well as in condensed matter, produce *defects*, such as domain walls, strings or vortices, and monopoles. Examples in condensed matter include quantised vortices in superfluid helium, quantised flux tubes in type-II superconductors, and disclination lines in nematic liquid crystals.

(1b) Cosmological Effects of Defects

Let us assume that there is an early phase transition, say at the GUT scale, at which defects are formed. As I will explain in more detail later, such defects are often stable for topological reasons, and could therefore survive long after their initial formation; some might even survive to the present day. If so, because they are extremely massive, they could have profound effects on the structure of the universe. Generally speaking, domain walls and monopoles can be ruled out: their effects are too extreme. Cosmic strings, however, are definitely a possibility. So too are 'textures', which although not strictly speaking defects are very similar in character. I will concentrate on cosmic strings. (For recent reviews, see Shellard and Vilenkin 1994 and Hindmarsh and Kibble 1995.)

Either strings or textures could provide the initial density perturbations from which galaxies eventually condense, and could explain the anisotropy in the cosmic microwave background radiation, first observed by the COBE satellite. Of course, there are other explanations for these small perturbations. The most popular theory at present is that they are generated by quantum fluctuations during a period of inflation. Defects do however provide a viable alternative scenario. In one respect they provide a better explanation, because the overall magnitude of the perturbations naturally comes out about right if the defects are formed at the GUT scale, whereas in most inflationary models it requires rather careful fine tuning. One of the exciting things about this field at the present time is that there is in prospect an avalanche of new data from satellites and ground-based observations which promises to resolve these issues once and for all within a few years.

Clearly it is very important to be able to make precise predictions to compare with the results of observation. Inflationary models have made quite specific predictions; and indeed, as a result, the simplest have been ruled out. It has proved harder to get accurate predictions from defect models, because defect formation and evolution are complex, non-linear processes, much harder to calculate. Considerable progress has been made, nevertheless. It has recently been shown that some predictions of defect models are rather robust, despite our ignorance of the details of how the defects are formed and evolve. In particular a lot of information can be gained from a search for the oscillations predicted by inflationary models in the angular power spectrum of the microwave anisotropy. Most defect models tend to predict qualitatively different oscillations.

It would be more satisfactory to be able to make definitive predictions of the effects of particular kinds of defects. To do that we have to try to follow the processes of defect formation and evolution. So far as evolution of cosmic strings is concerned, most of the information we have comes from computer simulations which have been carried out by three different groups (Albrecht and Turok 1989; Bennett and Bouchet 1990; Allen and Shellard 1990; Shellard and Allen 1990). I and my colleagues have also studied the problem analytically (Copeland *et al.* 1992; Austin *et al.* 1993, 1995). All the studies are consistent with the idea that the evolving string configuration reaches a 'scaling' regime, in which the strings in the universe would look essentially the same at all times, only scaled to the horizon distance at the time, though the conclusion does depend on the values of a couple of parameters that are as yet rather poorly known.

So far as formation is concerned, there has been a lot of debate recently about how to estimate the initial defect density, with widely divergent predictions. Here condensed matter has come to our aid. Surprisingly perhaps, the best analogue we have been able to find of phase transitions at these extremely high temperatures turns out to be a transition in the very lowest temperature range, the superfluid transition in liquid ^{3}He, which occurs at a few millikelvin. The Low Temperature Laboratory of the Helsinki University of Technology has been studying this transition for some years. They have recently performed an experiment that seems to provide a rather close analogue of an early-universe phase transition (Ruutu *et al*. 1996). [A related experiment has been performed by a group from Grenoble and Lancaster (Bäuerle *et al*. 1996).] The results of the Helsinki experiment are quite accurately fitted by the theory initially developed in the context of the early universe, and have definitely resolved the uncertainty about which prediction of the defect density is correct. I shall describe this in some detail later. First, I must set the scene by explaining the mechanism of defect formation.

2. Topological Defects

(2a) Domain Walls

The simplest field theory that exhibits a defect-forming phase transition is a model of a single scalar field ϕ described by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mathcal{V}(\phi), \qquad \mathcal{V} = \tfrac{1}{8}\lambda(\phi^2 - \eta^2)^2. \tag{1}$$

The potential $\mathcal{V}$ (Fig. 2) has degenerate minima, at $\phi = \pm\eta$, so there are two degenerate vacuum states, characterised by $\langle\phi\rangle \approx \pm\eta$.

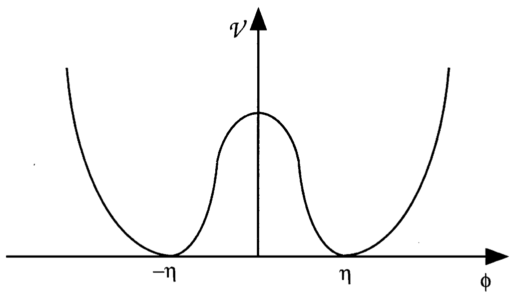

Fig. 2. Symmetry-breaking potential.

There is a phase transition in this model, at a critical temperature $T_c \sim \eta$. At high temperature, there are large fluctuations in ϕ, but on average it is zero. As the temperature falls below T_c, the field acquires a non-zero mean value, and has to choose between the two degenerate wells. This is a symmetry-breaking phase transition. The high-temperature phase is symmetric under the reflection $\phi \to -\phi$; the symmetry is spontaneously broken in the ordered low-temperature phase.

In a large system, such as the universe, the choice between $\pm\eta$ is made independently in widely separated regions, so we expect that some parts of the universe will have $\phi \approx \eta$ while others have $\phi \approx -\eta$. The two regions must then be separated by a *domain wall* across which ϕ varies as shown in Fig. 3*a*.

The domain wall is topologically stable. It can move; one domain can grow at the expense of the other — and in the special case where it completely surrounds one domain it could shrink to a point and disappear. But there is no local process by which part of a wall can disappear; it can never have an edge.

The wall represents trapped energy, because the field has to climb over the central hump in the potential in going from one side to the other. The energy density is shown in Fig. 3*b*. The energy per unit area of wall is of order η^3. For a phase transition at the GUT scale, where $\eta \sim 10^{15}$ GeV, this is a huge amount. Such massive domain walls certainly do not exist in the visible universe today. This has been used as an argument against theories with spontaneously broken discrete symmetries (Zel'dovich *et al.* 1975). If such domain walls were formed, they must have been very effectively eliminated, for example by a period of inflation following the phase transition, which would imply that our whole visible universe has come from a tiny part of a single domain, expanded by an enormous factor.

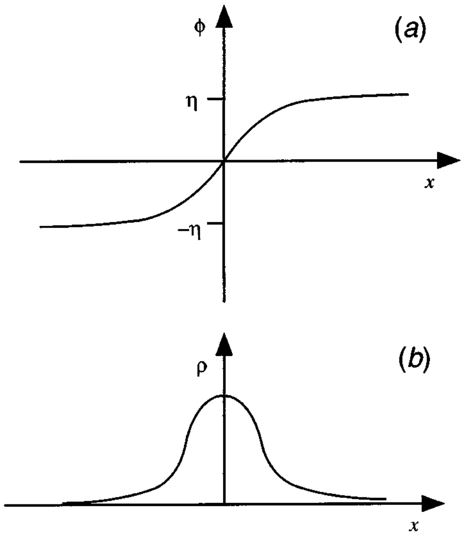

Fig. 3. A domain wall.

(2b) Cosmic Strings

Much more interesting from a cosmological point of view are cosmic strings. Consider for example a model of a *complex* scalar field with the *Mexican hat potential*

$$\mathcal{V} = \tfrac{1}{2}\lambda(\phi^*\phi - \tfrac{1}{2}\eta^2)^2, \tag{2}$$

illustrated in Fig. 4. Here the minima form a circle, parametrised by a phase angle α: $\phi = (1/\sqrt{2})\eta e^{i\alpha}$.

The theory is symmetric under the phase transformations $\phi \to \phi e^{i\alpha}$. It again exhibits a phase transition, at a temperature $T_c \sim \eta$. At high temperature, we have $\langle\phi\rangle = 0$, but below T_c, ϕ acquires a non-zero average value, choosing at random a point on the circle of minima.

Here too, the choice of phase angle will be made independently in widely separated regions. It may happen that as one traverses a large loop in space, the phase angle changes by 2π (see Fig. 5). Then somewhere within the loop ϕ must pass through zero. In fact that must happen all the way along a curve linking the loop — the core of a *cosmic string*.

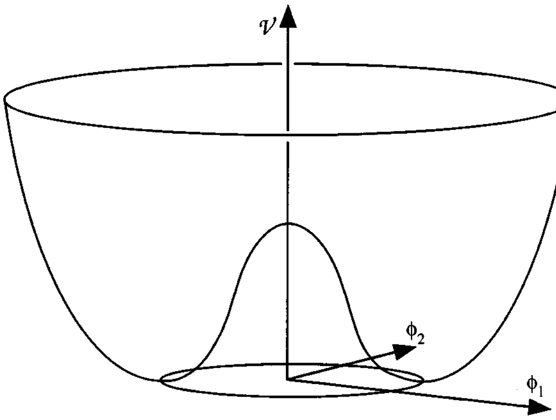

Fig. 4. Mexican hat potential.

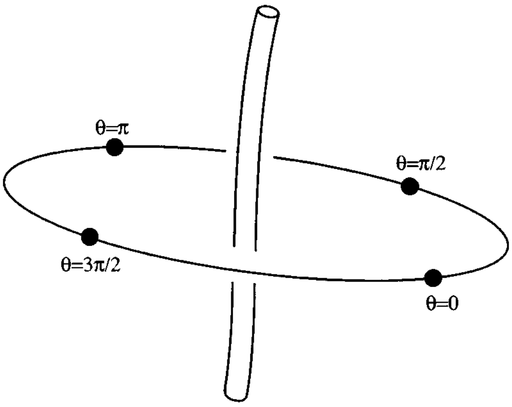

Fig. 5. A cosmic string.

Many of the models of interest are *gauge* theories, described for example by the Lagrangian

$$\mathcal{L} = D_\mu\phi^* D^\mu\phi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mathcal{V}(\phi), \tag{3}$$

where $D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. For energetic reasons, the configuration around a cosmic string in such a model will satisfy $D_\mu\phi \to 0$ at large distances. For a static string along the z axis, the typical form of the solution, in cylindrical polar coordinates ρ, φ, z, is

$$\phi = \frac{\eta}{\sqrt{2}} f(\rho) e^{ni\varphi}, \qquad A_\mu = -\frac{n}{e} k(\rho)\partial_\mu\varphi, \tag{4}$$

where n is an integer. The functions f and k satisfy the boundary conditions $f(0) = k(0) = 0$, $f(\infty) = k(\infty) = 1$; they are illustrated in Fig. 6. The string width is characterised by the inverse masses of the scalar and vector particles in the theory:

$$m_{\rm s} = \lambda^{\frac{1}{2}}\eta \quad \text{and} \quad m_{\rm v} = e\eta. \tag{5}$$

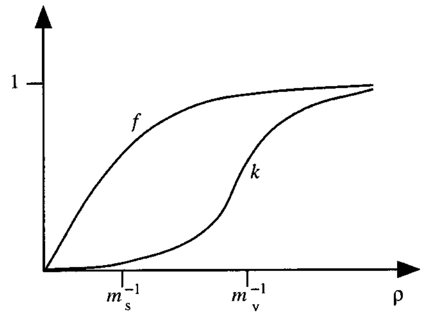

Fig. 6. Fields near a cosmic string.

Like domain walls, cosmic strings are topologically stable. They can move, and in the special case where a string forms a closed loop it can shrink to a point and vanish. But there is no way in which a small piece of a string can disappear. So if strings were formed at an early transition, some may still be around today.

For strings in simple relativistic models, the trapped energy (or mass) per unit length of string is equal to the string tension μ, and of order η^2. For GUT-scale strings this is a very large figure — a string of length equal to the sun's diameter would weigh as much as the sun itself — but as we shall see it not is so enormous as to be immediately ruled out. Strings in our universe could have very interesting observable effects.

(2c) Monopoles and Textures

I shall mainly discuss the effects of cosmic strings, but for completeness I should mention the other main types of defects. If we increase the number of independent real components of ϕ yet more, from 2 to 3, and choose

$$\mathcal{V} = \tfrac{1}{8}\lambda(\underline{\phi}^2 - \eta^2)^2, \qquad \underline{\phi}^2 = \sum_{k=1}^{3}\phi_k^2, \tag{6}$$

then we have a theory where the degenerate vacuum states are characterised by points on a 2-sphere, $\underline{\phi}^2 = \eta^2$. The broken symmetry here is O(3).

In this case, it is possible to have a point defect, a *monopole*. Around the point, the direction of ϕ in field space rotates; for example, we may take

$$\phi_k = \eta f(r)\frac{r_k}{r}, \tag{7}$$

where $f(0) = 0$ and $f(\infty) = 1$. This is sometimes called the 'hedgehog' configuration; the arrows point outwards all the way around, as in Fig. 7. The excess energy trapped in this configuration is the monopole mass, of order η, say 10^{16} for a GUT-scale monopole.

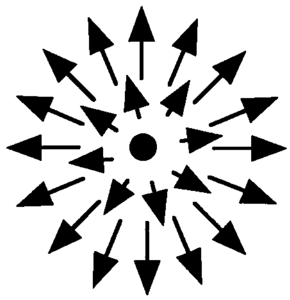

Fig. 7. Monopole configuration.

The monopole is topologically stable. It cannot disappear except by annihilation with an antimonopole. Bringing monopole pairs together is a slow process, so if monopoles are formed it is very difficult to find a way of reducing their number density now to a level consistent with observational bounds. This is a real difficulty, since most GUT models do predict monopoles. It was one of the main motivations for the idea of inflation — if the universe inflates by a huge factor, the monopole density is diluted essentially to zero.

Indeed, inflation dilutes *anything* present beforehand to insignificant levels, including for example cosmic strings. So if we want to use it to eliminate monopoles but at the same time have observable cosmic strings, we must assume that they are made *after* the period of inflation, or possibly during it — there are some very interesting models in which strings are formed during the late stages of inflation (Shafi and Vilenkin 1984; Shafi and Lazarides 1984; Vishniac *et al*. 1987; Yokoyama 1988).

We could extend the model further by increasing the number of fields ϕ_k from three to four or more. For the four-field model with potential analogous to (6), we find *textures*. These are strictly speaking not defects at all; the field is not required to vanish anywhere, but it may have a global topologically defined 'twist', which still has interesting dynamical effects. Indeed textures have been widely studied in recent years as a possible mechanism for structure formation in the universe (see for example Crittenden and Turok 1995). However, I shall not discuss them here.

(2d) Examples in Condensed Matter

There are many examples of defect-forming phase transitions in condensed matter systems. Cosmic strings are analogous to vortex lines in superfluid helium or to magnetic flux tubes in type-II superconductors.

Liquid helium enters a superfluid phase below the λ-point transition at $2{\cdot}17$ K. Even if the container is rotated, the motion of the superfluid component remains irrotational, and vorticity is concentrated in vortex filaments carrying quantised vorticity, which tend to run parallel to the axis and form a lattice (see Fig. 8).

Similarly, if a type-II superconductor is placed in a moderate magnetic field (between the lower and upper critical fields), the field will penetrate into the superconductor, but only in the form of narrow flux tubes carrying quantised magnetic flux, which again tend to form a lattice.

Very interesting phenomena are found in liquid crystals. For example a nematic liquid crystal consists, roughly speaking, of rod-shaped molecules that like to line

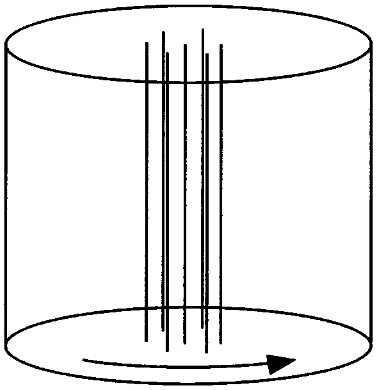

Fig. 8. Vortex lines in rotating superfluid helium.

up parallel to one another. Locally the state of the molecules is characterised by a dominant direction, the 'director', but opposite directions are equivalent. Such a system exhibits both line and point defects. For example one can have a disclination line around which the director rotates by 180° (see Fig. 9). This is an example where the 'topological quantum number' is only two-valued (it corresponds to integers modulo 2, rather than integers); a configuration in which the director rotates by 360° is topologically equivalent to the ground state, because the molecules near the line can all turn their inner ends upwards and 'escape into the third dimension'.

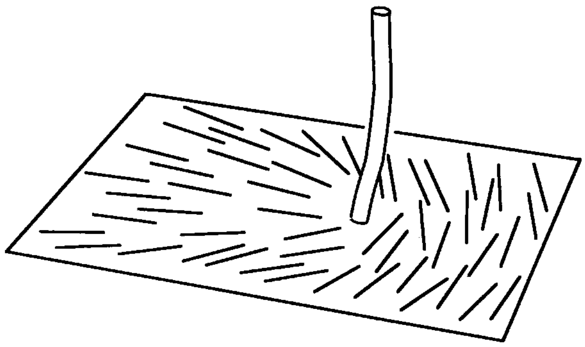

Fig. 9. Linear defect in a nematic liquid crystal.

The point defects are usually called 'hedgehogs'; around a central point the director points outwards (or equivalently inwards) everywhere. There are also defects attached to surfaces, for example 'boojums' which are roughly like half a hedgehog. In addition there are other kinds of liquid crystals with their own peculiar defects.

A particularly rich source of defects is superfluid helium-3. The lighter isotope ^{3}He also forms a superfluid, though at a much lower temperature than ^{4}He, between 2 and 3 millikelvin. Helium-3 is of course a *Fermi* liquid; like the electrons in a superconductor, the helium-3 atoms have spin one-half. In the BCS theory, superconductivity depends on the electrons forming bound Cooper pairs. Similarly the helium-3 atoms form bound pairs. However, it turns out that the binding occurs not in a ^{1}S state but in a ^{3}P state, i.e. both the orbital and spin angular momenta are equal to 1. Because they each have three components, the order parameter in helium-3 is a complex 3×3 matrix, with eighteen real components. It is hardly surprising that very complicated structures can emerge, including line and point defects, and various combinations (Volovik 1992).

Helium-3 has a complicated phase diagram (see Fig. 10) with two distinct superfluid phases, ^{3}He-A and ^{3}He-B. (In addition, in a magnetic field, an intermediate A1 phase interposes between the normal fluid and the other superfluid phases.)

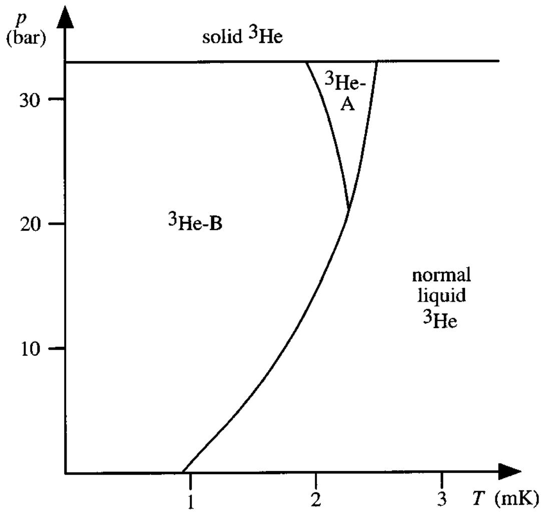

Fig. 10. Phase diagram of ^{3}He.

Some extremely interesting experiments have been performed on helium-3 recently by the Low Temperature Group in Helsinki, about which I shall have more to say later.

3. Formation and Evolution of Defects

Let us suppose that there is a phase transition, say at the GUT scale, at which cosmic strings can appear. What effects would this have? To answer that question we need to look at the formation and subsequent evolution of defects.

(3a) Defect Formation

Suppose that before the phase transition, when the universe was in the high-temperature symmetric phase, it was close to thermal equilibrium. In a rapidly expanding system, this is by no means self-evident. However, if there had been a previous inflationary period, it would have left the universe in a very homogeneous state, and therefore at nearly uniform temperature. (A similar homogeneity might conceivably be the result of other scenarios, such as the birth of the universe in some special quantum state.) In such a case, all parts of the universe would go through the transition at almost the same time. As they do so, the field in each region must choose a phase angle, and the choice will be made independently in widely separated regions. So we should expect to find trapped defects — in our case cosmic strings — forming a random tangle.

To decide what effects the strings might have, we have to ask how many are formed, and how they subsequently evolve. The characteristic scale $\xi_{\rm str}$ of the initial string configuration may be defined in terms of the string density: on average a length $\xi_{\rm str}$ of string is to be found in any randomly chosen volume $\xi^3_{\rm str}$, so the string contribution to the density is

$$\rho_{\rm str} = \mu/\xi^2_{\rm str}\,, \tag{8}$$

where μ is the mass or energy per unit length.

For reasons of causality, we certainly expect $\xi_{\rm str}$ at the time of string formation to be less, perhaps much less, than the horizon distance then (which is $2t$ in the early radiation-dominated universe — twice its age). It seems obvious that $\xi_{\rm str}$ should be related to the correlation length ξ of the underlying field, but near a phase transition that is of course changing rapidly, so this does not really answer the question. What fixes this initial scale is an important question to which I shall return shortly.

Initially, the characteristic scale $\xi_{\rm str}$ of the string network will grow rapidly. The strings are moving in a dense environment and are heavily damped. There is a tendency for the strings to shorten; small kinks will be pulled straight (see Fig. 11*a*). Occasionally two strings will cross and exchange partners (Fig. 11*b*) — a process often known as 'intercommuting'. This introduces new kinks that straighten in turn. Occasionally a string may cross itself, forming a small loop (Fig. 11*c*), which then oscillates and shrinks until it finally disappears. The net effect is to reduce the overall density of the string and increase the scale $\xi_{\rm str}$. In fact, it continues to increase until it becomes comparable with the horizon distance. At that point, string damping ceases to be important, and strings start to move with relativistic speeds. From then on their evolution is essentially independent of other matter in the universe.

(3b) The Scaling Regime

What happens thereafter? There are essentially only two possibilities. The characteristic length scale $\xi_{\rm str}$ cannot grow faster than the horizon distance. One possibility is that it reaches a 'scaling' regime, in which $\xi_{\rm str}/t$ approaches a constant. Then, so far as its string content is concerned, the universe will look much the same at any subsequent epoch, merely scaled in proportion to the horizon size. In the radiation-dominated universe, the total density scales as $1/t^2$; in fact,

$$\rho_{\rm tot} = \frac{3}{32\pi G t^2}. \tag{9}$$

Thus in a scaling regime, the strings form a fixed fraction of the total density: if $\xi_{\rm str}/2t \to 1/\gamma$, then

$$\frac{\rho_{\rm str}}{\rho_{\rm tot}} \approx \frac{8\pi}{3}\gamma^2 G\mu. \tag{10}$$

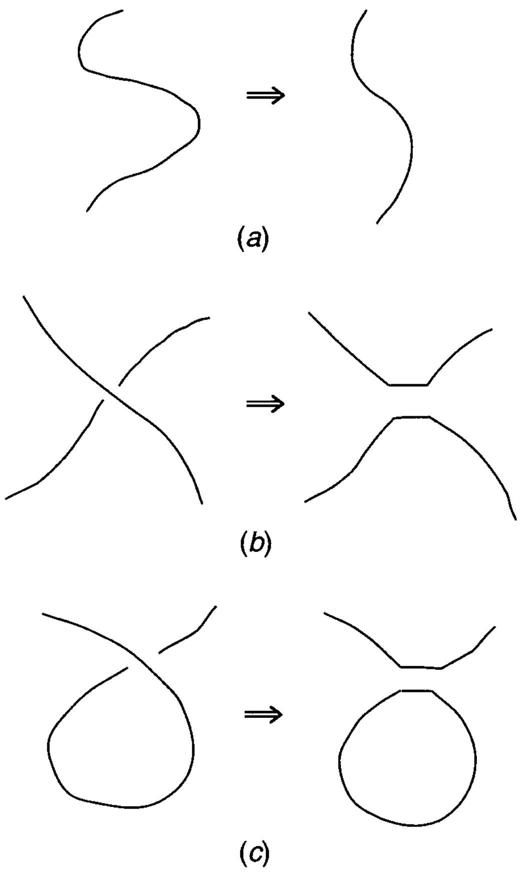

Fig. 11. Straightening and intercommuting of strings.

The dimensionless parameter $G\mu$, which governs the strength of the gravitational interactions of strings, plays an important role in the theory. For GUT scale strings, $G\mu \sim 10^{-6}$ or 10^{-7}, so (10) shows that strings may constitute a small but interesting fraction of the total density. Indeed this is one of the strengths of the cosmic string theory: there is a natural explanation for the order of magnitude of the density perturbations produced, whereas in the alternative inflationary scenario this requires fine tuning of the parameters.

The alternative end-point of string evolution is that $\xi_{\rm str}$ should increase more slowly than t. This means that eventually the strings must come to dominate the energy density of the universe. Such a universe would be very inhomogeneous — quite unlike ours. So if the cosmic string scenario is to have any chance of being correct, the strings must evolve towards scaling rather than string domination.

There are good reasons for believing that that is indeed the case. Most of our understanding of how strings evolve comes from computer simulations (Albrecht and Turok 1989; Bennett and Bouchet 1990; Allen and Shellard 1990; Shellard and Allen 1990), though my colleagues and I have also tried to model the system analytically (Copeland *et al.* 1992; Austin *et al.* 1993, 1995). The simulations show that in terms of its gross structure the string network does appear to reach scaling.

They have also revealed, however, that there is a lot of small-scale structure on the evolving network. Viewed from a large scale, there is a correlation of direction on the strings extending over distances of order $\xi_{\rm str}$, so they seem to be getting straighter with time. But there are also many small kinks, whose typical separation does not appear to grow over the timescale covered by the simulations. These kinks are generated by the intercommuting process: each time two strings exchange partners, a pair of new kinks, propagating in opposite directions, is created on each of the strings. The principal mechanism for damping this small-scale structure appears to be the emission of gravitational radiation. The analytic studies suggest that the small-scale structure too will eventually reach a scaling regime. The characteristic small scale, ζ, will start to grow in proportion to t, but with a large ratio (perhaps 10^4) between $\xi_{\rm str}$ and ζ. However, this conclusion still needs confirmation.

4. Defect Configuration at Formation

There has been a lot of discussion over the last few years about the characteristics of the initial string configuration. Two issues in particular have been raised: what determines the scale of the defect network at formation? And what is the proportion of 'infinite' string as opposed to small loops?

(4a) The Initial Defect Density

This depends strongly on the nature of the phase transition. If it is a strongly first-order transition proceeding by the nucleation of bubbles of the ordered phase, then it is reasonable to assume that within each bubble the field ϕ makes an independent random choice of phase angle. Where two bubbles come together, the phase will tend to smooth out between them, but where three meet along a line, it may happen that the net phase change around it is $\pm 2\pi$; in that case a string is trapped. Roughly speaking, we expect this to happen along one in four edges, so in this case the initial scale $\xi_{\rm str}$ of the string network is essentially the typical bubble diameter.

At a second-order transition, the mechanism is different but the result is not dissimilar. The transition occurs everywhere at almost the same time, and the field has to make a random choice of phase angle. The choices in widely separated regions will be uncorrelated. There will be some characteristic length which determines the scale over which the angle is correlated, and this will play the same role as the bubble size in the first-order case.

What fixes this scale? It seems clear that it must be essentially the correlation length ξ of the thermal fluctuations in the scalar field. Near a second-order transition, the equilibrium correlation length $\xi_{\rm eqm}$ becomes large, approaching infinity at $T_{\rm c}$, so the question arises: what temperature should we choose?

In the early work on the cosmic string scenario (Kibble 1976), it was assumed that the answer is the Ginzburg temperature $T_{\rm G}$. This is the temperature below which thermal fluctuations on the scale of the correlation length over the central hump of the potential become rare. It is determined by the condition

$$\Delta\mathcal{V}(T_{\rm G})\xi^3(T_{\rm G}) = T_{\rm G}, \tag{11}$$

where $\Delta\mathcal{V}(T)$ is the difference between the central maximum and the minima of the 'effective potential' or free-energy density at temperature T. Typically, $T_{\rm G}$ is below, but not far below, the critical temperature. The idea was that above this temperature the field can still fluctuate back and forth across the central hump, so defects are not yet really trapped.

It has recently become clear, however, that this idea is wrong. The process is an inherently dynamical one, while the Ginzburg temperature is an equilibrium concept. Above $T_{\rm G}$ small loops of string are readily created by thermal fluctuations, and there are rapidly fluctuating wiggles on long strings. But $T_{\rm G}$ is essentially irrelevant to the creation of the long strings or large loops that can survive a long time.

In large part the change in our understanding has come about because of discussions of analogous processes in condensed matter systems. Some years ago Zurek (1983, 1985) proposed that the ideas about defect formation might be tested with experiments in liquid helium. Such experiments have in fact recently been performed in helium-4 and, more recently, in helium-3; I shall discuss them shortly. Zurek proposed a different method of calculating the initial string density.

There are several different ways of stating the argument. One is based on ideas of causality. At the critical temperature, $\xi_{\rm eqm}$ diverges, but of course in a real case where we go through the transition at finite rate, the true correlation length ξ can never become infinite. In fact, it can never increase faster than the speed of light. Beyond a certain point in the approach to the phase transition, ξ will be unable to keep up, and will be essentially 'frozen in', remaining roughly constant until the point after the transition when $\xi_{\rm eqm}$ again falls to this level (see Fig. 12). In the relativistic case, the relevant point is determined by the condition that $|d\xi_{\rm eqm}/dt| \approx 1$ (i.e. the speed of light). In a condensed matter system, there will also be an effective limiting velocity: the characteristic speed at which fluctuations in the scalar field can propagate, for example the speed of sound or second sound.

As I shall describe in the following section, it has recently been possible to test these ideas in liquid helium-3, with results that clearly confirm the ideas of Zurek.

(4b) The Long-string Fraction

The first simulations of cosmic string formation were performed by Vachaspati and Vilenkin (1984). They chose a phase randomly at points of a cubic lattice. They found a distribution of small loops and long strings. The number density of loops of length l was proportional to $l^{-5/2}dl$, but in addition some 20% of the total length of string was in the form of strings much longer than the size of the box. In the infinite-volume limit, there would apparently remain a fraction of infinite strings. Later simulations have shown that the long-string fraction is somewhat dependent on the choice of lattice (Hindmarsh and Strobl 1994) and on the symmetry-breaking pattern (Kibble 1986), but have confirmed the existence of the long strings.

The long-string fraction is important for the later evolution of the network. Small loops can quickly shrink and disappear, so if there were no long strings,

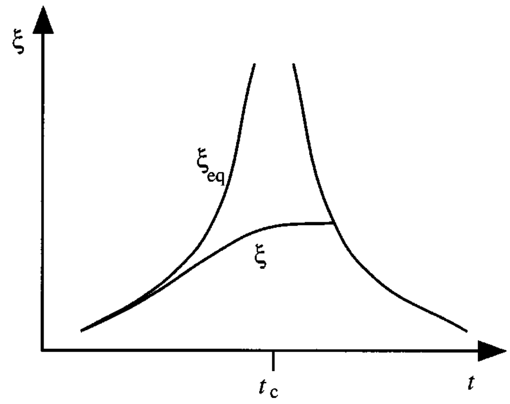

Fig. 12. Correlation length near a second-order phase transition.

it is questionable whether a scaling regime could be reached. Recent work by Ferreira and Turok (1995) suggests that without long strings the nature of the scaling solution might be very different.

The existence of long strings has been called in question by work of Borrill (1996). He performed a simulation, without introducing a lattice, of a first-order phase transition proceeding by bubble nucleation. His results suggest that the long-string fraction in that case is close to zero. On the other hand, other recent work by Robinson and Yates (1996) suggests that it is very difficult to avoid long strings if the transition proceeds by any causal mechanism. How this fits in with Borrill's work is unclear. This is a very interesting topic on which the last word has certainly not been said.

5. Tests in Condensed Matter Systems

Zurek's idea of testing the defect-formation scenario in condensed-matter systems has been a very fruitful one.

(5a) Liquid Crystals

In seeking analogues of the Big Bang phase transitions, the main difficulty is to arrange that all parts of the system go through the transition more or less simultaneously, or at least fast enough so that different parts of the system are causally unrelated. This can be more easily achieved in a nearly two-dimensional system. The earliest observation of a process similar to cosmic string formation was in thin films of nematic liquid crystals (Chuang *et al.* 1991; Bowick *et al.* 1994). Liquid crystals have the virtue that the transition is within the ordinary range of room temperatures, and that it is easy to see what is going on.

It is possible, for example, to observe the process of 'intercommuting' taking place (Chuang *et al.* 1991). It has also been possible to watch the emergence of a scaling regime. Of course, the analogy with the evolution of a cosmic string network in the early universe is not very close; there is no analogue of the universal expansion, and so the scaling exponent is different ($\xi_{\rm str}$ scales as the square root of the elapsed time, not as the age of the universe).

(5b) Helium-4

Zurek's original proposal was for a pressure quench in liquid helium. This experiment has been performed by a group in Lancaster (Hendry *et al.* 1994).

In this experiment a small sample of liquid helium is held at high pressure at a temperature just above the λ-point transition line (see Fig. 13). Then by a rapid expansion it is taken downwards through the transition. The defect-formation scenario suggests that this process should generate a population of vortices — the analogues of cosmic strings. This has indeed been confirmed. The presence of vortices was verified by observing the attenuation of second sound. Decreasing attenuation in the period following the transition shows that the defect density falls as $t^{-1/2}$ as expected. Unfortunately, it is not possible to measure the defect density immediately after the transition, because for a period of 4 ms or so the detector is swamped by noise. However, extrapolating backwards gives a defect density consistent with the Zurek estimate.

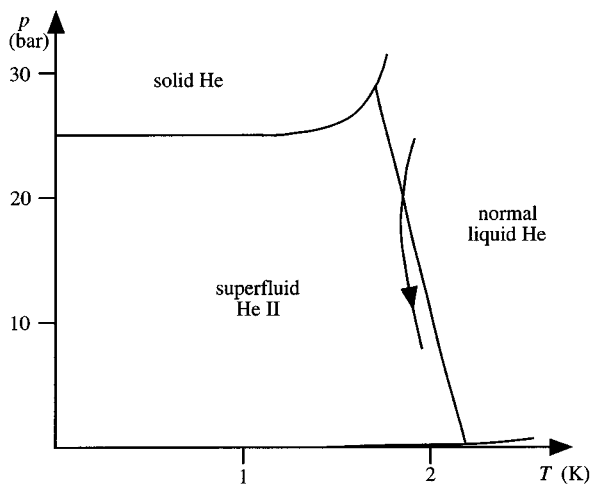

Fig. 13. A pressure quench in ^{4}He.

This is very encouraging, but cannot be regarded as a definitive test, because there are other possible explanations for the formation of vortices. Some vortices might be created at the walls of the container, particularly the bellows. Possibly more important is the role of the capillary used to fill the container. In the original design, the capillary tube is closed by a valve at the far end from the container. Thus when the system is expanded, a quantity of helium is ejected from the capillary, possibly creating some vorticity in the process. In a later version of the experiment, it is hoped to remove this problem by placing the valve at the other end of the tube.

Nevertheless, despite these interpretational questions, the most likely explanation of the results is that they reveal the formation of a random network of vortices during the process of going through the phase transition.

It is especially clear in this case that the Ginzburg temperature is irrelevant, because in superfluid helium, T_{G} is about half a degree below the critical temperature, and the experiments never get down to that level.

(5c) Helium-3

Although helium-3 is expensive, and working with it requires extremely low temperatures, it has some very important advantages. One of the most significant is that because the nucleus has spin $\frac{1}{2}$, vortices can be detected by using nuclear magnetic resonance. This means that in suitable cases we can detect *individual* vortices. Morever, the zero-temperature correlation length is much longer than in helium-4 — of order 100 nm rather than 1 nm — which means that the vortex core is many atomic spacings wide. Because of this, the simplest Landau–Ginzburg theory is quite accurate for helium-3, while for helium-4 it is rather poor. Moreover, it requires relatively much more energy to create a vortex in helium-3, so it is easier to rule out alternative explanations of vortex formation.

Another important practical advantage of helium-3 is its large neutron-absorption cross section. This is exploited in a recent experiment at the Low Temperature Laboratory in Helsinki (Ruutu *et al.* 1996). A small neutron source is placed next to a container of helium-3 in a rotating cryostat (see Fig. 14). The mean free path of a neutron in helium-3 is of order $0\cdot 1$ μm, so neutrons entering the chamber are absorbed near the walls. Each neutron releases 760 keV of energy, via the reaction

$$n + {}^3\mathrm{He} \rightarrow p + {}^3\mathrm{H} + 760\ \mathrm{keV}. \tag{12}$$

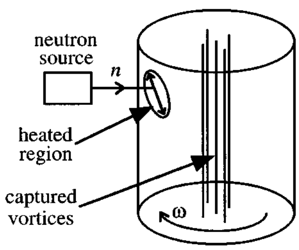

Fig. 14. Vortex formation by neutron-irradiated superflow in ^{3}He.

The energy goes into kinetic energy of the proton and triton, and is then dissipated by ionisation, heating a cigar-shaped region of the sample, about 70 μm long and 10 μm wide, raising it temporarily above the transition temperature. The heated region then cools back through the transition temperature, within about 1 μs, by quasiparticle diffusion out of the cigar. This generates vortices which can be detected by an NMR absorption measurement.

In the absence of the neutron source, there is a critical angular velocity ω_{c} below which no vortices are formed. If the container is kept rotating at angular velocity $\omega > \omega_{\mathrm{c}}$, vortices are nucleated at sites on the container walls, and then expand and migrate to a central cluster, parallel to the axis. However, if $\omega < \omega_{\mathrm{c}}$ no vortices are created. The normal fluid component will be rotating with the container while the superfluid component is at rest. Thus there is a counterflow velocity v_{s}.

At this point the neutron source is brought alongside. Each time a neutron is absorbed, it heats up a small region of the fluid. As it cools, vortices are formed, creating a random tangle of loops. Because of the counterflow velocity, the Magnus force will act to expand vortices above a certain critical size, given by

$$r_0 = \frac{\kappa}{4\pi v_{\mathrm{s}}} \ln \frac{r_0}{\xi}, \tag{13}$$

where κ is the circulation quantum, $\kappa = \pi\hbar/m_3$, and $\xi = \xi_0(1 - T/T_{\mathrm{c}})^{-1/2}$ is the superfluid correlation length. Vortices that are captured in this way eventually migrate to join the central cluster. Their effect is visible in the NMR signal, so that one can actually count individual vortices. This is shown in Fig. 15. Each of the steps in the figure represents a neutron event at which a single vortex is produced — or in some cases more than one; the step height shows this clearly.

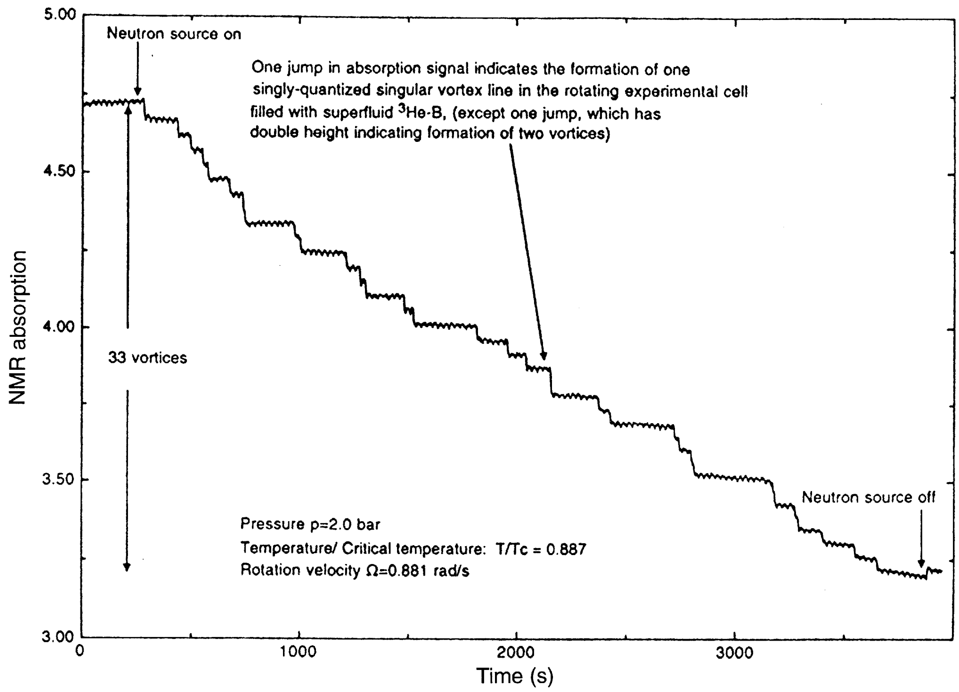

Fig. 15. NMR signal showing detection of individual vortices.

As the rotation speed is increased, the critical radius decreases, so more vortices are captured. We expect the number density of vortices of length l to be

$$n(l)dl = C\tilde{\xi}^{-3/2} l^{-5/2} dl, \tag{14}$$

where $\tilde{\xi}$ is the intervortex distance, and c is a constant of order unity. The number N of vortices captured is the number within the bubble volume $V_{\mathrm{b}} \sim R_{\mathrm{b}}^3$

with length between $\tilde{\xi}$ and the maximum size we expect to find inside the bubble, namely $R_{\rm b}^2/\tilde{\xi}$. This gives

$$N = \frac{\pi C}{9}\left[\left(\frac{\nu_{\rm s}}{\nu_{\rm cn}}\right)^3 - 1\right], \tag{15}$$

where $v_{\rm cn}$ is the critical counterflow velocity at which the largest vortices start to be captured, namely

$$v_{\rm cn} = \frac{\kappa}{4\pi R_{\rm b}} \ln \frac{R_{\rm b}}{\xi}. \tag{16}$$

The results very nicely confirm the characteristic cubic dependence of the number of vortices per neutron capture event on the angular velocity (see Fig. 16). Note that the critical velocity $v_{\rm cn}$ for vortex nucleation and capture is less than that, $v_{\rm c}$, for spontaneous vortex formation at the walls, and moreover has a different dependence on the temperature: it behaves like $(1-T/T_{\rm c})^{\frac{1}{3}}$ rather than $(1-T/T_{\rm c})^{\frac{1}{4}}$. In Fig. 17, I show the rate of vortex formation $\dot{N}$ plotted against $v_{\rm s}^3$ for several different values of the pressure, magnetic field and background temperature. The only dependence on these variables in (15) is in the value of $v_{\rm cn}$. As predicted, the data are well fitted by straight lines with a common intercept, determined by the constant factor in (15) and the neutron absorption rate.

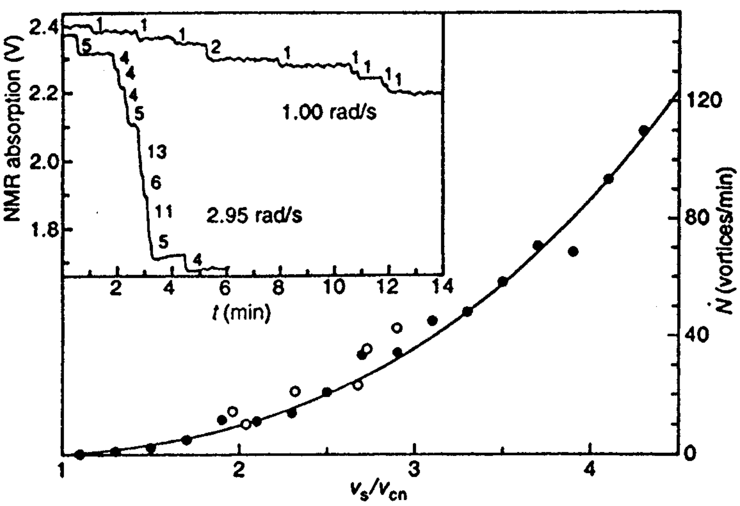

Fig. 16. Dependence of vortex formation rate on superflow velocity $v_{\rm s}$. The filled circles are values taken at $T = 0{\cdot}96T_{\rm c}$ and the open circles at $T = 0{\cdot}90T_{\rm c}$ (from Ruutu *et al.* 1996).

The agreement here is impressive, and must be counted as good evidence both for the basic mechanism of vortex formation at a second-order phase transition, and also for the detailed estimates made using the Zurek picture.

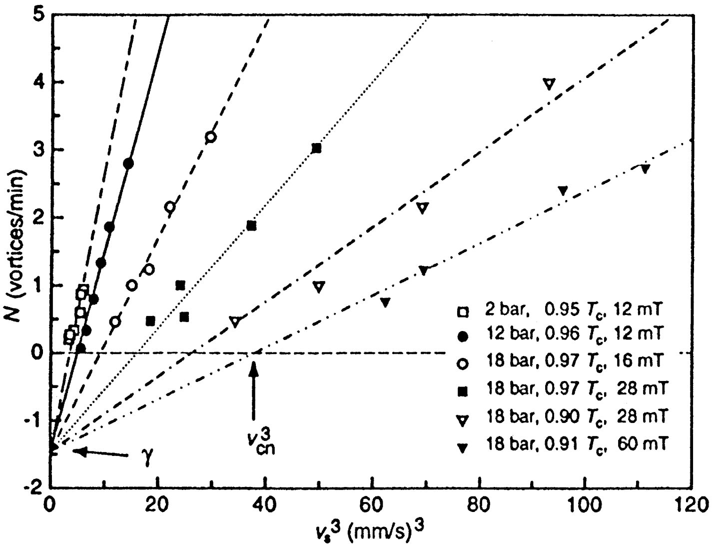

Fig. 17. Vortex formation rate against v_s^3 (from Ruutu *et al.* 1996).

6. Cosmological Effects

The real test of the cosmic string scenario must come from astronomical observations. We are fortunate that there is a huge amount of new data becoming available on such things as the cosmic microwave anisotropy and the large-scale galaxy distribution.

(6a) Gravitational Lensing

Most of the cosmologically interesting effects of strings are due to their gravitational fields. Around a straight, static string, the gravitational field is actually rather odd. A test particle in the neighbourhood of such a string experiences no gravitational acceleration, essentially because the tension acts as a negative source of the field and exactly cancels the effect of the string's energy. Space–time around a string is locally flat, except in the core of the string itself. But it is not globally flat. There is a deficit angle at the string; it is as though a wedge of space–time had been removed, of angle $\delta = 8\pi G\mu$, and the two edges glued together.

One of the observable effects of this is that the string would act as a cylindrical gravitational lens (see Fig. 18). A source, such as a distant quasar, behind the string, would yield two images separated by an angle of order δ, which for GUT-scale strings would be a few seconds of arc. Of course many other astronomical objects act as gravitational lenses, but a string across the sky would yield a characteristic pattern of lensed pairs strung out in a roughly linear array.

At first sight lensing seems a very promising way of identifying cosmic strings, but there are serious problems. Firstly, strings in the universe today would be very rare, so that the chance of finding one by random search is low. Secondly,

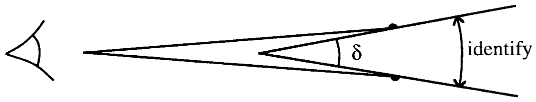

Fig. 18. Cosmic string as a gravitational lens.

the strings that are present would be far from straight, so the linear pattern would be much less obvious.

The absence of a local gravitational field is a peculiarity of a straight, static string. Strings that are wiggling about and have lots of small-scale structure, do attract the surrounding matter. Thus loops of string would act as gravitational lenses of more conventional type; however, they would not be particularly distinctive.

(6b) Density and Microwave Background Perturbations

A potentially more important implication of the string's deficit angle is that matter streaming past the string would be pulled inwards behind it (see Fig. 19). Thus a string moving through the universe would create an over-dense wake behind it. This is an important contribution to the density inhomogeneity. For wiggly strings, the direct gravitational attraction is another contribution to the density inhomogeneity. Which is more important depends on the speed of the string.

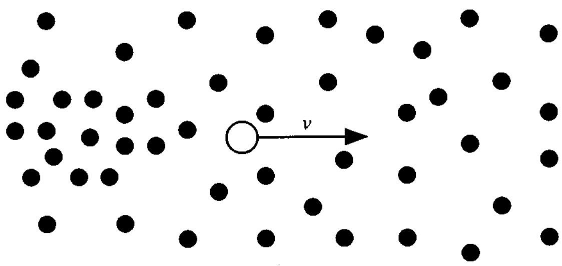

Fig. 19. Formation of a wake behind a moving cosmic string.

Strings would also induce anisotropies in the cosmic microwave background radiation (CMBR). If a string is moving across the sky, then the radiation ahead of it is red-shifted and that behind it is blue-shifted. There should be a discontinuity in the measured temperature of the CMBR of magnitude

$$\frac{\Delta T}{T} \sim 8\pi G\mu v_\perp, \tag{17}$$

where $v_\perp$ is the transverse velocity of the string.

If cosmic strings do indeed reach a scaling regime, then the power spectrum of the density perturbations on large scales will be close to the 'scale-invariant' Harrison–Zel'dovich form, i.e. the Fourier components of the density contrast $\delta = \delta\rho/\rho$ will scale as $\langle|\delta_k|^2\rangle \propto k$. As I mentioned earlier, one of the attractive features of the cosmic string scenario is that it naturally predicts about the right magnitude for these perturbations to allow galaxies to form — the magnitude is proportional to the dimensionless parameter $G\mu \sim (T_{\rm c}/M_{\rm Pl})^2$.

Making more precise predictions, particularly for smaller scales, requires detailed understanding of the scaling regime of the string network. This has proved hard. It requires precise knowledge of the parameters of the scaling regime. Numerically, this is difficult because the problem involves two very different scales, the characteristic scale $\xi_{\rm str}$ of the network and the scale ζ of the small-scale structure. Most of the predictions that have been made have relied on extrapolations from the parameters found in simulations of string evloution, but this is somewhat unreliable, because the simulations can run at most for a few tens of expansion times, whereas full scaling could only be established after much longer periods, say 10^4 expansion times or more.

However, a lot of work has been done recently to try to identify those features of the predictions that are robust and model-independent. The most significant observations to be expected within the next few years are accurate maps of the CMBR anisotropy. Predictions of inflationary and defect-based models are significantly different particularly at small angular scales.

The most popular type of model at present is an inflationary model within the context of a cold dark matter universe — in other words, most of the matter in the universe is in the form of exotic particles that were already non-relativistic at the time of decoupling from radiation. The primordial perturbations in the dark matter stem from quantum fluctuations during the inflationary epoch. The data are usually analysed in terms of the angular power spectrum of $\Delta T/T$, i.e. its decomposition into spherical harmonics. Inflationary models generally predict a peak in the angular power spectrum at $l \approx 200$, with a series of smaller secondary peaks at higher values of l. These peaks (often called 'Doppler peaks') stem from the fact that the acoustic density oscillations of a given wavelength have a known temporal phase, because they begin at a definite epoch, the 'horizon-crossing' time when the wavelength is comparable to the horizon.

A peak would generally be expected in defect models too, but often at a larger value of l, or smaller angular scale. The difference arises because in inflationary models perturbations on all scales, including super-horizon scales, are already present, whereas in defect models they can only start forming inside the horizon. Moreover, in cosmic string models, the secondary peaks are usually weak or absent (Albrecht *et al*. 1996; Magueijo *et al*. 1996). It is apparently possible to invent a defect model that would predict a main peak in the same position (Turok 1996), but only by making the model rather artificial. Equally, it might be possible within the context of an inflationary model to displace the peak to larger l, but it seems difficult to do so in a natural way.

The secondary peaks are important too. Secondary peaks may appear in some defect models, particularly texture models, but in a cosmic string scenario, they do not seem very likely, and certainly should not be as prominent as they would be in inflationary models.

Another important feature of defect models is that they predict non-Gaussian perturbations. In inflationary models, the different Fourier modes of the density contrast $\delta\rho/\rho$ are to a good approximation independent Gaussian random variables, but this is not true in defect models. For example, the temperature discontinuity across a cosmic string represents a correlation of the phases of many different modes. On large scales, the perturbations would still be nearly Gaussian, but on small angular scales, much less than 1°, non-Gaussian features should be visible. It has been suggested that the kurtosis of the temperature gradient would be a particularly sensitive measure (Moessner *et al*. 1994). There is no doubt that detection of non-Gaussian features in the CMBR anisotropy would be very strong evidence for a defect model.

(6c) Gravitational Waves

Oscillating loops of string lose energy primarily by emission of gravitational radiation. Collectively, they represent a significant contribution to the expected gravitational-wave intensity in the universe. Bounds on that intensity impose quite stringent limits on a cosmic string model.

One way of detecting gravitational waves is by observing their perturbing effect on the timing of the millisecond pulsars. These are believed to be very rapidly rotating neutron stars. They are among the most accurate astronomical clocks available to us. Gravitational waves in the intervening space between us and a pulsar would yield random fluctuations in the timing. These pulsars have been under observation now for a decade. The measured upper limit on random fluctuations yields an upper limit on the intensity of gravitational waves with periods of a few years. This in turn imposes a limit on any source of gravitational waves. In the case of cosmic strings, it limits the value of the parameter $G\mu$. The upper bound is unfortunately rather model-dependent. It is close to the value needed to explain the density pertubations in the universe, but there is at present no real conflict (for discussion and references, see Hindmarsh and Kibble 1995).

There is also a limit on the gravitational wave intensity in the universe at the time of nucleosynthesis, which imposes a rather similar bound. In that case, however, the prediction is less certain, because it is sensitive to gravitational waves emitted at much earlier times, shortly after string formation—it depends on the approach to scaling as well as the scaling regime itself. This is one prediction where our ability to estimate the initial defect density, as tested recently in liquid helium, is of particular importance.

(6d) Present Observational Status of the Theory

The inflationary cold dark matter model provides a good qualitative fit to the data on galaxy distributions and the CMBR anisotropy, but to fit both quantitatively seems to require some extra feature — a small admixture of hot dark matter (perhaps massive neutrinos), or a cosmological constant, perhaps. Inflation certainly does not work in the context of a hot dark matter universe.

Cosmic strings can also provide a good qualitative fit to the data (Albrecht and Stebbins 1992). In this case too, it is not easy to fit both the galaxy distribution and CMBR data quantitatively. If we normalise from the COBE CMBR data, the inflationary cold dark matter model predicts fluctuations on small scales that are too large to fit the galaxy correlation data. If we stick to cold dark matter, then defects generically are even worse. On the other hand, inflation does not work at all in a hot dark matter universe, whereas cosmic strings and hot dark matter do provide a viable model. In this case, the problem is if anything in the other direction: the fluctuations predicted on small scales tend to be somewhat low. At present, the simplest forms of both defect and inflationary models seem to be in trouble, but there is room within both to accommodate the discrepancy.

The big problem for the defect models, however, including the cosmic string scenario, is that so far we are unable to make the predictions as firm as is possible within the simpler context of inflation. It is for that reason that attempts to find generic, model-independent predictions from defect models are so important. Within the next decade, these predictions will be well tested by new satellites — notably COBRAS-SAMBA, recently chosen by ESA for its future programme — and by ground-based measurements.

Confirmation of some of our basic ideas from condensed matter experiments has given us more confidence, and should allow us to improve our theoretical estimates of observable effects, though there are still significant difficulties in handling the small-scale structure.

If the predominant form of dark matter in the universe were indeed 'hot' (relativistic at the time of decoupling), some kind of defect model would be the only possible explanation for the observed density and CMBR perturbations. Cosmic strings plus hot dark matter provide an interesting and viable alternative to the standard inflationary picture.

7. Conclusions

If we accept both the standard models of particle physics and cosmology — based on unified gauge theories and the 'hot big bang' — we are led to conclude that the universe would have gone through a sequence of phase transitions very early in its history. It is likely that topological defects of various forms would have been created at some of these transitions. Because of their inherent stability, some of these might have survived long enough to produce observable effects in the universe today. Our ideas about defect formation at phase transitions have been confirmed by recent condensed matter experiments.

In particular, cosmic strings formed at the time of grand unification could provide the explanation for the primordial density perturbations from which galaxies eventually form, and for the observed anisotropy in the cosmic microwave background. They provide an alternative to the popular inflationary model. At present both models are viable. Definitive tests should be possible within the next ten years, when good measurements of the small-scale anisotropy become available.

It should also be said that although cosmic strings and inflation are rival models, they are not incompatible. There are interesting models in which strings would be formed during the later stages of inflation. It is perfectly possible that strings and inflation *both* contribute to the density perturbations, although if their contributions are of the same order that would seem a rather remarkable coincidence.

The next few years promise to be exciting ones for this field.

Acknowledgments

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References

Albrecht, A., Coulson, D., Ferreira, P., and Magueijo, J. (1996). *Phys. Rev. Lett.* **76**, 1413.
Albrecht, A., and Stebbins, A. (1992). *Phys. Rev. Lett.* **68**, 2121; **69**, 2615.
Albrecht, A., and Turok, N. (1989). *Phys. Rev.* D **40**, 973.
Allen, B., and Shellard, E.P.S. (1990). *Phys. Rev. Lett.* **64**, 119.
Austin, D., Copeland, E., and Kibble, T.W.B. (1993). *Phys. Rev.* D **48**, 5594.
Austin, D., Copeland, E., and Kibble, T.W.B. (1995). *Phys. Rev.* D **51**, R2499.
Bäuerle, C., Bunkov, Yu.M., Fisher, S.N., Godfrin, H., and Pickett, G.R. (1996). *Nature* **382**, 332.
Bennett, D.P., and Bouchet, F.R. (1990). *Phys. Rev.* D **41**, 2408.
Borrill, J. (1996). *Phys. Rev. Lett.* **76**, 3255.
Bowick, M.J., Chandar, L., Schiff, E.A., and Srivastava, A.M. (1994). *Science* **264**, 943.
Chuang, I., Durrer, R., Turok, N., and Yurke, B. (1991). *Science* **251**, 1336.
Copeland, E., Kibble, T.W.B., and Austin, D. (1992). *Phys. Rev.* D **45**, R1000.
Crittenden, R., and Turok, N. (1995). *Phys. Rev. Lett.* **75**, 2642.
Ferreira, P., and Turok, N. (1995). Private communication.
Hendry, P.C., Lawson, N.S., Lee, R.A.M., McClintock, P.V.E., and Williams, C.D.H. (1994). *Nature* **368**, 315.
Hindmarsh, M.B., and Kibble, T.W.B. (1995). *Rep. Prog. Phys.* **58**, 477.
Hindmarsh, M.B., and Strobl, K. (1994). Statistical properties of strings. Cambridge/Sussex preprint, hep-th/9410094 (unpublished).
Kibble, T.W.B. (1976). *J. Phys.* A **9**, 1387.
Kibble, T.W.B. (1986). *Phys. Lett.* B **166**, 311.
Magueijo, J., Albrecht, A., Coulson, D., and Ferreira, P. (1996). *Phys. Rev. Lett.* **76**, 2617.
Moessner, R., Perivolaropoulos, L., and Brandenberger, R. (1994). *Astrophys. J.* **425**, 365.
Robinson, J., and Yates, A. (1996). *Phys. Rev.* D **54**, 5211.
Ruutu, V.M.H., Eltsov, V.B., Gill, A.J., Kibble, T.W.B., Krusius, M., Makhlin, Yu.G., Plaçais, B., Volovik, G.E., and Xu, W. (1996). *Nature* **382**, 334.
Shafi, Q., and Vilenkin, A. (1984). *Phys. Rev.* D **29**, 1870.
Shafi, Q., and Lazarides, G. (1984). *Phys. Lett.* B **148**, 35.
Shellard, E.P.S., and Allen, B. (1990). *In* 'The Formation and Evolution of Cosmic Strings' (Eds G.W. Gibbons, S.W. Hawking and T. Vachaspati), p. 421 (Cambridge University Press).
Shellard, E.P.S., and Vilenkin, A. (1994). 'Cosmic Strings and Other Topological Defects' (Cambridge University Press).
Turok, N. (1996). *Phys. Rev.* D, in press.
Vachaspati, T., and Vilenkin, A. (1984). *Phys. Rev.* D **30**, 2036.
Vishniac, E.T., Olive, K.A., and Seckel, D. (1987). *Nucl. Phys.* B **289**, 717.
Volovik, G. (1992). 'Exotic Properties of Superfluid ^{3}He' (World Scientific: Singapore).
Yokoyama, J. (1988). *Phys. Lett.* B **212**, 273.
Zel'dovich, Ya.B., Kobzarev, I.Yu., and Okun', L.B. (1975). *Sov. Phys. JETP* **40**, 1 [*Zh. Eksp. Teor. Fiz.* **67**, 3 (1974)].
Zurek, W.H. (1983). *Acta Phys. Polon.* B **24**, 1301.
Zurek, W.H. (1985). *Nature* **317**, 505.

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