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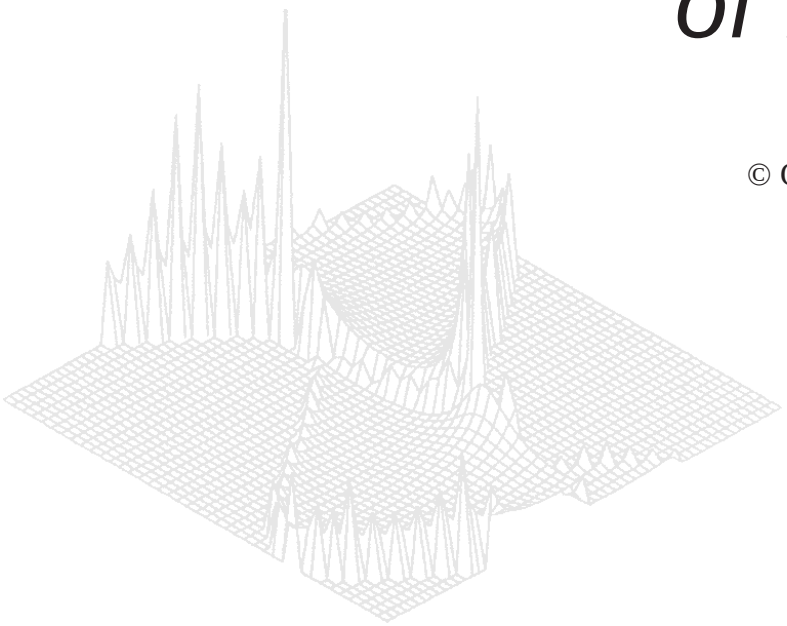

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Driven Granular Media and Dissipative Gases: Phase Transitions and Instabilities*

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Abstract

We study a simple model of a granular material or powder where the particles are excited by an external noise source and dissipate energy by inelastic collisions. Due to the inelastic collisions between particles there is an effective interaction between them. In one dimension this leads to long-range correlations between the particles in a gas phase despite the absence of long-range forces between the particles. In two dimensions the dissipative effects cause a very sharp liquid–gas phase transition at which the susceptibility has a pronounced peak. In the presence of a double-welled potential the inelasticity causes a symmetry-breaking instability where all the particles cluster into one of the wells.

1. Introduction

Granular materials or powders may be defined as any material which consists of 'grains' or small particles. Examples include many products of importance to the agricultural, mining and food industries, such as rice, wheat, coal, most ores and most breakfast cereals. Although granular materials have long been of interest to engineers they have only attracted significant attention from the physics community over the past decade (Jaeger and Nagel 1992). To physicists these materials are interesting because they exhibit a wide range of interesting phenomena, many of which remain either unexplained or not fully understood. These include, heaping, clustering, size-segregation and the breakdown of macroscopic hydrodynamics (Babić 1993; Bernu and Mazighi 1990; Campbell and Brennen 1985; Du *et al.* 1995; Goldhirsch and Zanetti 1993; Haff 1983; Hopkins and Louge 1991; Jaeger and Nagel 1992; Jenkins and Savage 1983; Luding *et al.* 1994; McNamara and Young 1992; Schmittmann and Zia 1991; Sela and Goldhirsch 1995; Williams 1997; Williams and MacKintosh 1995, 1996). These effects and others have been much studied experimentally and by computer simulation. However, despite extensive studies a reasonable understanding of granular materials is still lacking. This is presumably in part because the individual grains undergo complex interactions, where short range elastic forces and nonlinearities become important. However, there is clearly also a more fundamental reason. The vast majority of the experiments and simulations on granular materials involve several separate and identifiable influences. These are:

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(i) The interactions between the particles are inelastic. Thus when two particles collide, energy is dissipated.
(ii) The systems studied usually include a boundary such as the walls of the container.
(iii) Energy normally flows from the boundaries into the granular media.
(iv) The system is placed in a gravitational field.

Each of these influences obviously complicates the system and it is thus not surprising that many of the experiments and simulations show novel and rich behaviour. There are two obvious analogs here. The first is with the dynamics of simple fluids. In general the flows of such fluids away from fields and boundaries are relatively easy to understand. However, in the presence of boundaries and fields simple fluids exhibit convection and turbulence and are still not perfectly understood. The second analogy is with the theory of magnetism. If one wants to understand ferromagnetism it is simplest to study the Ising model on an infinite lattice in the absence of any external fields, boundaries or imposed gradients. The situation with granular materials is even more complicated. In general, influences (ii) to (iv) artificially break the symmetry and induce spatial gradients in the system which make a simple understanding somewhat difficult. In this paper we present a system which has some or all of these undesirable influences removed. Indeed a step in that direction has already been made (Goldhirsch and Zanetti 1993; Hopkins and Louge 1991; McNamara and Young 1992). The system is a dissipative gas, which is simply a gas in which the particles undergo inelastic collisions. Thus far the studies have concerned 'cooling' gases which are not heated in anyway. These gases can show interesting dynamics, but they have no non-trivial steady-state behaviour: i.e. all the particles just cool down and stop moving. The simplest system for which there is any non-trivial steady-state is the one we shall study here: a driven dissipative gas. This system was introduced in Williams and MacKintosh (1995) and was studied further by Williams and MacKintosh (1996) and Williams (1997).

In this system the particles undergo inelastic collisions but each particle is heated individually and continuously. It thus includes the important difference between granular and other fluids (inelasticity) whilst ignoring the complicating boundary and field effects. This system is intrinsically much simpler than traditional granular materials. However, as we shall see here, an examination of driven dissipative fluids also reveals non-trivial behaviour and can lead to a better understanding of material properties.

This paper is partly a review and partly a presentation of new results. It is laid out as follows. We first present an off-lattice study of a one-dimensional driven dissipative gas. This gas shows clustering and long-range correlations between the particles. In Section 3 we derive an equation of state for this gas. In Section 4 we present results from a lattice simulation of a two-dimensional driven dissipative gas. At a certain density this gas suddenly nucleates a liquid phase which coexists with the gas phase. In Section 5 we begin to introduce one of the complications mentioned above, i.e. we apply a potential. This leads to an interesting symmetry-breaking instability. We conclude in Section 6.

Throughout this study we have used various computer codes. These were checked by calculating by hand what occurs during several collisions and comparing it with the computed results.

2. Clustering in One-dimensional Gases

Clustering is observed when granular media are sheared (Campbell and Brennen 1985; Hopkins and Louge 1991; Babić 1993). In general clustering is driven by inelastic collisions. When two particles collide inelastically they dissipate energy, slow down and hence remain close to one another. Here we investigate numerically the effect of such inelastic collisions in a one-dimensional system of point-like particles that are excited by a thermal reservoir. We show that clustering, as described by the two-point correlation function, occurs even in the absence of any other forces between the particles. There have been several studies of systems which are started in a 'hot' state and then slowly cool (Goldhirsch and Zanetti 1993; Hopkins and Louge 1991; McNamara and Young 1992; Luding *et al.* 1994; Bernu and Mazighi 1990), as there is no energy input. In one and two dimensions these can show 'inelastic collapse'. The novel feature of the collapse is that for coefficients of restitution η below a critical value η_c the kinetic energy is dissipated in a finite time. For $\eta > \eta_c$ the kinetic energy dissipates gradually.

In the model considered here (Williams and MacKintosh 1995), we consider a continuous input of energy locally to each particle, as well as dissipative collisions. This might model for instance a system of particles confined to a line on a vibrating plate. For a coefficient of restitution $\eta < 1$, the system eventually settles down to a 'steady-state'. For $\eta \approx 1$, this steady-state looks, at least superficially, like an ideal gas, in which there are no significant spatial correlations between the particles. However, we show that as η is reduced, even in the absence of any long-range interactions, the system develops a structure factor that is characteristic of an equilibrium system with long-range interactions. In particular, the dissipative interactions lead to a correlation function $g(x)$ that is no longer a constant as it would be for an ideal gas, but shows a peak near the origin. Furthermore, this enhancement of $g(x)$ near $x = 0$ follows a power-law. Thus, the system behaves as if it there were long-range attractive interactions between the particles. Only in the limit $\eta \to 1$ does the correlation function become uniform. As η decreases $g(x)$ becomes more and more sharply peaked about $x = 0$. Note that in recent years there has been much work on the critical properties of driven gases (see e.g. Schmittmann and Zia 1991). Our work differs from these works in that it specifically uses dissipative collisions.

We consider N point particles of unit mass, $m = 1$, confined to a line of length $L = 1$ (Fig. 1). We use periodic boundary conditions, so that the particles lie on a circle of unit circumference. When two particles i and j collide in this one-dimensional system, the final (primed) velocities are given in terms of the initial (unprimed) velocities by

$$v_i{}' = \tfrac{1}{2}(1-\eta)v_i + \tfrac{1}{2}(1+\eta)v_j, \quad v_j' = \tfrac{1}{2}(1-\eta)v_j + \tfrac{1}{2}(1+\eta)v_i\,. \tag{1}$$

The main difference between this and previous studies of dissipative gases is that each individual particle is 'heated' at a constant rate. This is done by adding a random amount to the velocity of each particle during a time-step Δt via the Langevin equation

$$v_i(t+\Delta t) = v_i(t) + \sqrt{r}\sqrt{\Delta t} f(t)\,, \tag{2}$$

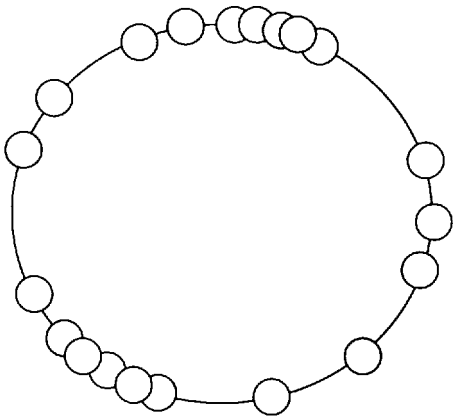

Fig. 1. Picture of the one-dimensional system of particles which are uniformly and individually heated. Periodic boundary conditions are used, and the particles are point-like. The system forms cool clusters surrounded by hot gas-like regions. As the coefficient of restitution is reduced these clusters become more pronounced.

where $f(t)$ is a random number chosen uniformly between $-\frac{1}{2}$ and $\frac{1}{2}$ and r is a number proportional to the heating rate. After the velocities are adjusted the system is transferred to the centre of mass frame, so that $v_i \to v_i \overline{v}$, where $\overline{v}$ is the average velocity of all the particles in the system. The algorithm (2) ensures that the velocities undergo a random walk, whilst the transfer to the centre of mass frame ensures that the particle speeds do not increase indefinitely. This transfer step is for convenience only, since the properties of collisions do not depend on the absolute speeds, only on the relative speeds. Before the heating step the kinetic energy of the system is $K = \frac{1}{2}\Sigma_{i=1}^{N} v_i^2$ and after heating it is

$$K' = \tfrac{1}{2}\sum_{i=1}^{N}(v_i + \delta v_i)^2 = K + \sum_{i=1}^{N} v_i \delta v_i + \tfrac{1}{2}\sum_{i=1}^{N}(\delta v_i)^2\,, \tag{3}$$

where $\delta v_i = \sqrt{r}\sqrt{\Delta t} f(t)$. On average the term linear in v_i vanishes and the average of the remaining term, $\frac{1}{2}\Sigma_{i=1}^{N}(\delta v_i)^2$, is $\frac{1}{2} rtN\langle f^2\rangle = \frac{1}{24} rtN$. Hence the heating rate is $\Omega \equiv \frac{1}{24} r$, which is the energy input per unit time per particle. Provided $\eta < 1$, this system, started with some initial random speeds, rapidly reaches a steady-state configuration.

Qualitatively the system appears to form liquid-like clusters of high density surrounded by a gas-like 'phase' of lower density (Fig. 1). In order to study this effect quantitatively we use the two-particle correlation function $g(x)$. We select a particle and ask what is the density of particles at distance x from it. This, suitably normalised, is $g(x)$. For an ideal gas of point particles the answer is $g(x)$ a constant. For a gas which has some attractive potential $U(x)$ acting between the particles, $g(x)$ will be peaked about $x = 0$ and will decay to a constant as $x \to \infty$. For the dissipative gas discussed here we find $g(x)$ shows a peak at the origin, even though we have no potential acting between the particles. This peak is caused by the dissipation effect discussed above, and can be thought of as a steady-state version of the collapse and clustering seen in cooling gases (Goldhirsch and Zanetti 1993; Hopkins and Louge 1991; McNamara and Young 1992; Luding *et al.* 1994; Bernu and Mazighi 1990). As $\eta \to 1$ the structure becomes less pronounced and $g(x)$ approaches a constant. However, as

η becomes small, $g(x)$ becomes very sharply peaked. Some characteristic results are shown in Fig. 2. We find that the correlation function depends only on the density and on η and is independent of the heating rate. At least for small x, where the finite size of the system has little effect, $g(x)$ can be approximated by a power law $g(x) \sim x^{\alpha(\eta)}$. Here $\alpha(\eta)$ is a monotonically increasing function of η. In the limit of a perfectly elastic system $\eta \to 1$ and $\alpha \to 0$. However, for perfectly inelastic systems where $\eta \to 0$ we find $\alpha \to \frac{1}{2}$.

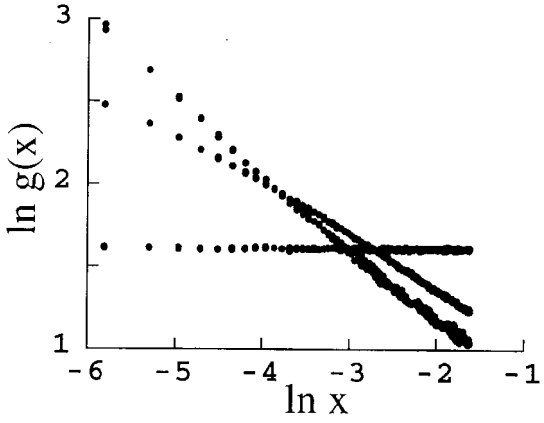

Fig. 2. A log–log plot of the two-point correlation function versus distance for three values of the coefficient of restitution for $N = 10$ particles. For $\eta = 0{\cdot}99$ (lowest curve) the correlation function is almost a constant. For $\eta = 0{\cdot}5$ (middle curve) strong correlations have developed, and for $\eta = 0{\cdot}01$ (upper curve) the function is clearly a power-law with exponent $-\frac{1}{2}$. The correlations arise because of the effect of inelasticity, i.e. when two particles collide they move more slowly and hence stay near each other, thus causing a correlation. The correlation function is independent of the heating rate Ω. Here we have superimposed data from two heating rates which differ by a factor of 100, $\Omega = 0{\cdot}0017$ and $\Omega = 0{\cdot}17$, and each point represents an average over 2×10^5 collisions.

3. Equation of State for One-dimensional Gases

Despite the long-range correlations in the one-dimensional system we can write down an equation of state by a simple energy argument (Jenkins and Savage 1983; Haff 1983). By 'equation of state' we mean the relation between the heating rate, the number of particles, the system size, and the kinetic energy of the particles. Let us ignore correlations between the particles. The average distance a particle must move between collisions is $L/2N$. Then, for particles which have some average speed v, the time between a collision (for each particle) is approximately $t = (L/2N)v^{-1}$. Since there are $N/2$ pairs of particles the rate of collisions is $\approx (N/2)t^{-1} = vL^{-1}N^2$. During each collision an amount of energy $(1-\eta^2)v^2$ is dissipated. Thus the rate of energy dissipation is approximately

$$W = v^3(1-\eta^2)L^{-1}N^2\,. \tag{4}$$

However, the rate of energy input is ΩN. In the steady state these must be equal. Thus we find an equation of state

$$K^{3/2}(1-\eta^2)N = C\Omega L\,, \tag{5}$$

where K is the kinetic energy per particle and C is a numerical constant. The dependencies on K, Ω and L are somewhat trivial since they can be derived independently by dimensional analysis. However, the dependence upon η and especially N are less trivial. We can test how accurately this equation describes the system by comparing it with the results of our computer simulation. The scaling of the kinetic energy with the coefficient of restitution is exactly as predicted by the theory (5) (Fig. 3). The scaling with N also agrees with theory (Williams and MacKintosh 1995, 1996; Williams 1997).

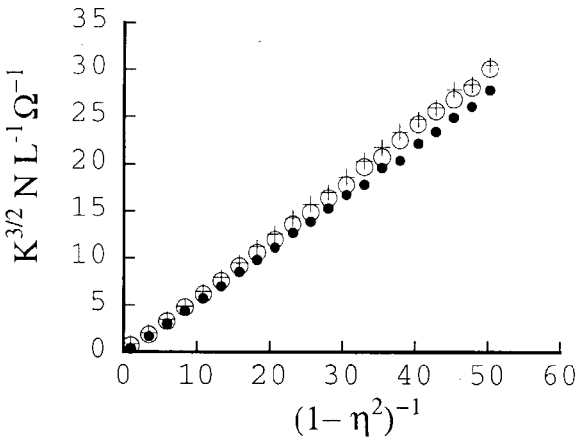

Fig. 3. Test of the η dependence in the equation of state (5). Here we have plotted $K^{3/2}NL^{-1}\Omega^{-1}$ versus $(1-\eta^2)^{-1}$. As predicted by theory (5) the relation is close to linear. Note that because the scaling with N in (5) is not precise the lines for different N are not exactly superimposed. The data are points from the computer simulation and each point represents an average over 10^6 collisions. The following pairs of the particle number and heating rate (N, Ω) were used: $(10,\ 0{\cdot}0017) = \bullet$, $(20,\ 0{\cdot}0017) = \bigcirc$, $(40,\ 0{\cdot}0017) = +$.

We can use the equation of state to obtain the force on a wall placed at the boundary of a finite system. We assume that the particles undergo perfectly elastic collisions with the wall. The time averaged force F exerted by the particles is the 'pressure' for a 1D gas. The current of particles hitting the wall is $J = Nv/L$. Each particle imparts momentum $2v$ to the wall. Thus the time-averaged force is $F = pv^2$, where $\rho = 1/L$ is the linear density. Thus the relation between the force and the density is

$$F = \Omega^{2/3}\rho^{1/3}(1-\eta^2)^{-2/3}\,. \tag{6}$$

This is very different from the result for an ideal gas where the pressure is proportional to the first power of the density.

The long-range correlations introduced in the previous section can be thought of as being induced by an effective potential of mean force $U(x)$, where $g(x) \propto \exp[-U(x)/kT]$. Here kT can be taken as the kinetic energy of the particles. This potential is approximately logarithmic. Note however that, since

the only energy occurring in the problem is the kinetic energy of the particles, the potential is just proportional to the temperature.

4. Two-dimensional Gases: A Liquid–Gas Phase Transition

In this section we study a two-dimensional excited dissipative gas. It is well known from the theory of phase transitions that the one-dimensional case is special. It is thus important to study the above effects in two dimensions. A logical question to ask is how do the long-range correlations affect the phases of this system in two or three dimensions? Here we show that they can induce a liquid phase in coexistence with a gas phase. The system we have in mind is a two-dimensional excited powder, i.e. a group of grains laid upon a surface, and gently randomly vibrated. The particles reside on a square $L \times L$ lattice with lattice constant unity, and move in the x and y directions, with velocity components v_x and v_y. The particles undergo inelastic collisions with each other with coefficient of restitution η. For simplicity we assume that the collision occurs only in the x direction or the y direction at any one time. Thus if the x direction is chosen the final (primed) and initial (unprimed) velocities of the two colliding particles, i and j, are given by

$$v'_{ix} = \tfrac{1}{2}(1-\eta)v_{ix} + \tfrac{1}{2}(1+\eta)v_{jx}, \quad v'_{jx} = \tfrac{1}{2}(1-\eta)n_{jx} + \tfrac{1}{2}(1+\eta)v_{ix}\,, \tag{7}$$

$$v'_{iy} = v_{iy}, \qquad v'_{jy} = v_{jy}\,. \tag{8}$$

The particles are continuously heated by changing either their x or y velocities as in the one-dimensional case. If the x direction is chosen then

$$v_x(t+\Delta t) = v_x(t) + \sqrt{r\Delta t} f(t)\,, \quad v_y(t+\Delta t) = v_y(t)\,, \tag{9}$$

where r and f have the same meanings as in the one-dimensional model.

We use periodic boundary conditions. The algorithm for particle motion is as follows: (1) A particle is chosen at random and either the x or y direction is then chosen, also at random. The time step is fixed originally at $\Delta t = 1$ and is decremented as the particle moves. (2) If say the x direction is chosen and $|v_x t| > 1$ the particle attempts to move to the next lattice site. If this is unoccupied the time is decremented and step (1) is repeated. (3) If the chosen lattice site is occupied a collision takes place and the x velocities of the two particles are adjusted according to (7) and (8) and the direction is again chosen randomly. (4) Once the particle has finished it motion, i.e. $|v_x t| < 1$, another particle and direction is chosen at random and heated according to (9). The process then repeats.

The advantage of using a lattice is clear in the collision step. Searching for collisions is trivial on a lattice, but would be very time-consuming in an off-lattice simulation. For off-lattice simulations with no heating an 'event-driven' algorithm can be used, but for a heated system the random heating invalidates this algorithm. The particles undergo continual collisions and continuos heating. Eventually these two effects balance out and the system reaches a steady state. At low densities the system behaves like a gas. However, the gas is not ideal. This is because the inelastic collisions between particles cause correlations between

them. The simplest way of seeing this is to consider two particles colliding in the centre of mass frame. Their initial speeds are v and their final speeds are $\eta v < v$. Thus the particles recede from each other more slowly than they approached and indeed more slowly than if they had not collided. They thus spend more time together and a correlation is induced. As shown in Section 2, in one dimension this correlation takes a power-law form.

Some characteristic results are shown in Fig. 4. At low densities the inelastic correlation causes one to have a relatively uninteresting correlated gas. To the naked eye this looks very similar to an ideal gas, and only by looking in detail at the correlation function can one distinguish the two. However, at higher densities the effective 'potential' induced by the inelastic collisions becomes very important. In particular, it leads to a first-order phase transition from a gas phase to a dense liquid phase which coexists with a vapour phase. This is clear in Fig. 4 where the behaviour of the system suddenly changes when the density is changed by of order 0.04%.

Although the transition is clear to the naked eye in large lattices, it is not so clear on smaller lattices. For this reason and to describe the transition quantitatively, it is useful to introduce the susceptibility χ (Binder and Herman 1988), defined by

$$\chi = \phi^{-1} {\sum_{s}}' s^2 n_s \,, \tag{10}$$

where n_s is the number of clusters of s particles and the prime means that the largest cluster is omitted. Here ϕ is the fraction of sites occupied by the particles, which we also call the density.

For small lattices where $L = 30, 40$ the transition is not clear and manifests itself as a pronounced peak in the susceptibility (Fig. 5). However, for larger lattices the peak is very pronounced, and there is a clear transition density ϕ_t beyond which the susceptibility is zero. This, together with the visual evidence shows that the system undergoes a liquid–gas phase transition at this point.

5. One-dimensional Gas in a Potential: A Symmetry Breaking Instability

The two systems studied above constitute possibly the simplest of all models for granular materials. Although we still do not understand them completely we do at least have a semi-quantitative understanding of some of their properties. It is thus not unreasonable to now include some of the complications often found in real granular materials. The existence of fixed boundaries has for instance been studied by Du *et al.* (1995) who found a breakdown on classical hydrodynamics caused by dissipative effects. In this section we wish to discuss, somewhat tentatively, the effect of another complication, i.e. the presence of a potential. In this section we work entirely in one dimension. It is well known that a classical ideal gas in equilibrium in a potential $U(x)$ has a density which scales as $\rho(x) = \rho_0 \exp[-U(x)/k_B T]$, where ρ_0 is a normalising constant and T is the temperature of the entire system. Consider what occurs for our dissipative gas. In the absence of any potential the average temperature and density are uniform over the system and are related by (5), $\rho = C\Omega(1-\eta^2)^{-1}T^{-3/2}$, i.e. hotter

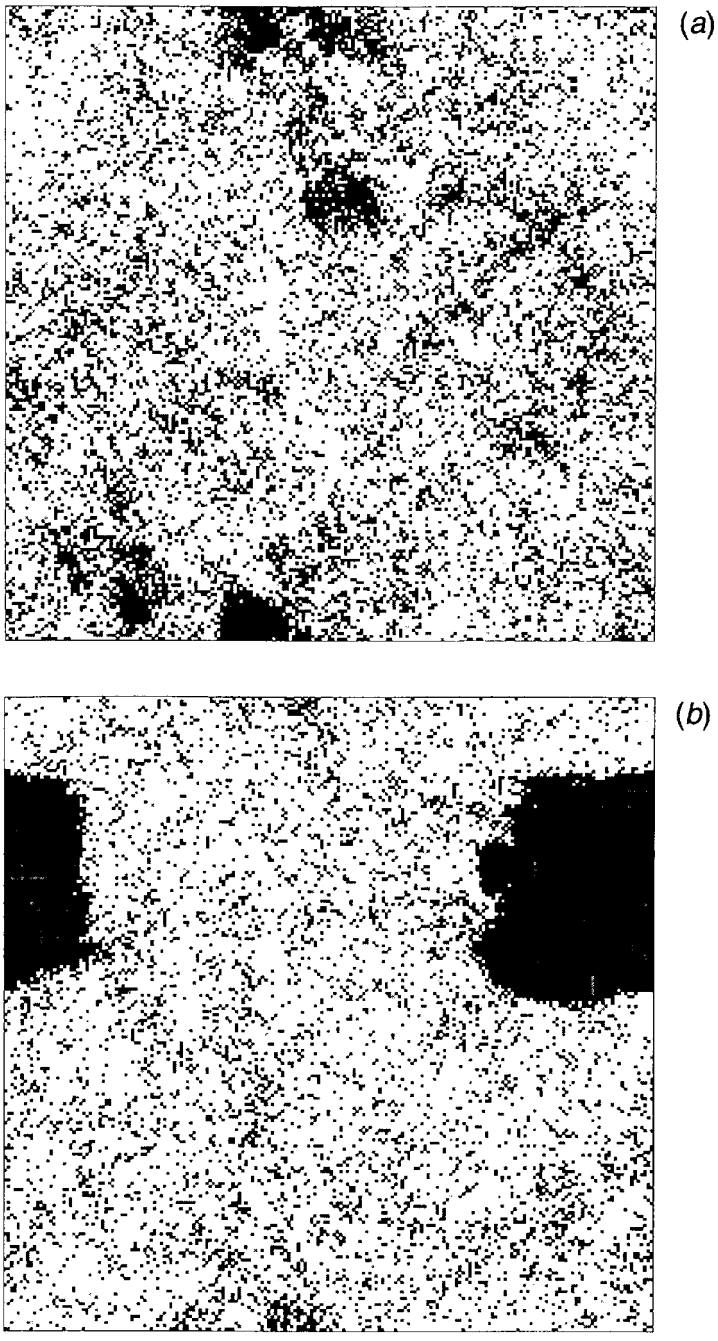

Fig. 4. Snapshots of the 2D lattice model for a 200×200 lattice with $\eta = 0$ and $r = 5$. The area fractions covered by the particles are (a) $\phi = 0{\cdot}259125$ and (b) $\phi = 0{\cdot}25925$. Between (a) and (b) the system undergoes a sharp transition from gas to liquid–gas coexistence. Note that periodic boundary conditions are used so there is only one droplet. The different shades of grey in the pictures correspond to different temperatures. Note that in going from (a) to (b) only five new particles are added, i.e. a change of area fraction of $\Delta\phi = 1{\cdot}25 \times 10^{-4} = 0{\cdot}04\%\phi$.

regions are less dense. An applied potential tends to confine the gas to regions where the potential is low. Thus in a parabolic potential, $U = x^2$, the particles will tend to cluster around the origin. The origin will thus be cooler than the surroundings and there will be a temperature gradient in either direction away from it. Thus a potential applied to an excited dissipative gas will induce a permanent temperature gradient. Of course, in a potential we cannot guarantee that (5) is correct, however, a qualitatively similar relation should hold. This cooling of the system in regions of low potential may lead to several interesting effects. The first of these is that the density may be a non-monotonic function of the potential. Thus, in some cases regions of high potential may have an increased density.

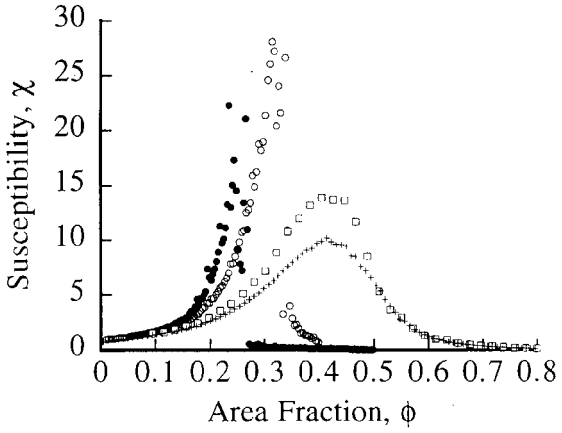

Fig. 5. Susceptibility χ versus the area fraction of particles, ϕ, for different $L \times L$. Here $\eta = 0$ and $r = 5$. The different points are as follows: $L = 30 \rightarrow +$, $L = 40 \rightarrow \square$, $L = 100 \rightarrow \bigcirc$, $L = 200 \rightarrow \bullet$. The position of the beak gives the transition density at which the system changes from a gas to gas–liquid coexistence. Note that as the size of the lattice increases the sharpness of the peak increases. The fact that the peak is higher for $L = 100$ compared with $L = 200$ is probably a result of finite sample times. Near the transition point there are large fluctuations in the susceptibility and an accurate measurement requires many samples.

We wish to concentrate on a second, more novel effect of the density–temperature relation when it is coupled to a potential. This we call 'phase separation' or 'symmetry breaking', and it can be most readily seen in the simple double-well potential $U(x) = x^4 - x^2$. This potential has two equal wells separated by a barrier. For an ideal gas placed in this potential, the density of particles in each well would be equal and would undergo small fluctuations of order $\sqrt{N}$. Thus for large N, at any given time the number of particles in each well would be equal. A moments thought shows that this is not the case for our dissipative gas (Fig. 6). Suppose the system is started with $N/2$ particles in each well. Due to the random noise in the system the particles will begin to mount the barrier and particles will be exchanged between wells. If the left well gains one particle then the density in that well increases and the temperature of that well decreases. Thus particles will be less likely to leave the left well, as they are cooler and have less energy to mount the barrier. Conversely, particles in the

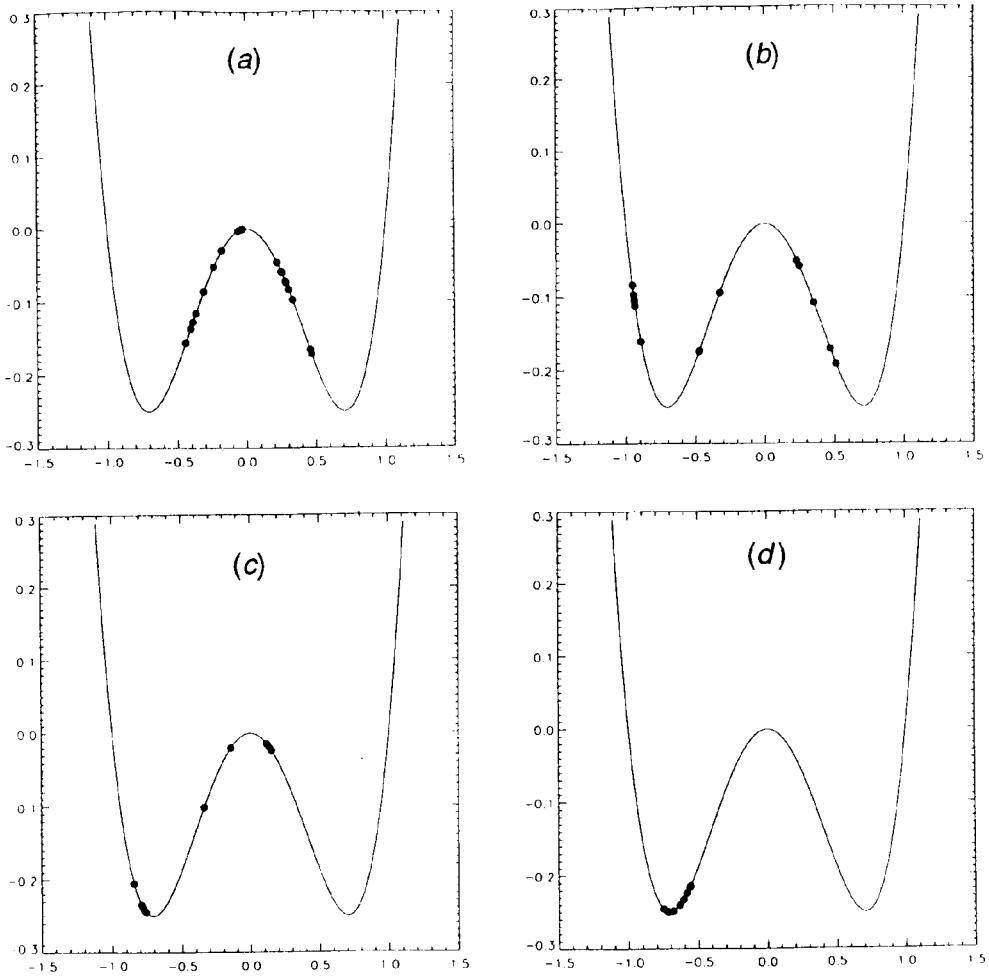

Fig. 6. Series of snapshots taken from a computer simulation of the particles in a one-dimensional double-well potential. Here we show the situation at low heating rates. In each case the potential is plotted against position. In (a), at the start of the simulation, the particles are roughly equally distributed between the wells. In going from (b) to (c) the particles begin to cluster, until in (d) they are all in the left well.

right well become hotter and begin to mount the barrier into the left well with ease. In this case there is a net current of particles and this process clearly describes an instability and leads to an avalanche of particles from one well to the other. The symmetry is thus broken in this system. At low heating rates one expects the particles to be mainly located on one side of the barrier, and the system phase separates. Of course this phase separation is not permanent, and after long intervals of time one expects a rapid transition from one well to the other. Nevertheless, at any one time the most likely state of the system is to

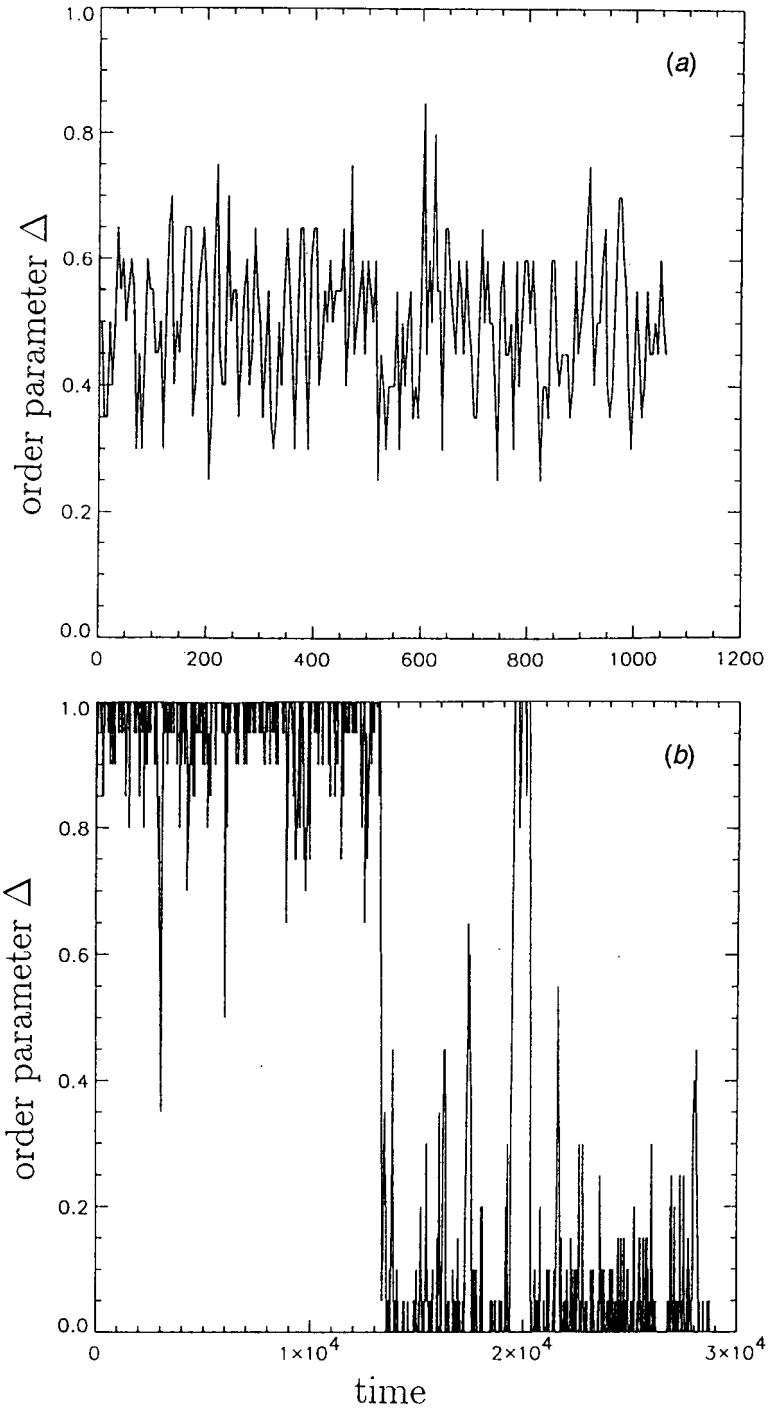

Fig. 7. Plots of the order parameter Δ as a function of time for the particles in a one-dimensional double-well potential. In (a) we see the high temperature case, where Δ fluctuates rapidly near $\frac{1}{2}$. In (b) we see the low temperature case, where Δ remains near 0 or 1. In this case there are rare switches between the two states.

have almost all the particles in one well. At higher heating rates this symmetry breaking ceases to be important. All the particles have enough energy to mount the barrier and the phase separated state disappears.

It is possible to test this idea by computer simulation. To this end we define an order parameter

$$\Delta = (1 - N_{\text{left}}/N)\,, \tag{11}$$

where N_{left} is the number of particles to the left of the origin. If all the particles are in one well then either $\Delta = 0$ or $\Delta = 1$. If the particles are equally distributed between the two wells then $D = \frac{1}{2}$.

For the high temperature phase we expect rapid time variations in Δ about $\Delta = \frac{1}{2}$. In the low temperature phase we expect D to be confined near $\Delta = 0$ or $\Delta = 1$, with rare rapid variations between the two. This is in fact what is found (see Fig. 7).

6. Conclusion

In this paper we have a simple model for a granular material, a driven dissipative 'gas'. This model avoids many of the complications inherent in previous granular studies, but still shows non-trivial behaviour. In the absence of a potential this behaviour is entirely due to the inelastic collisions between the particles. In one dimension these lead to long-range correlations which have approximately a power-law form. Despite this the one-dimensional system can be readily described by a simple equation of state. In two dimensions the inelastic collisions lead to a transition from a gas phase to a liquid–gas coexistence. More complex behaviour arises when the system is coupled to a potential. The phase separation which occurs is as a result of the temperature–density relation for these gases. In particular, it arises because dense regions are cool.

The models presented here are perhaps the simplest possible for granular materials. It is clear that they are only partially understood at present. Thus, we have an equation of state in the one-dimensional case, but we have no accurate way of predicting the exponent α which describes the correlations. Although we know that the two-dimensional case exhibits a liquid–gas phase separation we have not predicted where this will occur. In general all the systems we have presented are simple enough that they should be amenable to a simple theoretical description. This represents the greatest challenge for future work. One thing however is clear. The inelastic collisions cause an effective attractive potential to act between the particles. This potential can be thought of as causing the correlations in one dimension, the gas–liquid transition in two dimensions and the symmetry-breaking instability in the presence of an external potential.

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