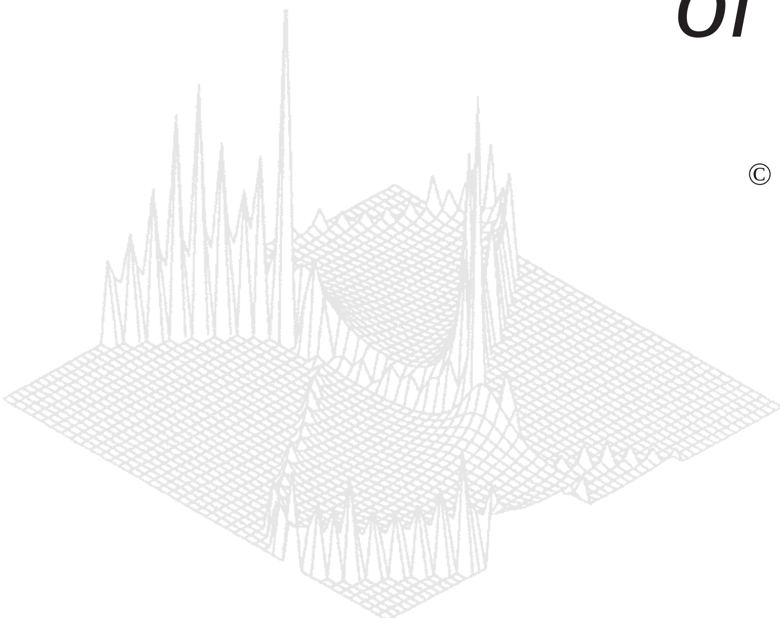
CSIRO PUBLISHING

Australian Journal of Physics

Volume 50, 1997



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood Telephone: 61 3 9662 7626
Vic. 3066 Facsimile: 61 3 9662 7611
Australia Email: peter.robertson@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO Australia and
the Australian Academy of Science

Aust. J. Phys., 1997, **50**, 935–40

Ballooning Modes in a Toroidally Rotating Plasma

S. Sen

Department of Physics, Flinders University of South Australia,
GPO Box 2100, Adelaide, SA 5001, Australia.

Abstract

The stability of ideal ballooning modes in the presence of a rigid toroidal rotation is revisited. Two stabilising contributions arising due to rigid toroidal rotation are identified. These stabilising contributions can be particularly significant for present and future generations of low and ultra-low aspect ratio tokamaks.

1. Introduction

It is well known that ideal ballooning modes can limit the beta values attainable in a tokamak operation in the usual first stability parameter regime, but the existence of a theoretically predicted second stability regime could give the possibility of high beta operation. Various methods, such as the use of highly energetic particles (Rosenbluth 1983), cross sectional shaping (Miller 1979; Chance 1983), current profile control (Sykes and Turner 1979; Coppi 1983) and the use of radio-frequency waves close to characteristic frequencies such as the lower hybrid frequency and the ion cyclotron frequency (D'ippolito 1987; Sen *et al.* 1986; Sen *et al.* 1991) have been shown to be capable of stabilising ballooning modes on individual flux surfaces for low values of shear, thus providing a route by which to access the second stability regime. Several other mechanisms have been known to affect the stability of ballooning modes. One of these is plasma rotation (Waelbroeck and Chen 1991 and references therein). In almost all the NBI heating experiments beam injection is unbalanced and, as a result, large plasma flows with toroidal and poloidal velocities which could be a significant fraction of poloidal Alfvén speed are observed (Brau 1983). However, poloidal flow is efficiently damped by magnetic pumping (Hassam and Kulsrud 1978). Indeed, as is observed experimentally, the damping time of poloidal flows is of the order of the ion–ion collision time (Brau 1983) and hence is much smaller than the damping time of toroidal flows. As a result, poloidal rotation dies away immediately after the beams are turned off, leaving the plasma rotation in the toroidal direction. Toroidal rotation, by contrast, is dissipated only through the diffusive transport of momentum. In a quiescent plasma, this viscous transport can be expected to reduce to low, neoclassical levels (Waelbroeck 1992).

The stability of MHD ballooning modes in the presence of toroidal flow is therefore an important issue. However, the presence of flow in the equilibrium

10.1071/PH96098 0004-9506/97/050935$05.00

state has a detrimental effect on the work of stability analysis. This is because it is usually impossible to give a necessary and sufficient condition for stability in the presence of flow. One can only state a sufficient condition for stability, typically as the positivity of the energy integral suggested by Frieman and Rotenberg (1960). Our main result in this work is to improve this criterion for ballooning perturbation by taking into account the effects of the Coriolis force and the modification in the pressure profiles due to rotation. This might bring the criterion closer to the actual stability boundary. These additional stability contributions can be particularly significant for the low (Peng and Strickler 1986) and the ultra-low (Shaing 1995) aspect ratio tokamaks. This is particularly important because one of the major approaches for extracting fusion power more economically is to reduce the aspect ratio in present and future generations of tokamaks.

2. Equilibrium and Stability

We consider a plasma confined in a perfectly conducting axisymmetric torus which obeys the ideal MHD equations

$$\mathbf{B}_t + \mathrm{curl}(\mathbf{B} \times \mathbf{V}) = 0\,, \tag{1}$$

$$\rho(\mathbf{V}_t + \mathbf{V}.\nabla\mathbf{V}) + \nabla p = \mathbf{J} \times \mathbf{B}\,, \tag{2}$$

$$\rho_t + \mathrm{div}(\rho\mathbf{V}) = 0\,, \tag{3}$$

$$S_t + \mathbf{V}.\nabla S = 0\,, \tag{4}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{J} = \mathrm{curl}\ \mathbf{B}$ is the current density, and $\mathbf{V}$, p and ρ are the plasma velocity, pressure and density respectively. Here S is a function of the specific entropy and γ is the ratio of specific heats, while the subscript t implies derivations with respect to t. We assume that the plasma satisfies the equation of state

$$p = S\rho^\gamma\,. \tag{5}$$

We consider the case of pure toroidal rotation where the plasma experiences a rigid rotation about its axis of symmetry, with a constant angular velocity $\Omega = V_0/R$ (where V_0 is the flow velocity and R is the radius). We assume the temperature to be a surface quantity (in view of the high thermal conductivity along the field lines) and replace equation (4) by $\mathbf{B}.\nabla T = 0$. With a further choice of $T = \mathrm{constant}$ and with $C_s^2 = \partial p/\partial\rho$ $(=\gamma\bar{R}T)$ denoting the square of the sound velocity, the parallel component of equation (2) can be solved to give the equilibrium pressure as (Sen 1994; Maschke and Perrin 1980):

$$p_0 = \bar{p}_0(\psi)\exp\left(\frac{\Omega^2 R^2}{2\bar{R}T}\right), \tag{6}$$

where $\overline{p_0}(\psi)$ is an arbitrary function of the poloidal flux. Note that rotation induces poloidal asymmetry in the equilibrium pressure on a flux surface. We will come back to this point in detail later.

To consider the linear stability of the equilibrium just described it is convenient to carry out the analysis in a frame rotating with the plasma. In this frame the governing equations differ from those in an inertial frame by the addition of two terms, the centrifugal and the Coriolis force, which arise because of the noninertial nature of the reference frame. Assuming that the perturbations have a time dependence of the form $e^{i\omega t}$, we can show that stability is given by (Bhattacharjee 1987; Hameiri and Laurence 1984)

$$\omega^2 K - 2\omega A - \delta W = 0\,, \tag{7}$$

where

$$K = \tfrac{1}{2}\int \rho\boldsymbol{\xi}^2 dv\,, \tag{8}$$

$$A = \tfrac{1}{2}\int i\rho\boldsymbol{\Omega}\,.(\boldsymbol{\xi}\times\boldsymbol{\xi}^*)\,, \tag{9}$$

$$\begin{aligned}\delta W = \int \frac{dl}{B}\bigg[X'\mathbf{N}^2 - X^2\bigg(&2(\mathbf{N}\,.\,\nabla p)(\mathbf{N}\,.\,\boldsymbol{\kappa}) - R\Omega^2(\mathbf{N}\,.\,\nabla R)(\mathbf{N}\,.\,\nabla\rho)\\ &+\frac{1}{\gamma p}(\rho R\Omega^2\mathbf{N}\,.\,\nabla R)^2\bigg)\bigg]\,,\end{aligned} \tag{10}$$

where $\boldsymbol{\xi}$ is the Lagrangian displacement of the fluid from its equilibrium position, X is the component of $\boldsymbol{\xi}$ in the direction $\mathbf{N}$, which is parallel to $\mathbf{B}\times\kappa$ and normalised such that $\mathbf{N}\,.\,\nabla\psi = 1$, and the prime indicates derivation with respect to l, the coordinate along the field-line, and κ is the curvature.

Now it is easy to see that the presence of the second term (the contribution due to the Coriolis force) in equation (7) does not allow a self-adjoint formulation. Thus, unlike in the static case, it is not possible to establish a necessary and sufficient condition for stability in a manner analogous to that used in the ideal MHD theory based on the energy principle. This difficulty is usually overcome by assuming a large aspect ratio ($\epsilon = a/R \ll 1$) and high beta tokamak ordering (Hameiri and Laurence 1984). Then, in calculating δW, only terms of the order of ϵ^2 are retained, so that the first and third terms in equation (7) being both of order of ϵ^2 are retained, while the middle term which turns out to be of order of $\epsilon^{\frac{5}{2}}$ is dropped to leading order. Once this term is dropped the problem then reduces to a self-adjoint formulation such that $\delta W < 0$ implies instability. Therefore, $\delta W < 0$ with $\delta W \sim \epsilon^2$ is not only a necessary but also a sufficient condition for instability. Hence, as evident from the expression of δW (equation 10), a rigid rotation in general has a destabilising effect on ideal ballooning modes (Bhattacharjee 1987).

However, we will now show that extending the stability analysis to higher order can make this picture different. First, we will show by a heuristic argument that the Coriolis force which is usually dropped in the stability analysis has a stabilising role. To show this we notice that in order for the mode to be stable (i.e. ω to be real) the discriminant of equation (7) has to be positive, i.e.

$$4A^2 + 4K\delta W > 0\,.$$

This shows that the Coriolis force (represented by $4A^2$ term in the above condition) has a stabilising contribution. This condition however is only sufficient for stability and not necessary. This was also noted by Frieman and Rotenberg (1960). This stabilising contribution should be taken into account while predicting the deleterious role of the rigid rotation. It should be pointed out that this improvement in the stability is of the order of $\epsilon^{\frac{5}{2}}$ (see e.g. Sykes 1979). However, one of the more economic approaches to extracting fusion power in the present and future generations of tokamaks is to reduce the aspect ratio to low, e.g. $(R/a) \sim 2{\cdot}5$, and ultra-low, e.g. $(R/a) \sim 1{\cdot}0$, levels (Peng and Strickler 1986; Shaing 1995), in which case the stabilising contribution of the Coriolis contribution can be significant.

Now we will show that there can be one more stabilising effect due to the rigid rotation. To show this we start with the expression of equilibrium pressure from equation (6), which can be written (assuming large aspect-ratio and circular cross section) as

$$p(r,\theta) = \bar{p}(r)\left(1 + \delta\frac{r}{R_0}\cos\theta\right), \tag{11}$$

where $\bar{p}(r) = p(r)e^{\delta}, \delta = \Omega^2 R_0^2/2\overline{R}T$ and $R = R_0 + r\cos\theta$. This gives

$$\begin{aligned}\nabla p &= \frac{\partial p}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial p}{\partial\theta}\mathbf{e}_\theta \\ &= \frac{\partial\bar{p}}{\partial r}[1 + \eta\cos\theta]\mathbf{e_r} - \frac{\bar{p}}{R_0}\delta\sin\theta\mathbf{e}_\theta\,,\end{aligned} \tag{12}$$

where $\eta = \delta\ (r{-}L_p)/R_0$ and $L_p = -\bar{p}(d\bar{p}/dr)$ is the pressure gradient scale length. An interesting point can be noted from equation (12). For $r < L_p (\eta < 0)$, i.e. for inner flux surfaces the pressure gradient is reduced in the outboard side of the plasma ($|\ \theta\ | \leq \pi/2$) where the ballooning mode is most unstable and the rigid rotation is therefore expected to have a stabilising influence on the ballooning mode. On the other hand, for $r > L_p(\eta > 0)$, i.e. for the outer flux surfaces, rotational drive adds to enhance the pressure gradient in the outboard side of the plasma and the rigid rotation is expected to have a destabilising influence on the ballooning mode. It is well known for a static plasma that if the equilibrium is such that the pressure surfaces are shifted inward (towards the favourable side) with respect to the flux surfaces, the stability of the ballooning mode can be improved (see e.g. Wang and Bhattacharjee 1990). From equations (11) and (12) it is evident that due to the effect of the centrifugal force, equilibrium pressure becomes poloidally asymmetric and $p(\psi)' =$ constant surfaces (here the prime refers to the derivations with respect to ψ) are inwardly shifted for $r < L_p$

($\eta < 0$). So, rigid rotation has a stabilising influence on the inner surfaces. On the other hand, for $r > L_p(\eta > 0)$, the $p(\psi)' =$ constant surfaces are shifted outward (towards the unfavourable side) and as a result instability is favoured (Bishop and Hastie 1985).

To show this more explicitly, let us assume the limit $\epsilon \ll 1$ for the sake of simplicity and write equation (10) in a more familiar form for circular flux surfaces (Sykes 1979):

$$\delta W = \int_{-\infty}^{\infty} [1 + (\theta s + \lambda)^2] \left(\frac{dX}{d\theta} \right)^2 - G[\cos\theta + (\theta s + \lambda)\sin\theta] X^2 d\theta \,, \qquad (13)$$

where

$$G = \alpha(1 + \eta \cos\theta), \qquad \alpha = \frac{2R_o q^2}{B_0^2} \left(\bar{p} + \frac{\rho R_o^2 \Omega^2}{2} \right)',$$
$$\lambda = \alpha \left(1 + \frac{\eta}{4} \cos\theta \right) \sin\theta, \qquad s = rq'/q$$

and B_0 is the toroidal magnetic field. It is straightforward to see that the corresponding Euler equation obtained from (13) is the $s - \alpha$ equation derived by Sykes (1979) in the static case (hence α in their equation does not have a flow pressure contribution and η, the poloidal asymmetry, is produced by the high power neutral beam injection). This $s - \alpha$ equation was solved numerically by Sykes (1979), who confirmed stability for $\eta < 0$. Later Bishop and Hastie (1987) extended the case for $\eta > 0$ and found destabilisation for this case. It is also worth noting that the pressure asymmetric effect (due to rotation) on the stability of the ballooning mode was also noted by Wang and Bhattacharjee (1990). What we have demonstrated here is that the rigid rotation can produce a *favourable* ($\eta < 0$) poloidal asymmetry in the equilibrium pressure and thereby can actually exert a stabilising contribution on the ballooning mode stability. As typically $L_p \sim L_n$ (the density scale length), the $r < L_p$ region encompasses the bulk of the plasma profile. So, the bulk of the plasma is expected to experience the stabilising effect of the rigid rotation. At the outer edge, the present theory which is limited to the case of rigid toroidal rotation is however not expected to be applicable. So, the case of $r > L_p$ should be treated with caution. Before leaving this section it is important to mention that the pressure asymmetry envisioned in this work is a next order effect (as $\eta \sim \epsilon$). However, as found by Sykes (1979) and also by Fielding and Haas (1978), the stabilisation can be substantial for rather modest values of 'η'. So even though the poloidal asymmetry in the pressure profile is of the next order, it can play an important role in the ballooning mode stability for a low aspect ratio tokamak. It can be shown that, although the higher order stabilising contributions due to the Coriolis force ($\sim \epsilon^{\frac{5}{2}}$) and due to the poloidal asymmetry in the pressure ($\sim \epsilon^3$) taken separately cannot compete with the lower order ($\sim \epsilon^2$) destabilising contribution, these two higher order stabilising contributions *taken together* can supersede the lower order destabilising contribution for low aspect ratio tokamaks ($\epsilon \geq 1/2{\cdot}5$).

3. Conclusions

In summary, we have found that the effect of rigid toroidal rotation on the stability of ballooning modes may not be all deleterious. We have identified two contributions arising due to the rigid rotation—one due to the Coriolis force and the other due to the modification of pressure—which can have stabilising influences on ballooning modes. These two stabilising contributions can add together to suppress the rotational drive. This improvement in stability can be particularly significant in low aspect ratio tokamaks. Our results therefore could be crucial in present and future generations of tokomaks which aim to reduce the aspect ratio to low and ultra-low values for extracting fusion power economically. This stabilisation of ballooning modes by toroidal rotation may be crucial in explaining the core confinement improvement (like VH modes) in tokamaks. This could also be a plausible cause for the H-mode transition induced by the NBI which introduces considerable toroidal momentum to the plasma.

Acknowledgments

The author would like to thank Professors R. L. Dewar and R. G. Storer for useful discussions. The research was supported by the Australian Research Grants Scheme.

References

Bhattacharjee, A. (1987). Proc. Workshop on Theory of Fusion Plasmas, Varenna (Eds A. Bonderson *et al.*), p. 47 (Edit. Compositori: Bologna).
Bishop, C. M., and Hastie, R. J. (1987). *Nucl. Fusion* **25**, 1443.
Brau, K. (1983). *Nucl. Fusion* **23**, 1643.
Chance, M. S. (1983). *Phys. Rev. Lett.* **51**, 1963.
Coppi, B. (1983). *Comm. Plasma Phys. Controll. Fusion* **8**, 11.
D'ippolito, D. A. (1987). *Phys. Rev. Lett.* **58**, 2216.
Fielding, P. J., and Haas, F. A. (1978). *Phys. Rev. Lett.* **41**, 801.
Frieman, E. A., and Rotenberg, M. (1960). *Rev. Mod. Phys.* **32**, 898.
Hameiri, E. J., and Laurence, P. J. (1984). *Math. Phys.* **25**, 396.
Hassam, A., and Kulsrud, R. (1978). *Phys. Fluids* **21**, 2271.
Maschke, E., and Perrin, H. (1980). *Plasma Phys.* **22**, 579.
Miller, R. L. (1979). *Phys. Rev. Lett.* **43**, 765.
Peng, Y.-K. M., and Strickler, D. J. (1986). *Nucl. Fusion* **26**, 769.
Rosenbluth, M. (1983). *Phys. Rev. Lett.* **51**, 1967.
Sen, A., Sundaram, A. K., and Kaw, P. K. (1986). *In* 'Plasma Physics and Controlled Nuclear Fusion Research', Vol. 2, p. 101 (IAEA: Vienna).
Sen, S. (1994). *Contribution Plasma Phys.* **34**, 265.
Sen, S., Janaki, M. S., and Dasgupta, B. (1991). *Phys. Lett.* A **157**, 411.
Shaing, K. C. (1995). *Phys. Plasmas* **2**, 349.
Sykes, A. (1979). *In* 'Plasma Physics and Controlled Nuclear Fusion Research', Vol. 1, p. 625 (IAEA: Vienna).
Sykes, A., and Turner, M. (1979). Proc. 9th European Phys. Soc., Oxford, Vol. 1, EP22.
Waelbroeck, F. L. (1992). *Phys. Fluids* **4**, 2441.
Waelbroeck, F. L., and Chen, L. (1991). *Phys. Fluids* B **3**, 601.
Wang, X. H., and Bhattacharjee, A. (1990). *Phys. Fluids* B **2**, 2346.

Manuscript received 18 September 1996, accepted 20 March 1997