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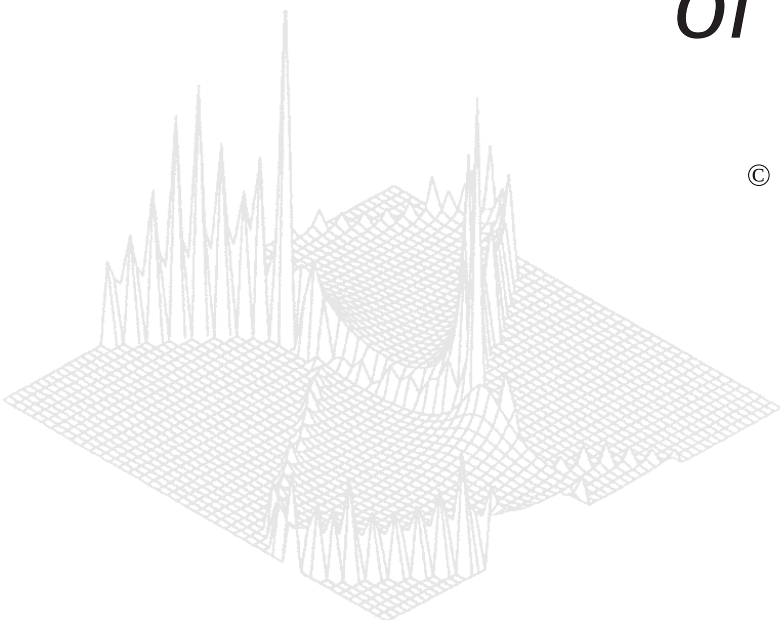

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Construction of Real Space–Time from Complex Linear Metric Connections

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Abstract

A construction of real space–time based on metric linear connections in a complex manifold is described. The construction works only in two or four dimensions. The four-dimensional case based on a connection reducible to group $U(2,2)$ can generate Riemann–Cartan geometry on the real submanifold of the original complex manifold. The possibility of connecting the appearance of Dirac fields with anholonomic complex frames is discussed.

1. Introduction

We cannot build physics on the basis of the matter-concept alone. But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics?

Thus wrote Albert Einstein and Leopold Infeld in their book 'The Evolution of Physics' in 1938 (Einstein and Infeld 1971). Einstein's original idea was to consider particles as regions with a high concentration of field energy, where the field itself would have some geometrical meaning. A considerable difficulty with such an approach lies in the existence of spin $\frac{1}{2}$ particles. Geometrical theories based on the structure of the real four-dimensional space–time manifold do not naturally contain functions that are needed to describe such particles. Wave functions describing spin $\frac{1}{2}$ particles have transformation properties of complex vectors transforming under the action of complex groups. They can be certainly described quite well within the real space–time, but they do not form a natural part of the geometry of space–time. Thus in order to give some simple geometrical explanation of the existence of spin $\frac{1}{2}$ particles, one should start with a more fundamental complex manifold, and explain, why in some circumstances it has an appearance of the real space–time and what sort of geometry would reveal its true complex structure.

The purpose of the present article is to show that there is a mathematically natural way to proceed in such a direction if one accepts that the fundamental manifold is not open to direct observations and its structure may be investigated only via cross sections in its bundle of frames. In fact, that is not really a new idea. In general relativity, one talks about local observers associated with the local Lorentz frames (Misner *et al.* 1970) and the geometrical properties of the

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base manifold are deduced from a smooth choice of such frames over a given region, i.e. from a local cross section of the bundle of linear frames of the base manifold. Of course, one takes the space–time manifold as the base manifold in general relativity. What is suggested here is to take one more step and start with a more fundamental base manifold which would appear as a four-dimensional real space–time only for a particular class of the local cross sections in its bundle of frames. The general features of such a construction have been described (Smrz 1987) and lead to a tentative geometrical interpretation for the complex phase used in quantum theory (Smrz 1995). Here we shall concentrate on the aspects mentioned above, namely tentative geometrical interpretation of the spinor fields describing spin $\frac{1}{2}$ particles.

We start with a linear connection in a complex n-dimensional manifold reducible to a unitary or pseudo-unitary subgroup of $Gl(n, C)$. This provides a natural way of reducing the observable real dimensions to n. It also defines a linear connection on the n-dimensional real submanifold of the base manifold reducible to $SO(r, s)$, $r + s = n$, but only when n is either 2 or 4. The case with $n = 4$ is obviously the one that seems to correspond to reality. The linear connections reducible to $U(2, 2)$ lead to connections on the real submanifold reducible to $SO(4, 1)$. They also provide possible reasons why in some regions of space–time we observe complex vectors (spinors). They may be associated with anholonomic complex coordinates, and appear only when the original connection goes beyond the 'pure torsion' connection defined on a pseudo-unitary complex affine space.

One should realise that when considering metric connections in a bundle of linear frames of any manifold, there are three stages:

(1) The manifold is an affine metric space and the connection is the natural flat connection defined by the parallelism in the affine space.

(2) The manifold is still an affine metric space (i.e. affine coordinates still exist), but the connection is not flat, though it preserves the metric.

(3) The manifold is not an affine space (i.e. affine coordinates can be defined only as anholonomic) and the connection is not flat.

From considerations of general relativity theory we consider (3) as more fundamental than (2) since the basic structure of space–time appears to be of type (3) with zero torsion, but from the mathematical point of view (2) is the natural step from (1). In the approach starting from the connections in complex manifolds, stage (2) corresponds to a description of an empty space–time of general relativity, while stage (3) leads to the appearance of spinors.

2. Space–Time from Linear Connections on a Fundamental Manifold

In this section the basic idea is briefly reviewed (Smrz 1987). For an explanation of geometric concepts the reader may consult for example the book by Nakahara (1990). We assume the existence of a fundamental manifold which is not open to direct geometrical observations. Yet, the geometry of the fundamental manifold in the form of a connection in its bundle of frames is assumed to determine the physical properties of the Universe, including the apparent existence of the real four-dimensional space–time. Observers investigate the connection by building local cross sections in the bundle of frames and measure their departure from the horizontal directions determined by the connection. As mentioned before, this is a normal procedure in general relativity where, however, the fundamental manifold is space–time. In the present approach, the most basic geometrical properties of space–time, including being real and four-dimensional, are to result from such observations. This is achieved by including group elements which may act like translations within the structure group of the bundle. What follows is the mathematical formulation of the basic idea made as self-contained as possible.

Let M be the fundamental manifold with a linear connection. We shall consider M as real manifold of dimension m, since a complex manifold of dimension n is also a special case of a real $2n$-dimensional manifold. Let x^μ denote local coordinates in M. The bundle of frames P consists of all possible linear frames

$$\left\{ h_i^\mu \frac{\partial}{\partial x^\mu};\ i, \mu = 1, .., m \right\}$$

at each point $x \in M$. A local cross section in P is a smooth selection of a particular frame $h_i^\mu(x)\frac{\partial}{\partial x^\mu}$ for each x in a neighbourhood of a point in M. A point in P, i.e. a frame at $x \in M$, may then be characterised by the set of $m + m^2$ coordinates x^μ and a_j^i:

$$\left\{ h_i^\mu(x) a_j^i \frac{\partial}{\partial x^\mu};\ j = 1, .., m \right\}.$$

This is the local trivialisation of P with respect to the selected local cross section.

The linear connection in M is then characterised by the horizontal lift $X_\mu^{(h)}$ of $\dfrac{\partial}{\partial x^\mu} \in T_x M$ to $p \in P$:

$$X_\mu^{(h)} = \frac{\partial}{\partial x^\mu} - A_{\mu j}^i(x) Y_i^j\,, \tag{1}$$

where

$$Y_i^j = a_k^j \frac{\partial}{\partial a_k^i}$$

are the right-invariant vector fields of the general linear group $Gl(m, R)$. Functions $A_{\mu j}^i(x)$ are the components of the connection. When the reference cross section characterised by $h_i^\mu(x)$ is changed to $h_j^\mu(x)\alpha_i^j(x)$, where $\alpha_i^j(x)$ are elements of $Gl(m, R)$, the coordinates (x^μ, a_j^i) in P change to

$$\tilde{x}^\mu = x^\mu \qquad \tilde{a}_j^i = \alpha^{-1}{}^i_k(x) a_j^k\,,$$

and, accordingly,

$$\begin{aligned} X_\mu^{(h)} &= \frac{\partial}{\partial x^\mu} + \frac{\partial \alpha^{-1}{}^i_k(x)}{\partial x^\mu} a_j^k \frac{\partial}{\partial \tilde{a}_j^i} - A_{\mu j}^i a_k^j \frac{\partial \tilde{a}_m^l}{\partial a_k^i} \frac{\partial}{\partial \tilde{a}_m^l} \\ &= \frac{\partial}{\partial x^\mu} - [\alpha^{-1}{}^l_i(x) A_{\mu j}^i(x) \alpha_m^j(x) - (\partial_\mu \alpha^{-1}{}^l_i(x)) \alpha_m^i(x)] \tilde{a}_k^m \frac{\partial}{\partial \tilde{a}_k^l}\,. \end{aligned}$$

This is the gauge transformation of the connection components which may be also written in the matrix form as

$$\tilde{A}_\mu(x) = \alpha^{-1}(x)A_\mu(x)\alpha(x) + \alpha^{-1}(x)(\partial_\mu\alpha(x)) \,. \tag{2}$$

Even though the gauge transformation is only a special kind of a transformation of coordinates, it has physical significance, since it involves selection of a reference cross section and that forms a part of the process of measurement.

We assume that the connection in P is reducible to a subgroup G of $Gl(m,R)$, which contains a ten-dimensional subgroup H with the Lie algebra spanned by

$$\{L_{ij} = -L_{ji}, T_i;\ i,j = 1,..,4\}$$

and the commutation relations

$$[L_{ij}, L_{kl}] = g_{jk}L_{il} + g_{il}L_{jk} - g_{ik}L_{jl} - g_{jl}L_{ik} \,, \tag{3}$$

$$[L_{ij}, T_k] = g_{jk}T_i - g_{ik}T_j \,, \tag{4}$$

where $g_{ij} = diag(1,1,1,-1)$. Such a set-up may define a Riemann–Cartan geometry on a four-dimensional submanifold of M. Let N be such a submanifold, and select the coordinate system in M in such a way that $x^\mu, \mu = 1,..,4,$ are coordinates in N, and N is defined by fixing the values of $x^\mu, \mu = 5,..,m$. Denoting the components of the connection corresponding to L_{ij} and T_i by $A^{ij} = -A^{ji}_\mu$ and A^i_μ respectively, we suppose that the 4 $\times$4 matrix A^i_μ is invertible. Then a cross section in the bundle of frames of N is defined by

$$\left\{A^{-1}{}^\mu_i \frac{\partial}{\partial x^\mu};\ \mu, i = 1,..,4\right\} \,,$$

while A^{ij}_μ become the Lorentz components of a metric linear connection in N. The conserved metric is then given by

$$g_{\mu\nu}(x) = A^i_\mu(x)A^j_\nu(x)g_{ij} \,. \tag{5}$$

The identification described above does not depend on the choice of the reference cross section as long as the gauge transformation stays within the Lorentz subgroup. The commutation relations (3) and (4) guarantee the correct transformation of A^{ij}_μ and A^i_μ under the Lorentz gauge transformations.

We do not assume any particular form for $[T_i, T_j]$. That is needed only when a gauge transformation outside the Lorentz subgroup is applied. Thus one can generate a flat connection in N even if $[T_i, T_j]$ is not zero.

It should be expected that the submanifold N of M, i.e. the observed space–time manifold, is determined in some way by the horizontal lift. Since we assume that observers can measure a change of the position only by comparing their reference cross section with the horizontal direction determined by the connection, it is natural to demand that the horizontal lift is trivial in the unobserved directions:

$$X_\mu^{(h)} = \frac{\partial}{\partial x^\mu},\ \mu = 5, .., m. \tag{6}$$

We shall see in the next section how this can be achieved when the fundamental manifold is complex.

3. Linear Connections in Complex Manifolds

Let M be an n-dimensional complex manifold and $z^\mu = x^\mu + iy^\mu$ local coordinates in M. A linear connection may be defined in a way similar to the real case by the horizontal lift

$$Z_\mu^{(h)} = \frac{\partial}{\partial z^\mu} - A^i_{\mu j}(z) Y_i^j\,, \tag{7}$$

where $A^i_{\mu j}(z)$ and Y_i^j are now complex. In particular, we have

$$Y_j^i = a_k^i \frac{\partial}{\partial a_k^j}\,,$$

where a_j^i are elements of a complex invertible $n \times n$ matrix.

The above is in fact a special case of a linear connection in a $2n$-dimensional real manifold with local coordinates x^μ and y^μ. Writing

$$\frac{\partial}{\partial z^\mu} = \frac{\partial}{\partial x^\mu} - \frac{i}{2}\frac{\partial}{\partial y^\mu}\,,$$

$$A^i_{\mu j} = M^i_{\mu j} + iN^i_{\mu j}\,,$$

$$a_j^i = u_j^i + iv_j^i\,,$$

$$\frac{\partial}{\partial a_j^i} = \tfrac{1}{2}\left(\frac{\partial}{\partial u_j^i} - i\frac{\partial}{\partial v_j^i}\right),$$

$$Z_\mu^{(h)} = \tfrac{1}{2}(X_\mu^{(h)} - iY_\mu^{(h)})\,,$$

we have

$$X_\mu^{(h)} = \frac{\partial}{\partial x^\mu} - M^i_{\mu j} U_i^j + N^i_{\mu j} V_i^j\,, \tag{8}$$

$$Y_\mu^{(h)} = \frac{\partial}{\partial y^\mu} - M^i_{\mu j} V_i^j - N^i_{\mu j} U_i^j\,, \tag{9}$$

and where

$$U^i_j = u^i_k \frac{\partial}{\partial u^j_k} + v^i_k \frac{\partial}{\partial v^j_k}\,, \qquad V^i_j = v^i_k \frac{\partial}{\partial u^j_k} - u^i_k \frac{\partial}{\partial v^j_k}.$$

Let us now consider a reduction of the connection to a subgroup of $Gl(n, C)$. The analogue of a reduction to the orthogonal or pseudo-orthogonal subgroups in the real case is a reduction to the unitary or pseudo-unitary groups. Assume that the connection is reducible to $U(r,s)$, $r+s=n$, and denote by g_{ij} the diagonal matrix with r elements equal to $+1$ and s diagonal elements equal to -1. The subgroup is then defined by

$$\bar{a}^k_i g_{kl} a^l_j = g_{ij}$$

and the right invariant vector fileds of $U(r,s)$ may be written as

$$Y_{ij} = g_{ik} Y^k_j - g_{jk} \bar{Y}^k_i\,,$$

where the bar denotes complex conjugation.

In the real representation we have

$$Y_{ij} = \tfrac{1}{2} U_{ij} + \frac{i}{2} V_{ij}\,,$$

where

$$U_{ij} = g_{ik} U^k_j - g_{jk} U^k_i\,, \qquad V_{ij} = g_{ik} V^k_j + g_{jk} V^k_i.$$

The reduced horizontal lift of $\partial/\partial x^\mu$ can be written as

$$X^{(h)}_\mu = \frac{\partial}{\partial x^\mu} - \tfrac{1}{2} M^{ij}_\mu U_{ji} + \tfrac{1}{2} N^{ij}_\mu V_{ji}\,,$$

where

$$M^{ij}_\mu = M^i_{\mu k} g^{kj} = -M^{ji}_\mu\,, \tag{10}$$

and

$$N^{ij}_\mu = N^i_{\mu k} g^{kj} = N^{ji}_\mu. \tag{11}$$

The requirement of M^{ij}_μ being skew-symmetric and N^{ij}_μ symmetric is necessary for the reduction. At the same time, the reduced horizontal lift of $\partial/\partial y^\mu$ then automatically yields the trivial lift

$$Y^{(h)}_\mu = \frac{\partial}{\partial y^\mu}.$$

Of course, one could also consider the case of M_μ^{ij} being symmetric and N_μ^{ij} skew-symmetric, which would lead to the trivial lift of $\partial/\partial y^\mu$. In either case, n real dimensions of M have trivial horizontal lift once the connection is reduced from the $2n^2$-dimensional group $Gl(n, C)$ to the n^2-dimensional group $U(r, s)$. In this way a linear connection in M reducible to $U(r, s)$ may lead to an induced linear connection reducible to $SO(k, l)$ in the real submanifold of M as long as the Lie algebra of $U(r, s)$, $r + s = n$, contains the Lie algebra spanned by $\{L_{ij} = -L_{ji}, T_i; i, j = 1, .., n\}$ with commutation relations as in (3) and (4). The lowest dimension when this happens is $n = 2$ with $SU(2)$ locally isomorphic to $SO(3)$ or $SU(1, 1)$ locally isomorphic to $SO(2, 1)$. The next case is $U(2, 2)$ containing subgroups locally isomorphic to $SO(4, 1)$, $SO(3, 2)$, as well as the Poincaré group. For $n > 4$ no pseudo-unitary or unitary group contains subgroups locally isomorphic to $SO(k, l)$, $k + l = n + 1$, or to the corresponding Poincaré group. It means that if the Riemann–Cartan geometry on a real manifold is to be generated in the above described way, there is very little choice. It is, however, significant that the observed space–time geometry, i.e. Lorentz reducible metric connections on a 4-dimensional real manifold, is one of the allowed options.

4. Empty Space–Time and Connections in a Complex Four-dimensional Metric Affine Space

Let M be a complex four-dimensional hermitean metric affine space with canonical form of the metric $g_{ij} = diag(1, 1, -1, -1)$. This means that it is possible to choose an affine coordinate system z^i, $i = 1, .., 4$, such that for arbitrary tangent vectors we have

$$g\left(u^i \frac{\partial}{\partial z^i}, v^j \frac{\partial}{\partial z^i}\right) = \bar{u}^i g_{ij} v^j$$

at any point of M. Even such a simple space can have a non-flat connection defined in its bundle of frames. Thus if z^μ are general coordinates in M we can write the horizontal lift (7) defining the connection. If we assume that the connection is reducible to $U(2, 2)$, equations (10) and (11) can be written as

$$A_\mu^\dagger G + GA_\mu = 0\,, \tag{12}$$

where G denotes the matrix with elements g_{ij} and A_μ the matrix with elements $A^i_{\mu j}$. The basis for complex matrices satisfying (12) consists of sixteen matrices, of which eight are hermitean and anticommute with G, while the other eight are antihermitean and commute with G. In terms of the standard Dirac matrices $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ and $\gamma_5 = \gamma_4\gamma_1\gamma_2\gamma_3$, using the representation in which $\gamma_4 = G$ and $\gamma_4^2 = -\gamma_1^2 = -\gamma_2^2 = -\gamma_3^2 = 1$, and γ_4 is hermitean while the other gamma matrices are antihermitean, we can write

$$A_\mu = \tfrac{1}{2}A_\mu^{ij} L_{ij} + B_\mu^i P_i + C_\mu^i Q_i + F_\mu D + H_\mu E\,, \tag{13}$$

where $L_{ij} = \frac{1}{2}(\gamma_i\gamma_j - \gamma_j\gamma_i)$, $P_i = \dfrac{i}{2}\gamma_i$, $Q_i = \frac{1}{2}\gamma_i\gamma_5$, $D = \dfrac{i}{2}\gamma_5$, $E = \dfrac{i}{2}I$, $i, j = 1, .., 4$, and the functions A_μ^{ij}, B_μ^i, C_μ^i, F_μ, and H_μ are real. The subgroup L of $U(2, 2)$

generated by L_{ij} is locally isomorphic to the Lorentz group $SO(3,1)$, and together with P_i or Q_i we get groups locally isomorphic to the two de Sitter groups $SO(4,1)$ and $SO(3,2)$, while L together with $P_i + Q_i$ or $P_i - Q_i$ yields groups locally isomorphic to the Poincaré group.

A special case of (13), where

$$A_\mu = \tfrac{1}{2} A_\mu^{ij} L_{ij} + A_\mu^i (aP_i + bQ_i) \tag{14}$$

with a and b real constants such that at least one is different from zero and the matrix $[A_\mu^i]$ is invertible, corresponds to a Riemann–Cartan geometry induced on the real submanifold of M. Notice that even though we assumed the existence of the pseudo-unitary affine coordinates z^i in M, the Minkowski coordinates on the real submanifold of M will not in general exist. The curvature and the torsion of the induced connection is calculated in the usual way from A_μ^{ij} and A_μ^i. Under the complex gauge transformation (2) limited to group L the functions A_μ^{ij} and A_μ^i transform respectively as the Lorentz components of the induced connection and the tetrads identifying the reference cross section in the bundle of frames of the real submanifold of M. It is only the complex tetrads of the full complex manifold M which transform directly via the action of L, i.e. like the four-component Dirac spinors.

The assumption about the existence of the pseudo-unitary affine coordinates is not necessary for the geometrical interpretation of (14). Even when the coordinates z^i do not exist, the connection with A_μ given by (14) generates a Riemann–Cartan geometry. The idea behind the assumption is as follows. If z^i exist, the set of cross sections in the bundle of frames connected by gauge transformations limited to L, i.e. the system that the classical observer uses to make geometrical measurements, contains only the information inscribed in the comparison of the cross section with the horizontal direction given by the connection. Only the induced geometry on the real submanifold of M can be deduced from this, while the complex tetrads remain hidden for the observer. This aspect is further discussed in the next section.

A few words should be added about the 'non-geometric' fields present in the general form (13) of the matrix A_μ. At least one of them has a clear physical interpretation. Field H_μ should correspond to the electromagnetic field potential. The corresponding generator is that of the $U(1)$ subgroup of $U(2,2)$, and its trivial relationship to the other generators (it commutes with everything) agrees with the main characteristics of the electromagnetic interactions. One can say that the suggested approach leads also naturally to the appearance of the electromagnetic field. However, a physical interpretation of F_μ combined with the 'dilation' generator D is not clear.

5. Dirac Fields and Anholonomic Frames

Consider a linear metric connection in a manifold. At this stage it is irrelevant whether the manifold is real or complex. The metricity of the connection implies that the horizontal lift can be defined within the bundle of orthonormal (or in the complex case unitary or pseudo-unitary) frames taken as a subbundle of the full bundle of linear frames. Thus the connection carries two distinct sets of information. One is about the connection within the bundle of orthonormal frames, while the other is about the imbedding of such a bundle into the full bundle of frames. The two sets of information differ in their mathematical description. The connection within the bundle of orthonormal frames is investigated by comparing a selected cross section with the horizontally transported frames. The difference is expressed in terms of the structure group of such bundle, i.e. the group that transforms one orthonormal frame into another. On the other hand, the mathematical description of the imbedding must go outside the structure group. A non-trivial imbedding means that orthonormal (or pseudo-unitary) coordinates do not exist in the base manifold and that the orthonormal frames are anholonomic. For holonomic frames the functions h^i_μ are equal to the partial derivatives $\partial z^i/\partial z^\mu$ of the pseudo-unitary coordinates with respect to the general coordinates, while in the case of anholonomic frames we have $\partial_\mu h^i_\nu - \partial_\nu h^i_\mu$ different from zero.

In the present approach the action of the structure group of the subbundle of pseudo-unitary frames is limited to gauge transformations within the subgroup L. These translate into the induced real transformations within the induced real geometry and their complex origin disappears. It is feasible that when the imbedding of pseudo-unitary frames into the full bundle of frames is trivial, nothing else but the induced real geometry may be observed. However, when the pseudo-unitary frames are anholonomic, they play a non-trivial role and the complex tetrads h^i_μ must be included in the description of the geometry. In the standard theory, this may (at least approximately) correspond to the addition of the Dirac field to the pseudo-Riemannian space–time. A precise definition of the Dirac field in terms of the tetrads as well as a geometrical significance of the Dirac equation still needs to be discovered. One should look at expressions like

$$\int_C h^i_\mu dx^\mu ,$$

where the integral is taken along a closed curve.

One more comment should be added to the above discussion. As the gauge transformations controlled by the classical observer are assumed to be limited to subgrup L, one may have anholonomic frames even when the base manifold stays as the affine metric space and the pseudo-unitary coordinates exist. The object $\partial_\mu h^i_\nu - \partial_\nu h^i_\mu$ which measures anholonomity does not transform covariantly under general gauge transformations. If the pseudo-unitary coordinates exist, it is possible to make the object equal to zero by some gauge tranformation but, in general, it may not be possible when the tranformations are limited to L. Thus, in principle, both the case of the empty space–time as well as that of the space–time plus the Dirac field could be handled within the bundle of frames of an affine metric space.

6. Conclusions

Whether we believe in a total geometrisation of physics or not is a matter of personal taste. Still, the idea of representing particles by regions of special geometrical significance within a unified geometrical theory has always been attractive for a large number of physicists. Spin $\frac{1}{2}$ particles needing complex vectors for their description present a challenge, and in the past the necessary geometrical quantities were always added to the existing space–time structure. In the standard approach one deals with the spinor valued Dirac fields, while the attempts to include the spinors within the fundamental continuum were again extending the space–time structure by adding spinor coordinates (Smrz 1968; Salam and Strathdee 1974). Yet, it seems possible to start with a strictly complex manifold where the presence of the complex vectors is natural, and derive the observed real space–time with its geometrical properties via investigation of cross sections in its bundle of linear frames. The reduction of the connection to a pseudo-unitary subgroup is closely related to the reduction of the number of observed dimensions, and the mathematical character of the gauge transformations changes the original complex group into its real adjoint representation. At the same time, the approach offers further possibilities. The regions with anholonomic frames may force the inclusion of Dirac fields and thus give a special geometric meaning to the regions occupied by particles. The existence of wider gauge transformations leads to possible dramatic changes in the geometrical structure of the observed space–time (including the change of the dimension) and could help to understand the strange features of the quantum behaviour of particles (Smrz 1995).

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