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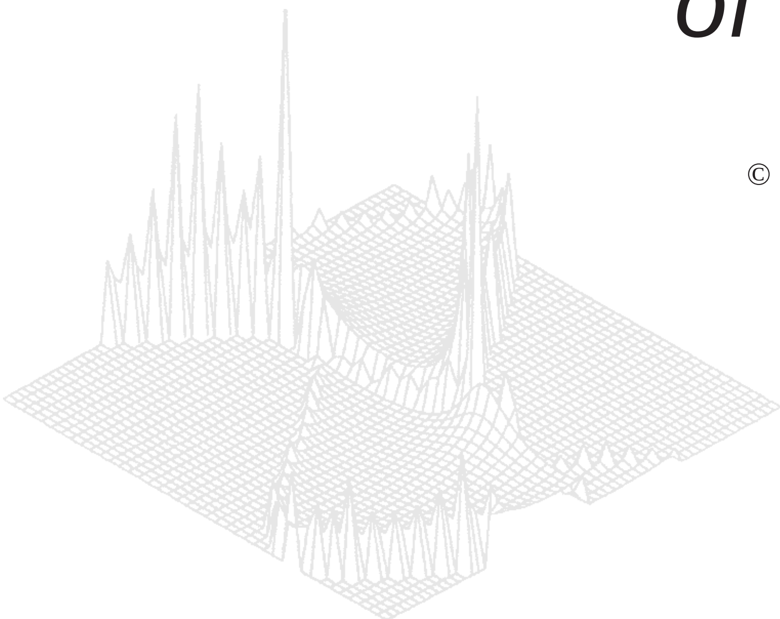

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Effects of the Dust Charge Fluctuation and Ion Temperature on Large Amplitude Ion-acoustic Waves in a Dusty Plasma

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Abstract

The effects of the dust charge fluctuation and ion temperature on large amplitude ion-acoustic waves are investigated in a plasma with a finite population of negatively charged dust particles, by numerical calculation. The nonlinear structures of ion-acoustic waves are examined, showing that the conditions for existence sensitively depend on the effects of the variable charge of dust grains and ion temperature, electrostatic potential and Mach number. The electrostatic potential on the surface of dust grain particles increases the dust charge number. The effect of the ion temperature increases the propagation speed of the ion-acoustic wave, and decreases the dust charge number. It is found that both compressive and rarefactive solitons can propagate in this system and the criterion for both solitons depends on the ion temperature. The region for existence of large amplitude ion-acoustic waves significantly depends on the dust charging. New findings of large amplitude ion-acoustic waves with variable charge dust grains and finite ion temperature in a dusty plasma are predicted.

1. Introduction

Dusty plasmas have been the subject of intensive investigation in astrophysics and plasma-aided engineering. In space, plasmas containing charged dust grains are ubiquitous (Grun *et al.* 1984; Goertz 1986; Hartquist *et al.* 1992), including asteroids, planetary rings, cometary tails, interstellar clouds, circumstellar and protoplanetary accretion disks, nova ejecta, and the Earth's space environment. Impurity-containing plasmas are also relevant to many laboratory and technological plasmas, such as low temperature rf and dc glow discharges, rf plasma etching and the wall region in fusion plasmas. The dust particles and negatively charged impurity ions in most plasmas of interest are of micron or submicron size, which is usually much less than the Debye length (Bingham *et al.* 1991; Tsytorich and Harnes 1993). They can have large mass (m_d is of the order of 10^6–10^{10} times the proton mass) and be negatively charged with large charge numbers (Z_d is of the order of 10^3–10^7 times the electron charge), where m_d is the dust mass. The grain charge can be due to field emission, ultraviolet irradiation, or microscopic plasma currents. Thus it is not surprising that astronomers and industrial researchers have investigated many physical processes (Goertz and Morfill 1983; Selwyn *et al.* 1990; Boufendi *et al.* 1993), especially including the problem of dust charging.

The presence of massively charged dust particles (impurity ions) in a plasma can drastically affect its dispersion and nonlinear properties. Since we look for the collective behaviour of the charged dust on the plasma, the dust can be

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described as one or more fluids. For the low frequency and long wavelength wave mode, the dust grains can be described as negative ions with large mass and large charge. Ion- and dust-acoustic wave modes in dusty plasmas have been treated by several authors (Rao *et al.* 1990; D'Angelo 1990; Nejoh 1997). Dust grain particles are usually negatively charged due to the flow of the plasma currents to their surface. The amount of charge accumulated on a dust particle is determined by its charging capacity as well as by the electron and the ion current balance. In a moderately hot plasma, this process dominates over other charging processes such as the photoelectric effect and secondary electron emission. In the presence of disturbances, a time-dependent quantity and, therefore, the grain charge perturbation should be included as a new variable in the dynamics of dusty plasmas. This feature constitutes essentially the qualitatively new aspect in the description of a dusty plasma as compared with that of a multi-fluid plasma. It has been known that high-speed particles have an influence on the excitation of various kinds of nonlinear waves in interplanetary space and the Earth's magnetosphere (Nejoh 1994, 1996*a*, 1996*b*; Nejoh and Sanuki 1994, 1995). On the other hand, in many laboratory and industrial plasmas, the ion-acoustic wave is more relevant than the dust-acoustic wave, since the latter involves a time scale which is much large than the time of the existence of the plasma. However, not many theoretical works on large amplitude ion-acoustic waves have been carried out for dusty (impurity-containing) plasmas. In particluar, the effect of the ion temperature on large amplitude ion-acoustic waves with dust charge fluctuations has not yet been investigated.

In this article, we make an attempt to theoretically investigate the possibility for existence of large amplitude ion-acoustic waves under the influence of dust particles in an unmagnetised plasma consisting of warm ions, hot and isothermal electrons and cold dust particles. We study the properties of nonlinear ion-acoustic waves accounting for the dust charge fluctuations in dusty plasmas. We also describe the dependence of the ion-acoustic waves on the ion temperature and demonstrate the region for existence of ion-acoustic waves and study the dependence of the region for existence on parameters such as the ion temperature, electrostatic potential, Mach number, and so on.

The layout of this article is as follows. In Section 2, we present the basic equations for a dusty plasma and derive a nonlinear differential equation for electrostatic nonlinear waves. Analysing the nonlinear structure of the pseudopotential, we show that the nonlinear wave is supersonic, and define the conditions for existence of ion-acoustic waves. A new nonlinear equation for the dust charge fluctuation with the ion temperature is presented for the first time. In Section 3, we show numerical results of the dust charging effect, and illustrate the dependence of the region for existence on parameters such as the ion temperature and Mach number. It turns out that the large amplitude compressive and rarefactive nonlinear waves can propagate. For such low frequency waves the variable charge on the dust grains is of crucial importance in considering the properties of the nonlinear electrostatic ion-acoustic waves. The dust charging will change the properties of these waves and dominate the wave characteristics. We investigate this in Section 3. In the small amplitude limit, we demonstrate that both compressive and rarefactive solitons can propagate and the criteria for both solitons exist under proper conditions. The last section is devoted to a concluding discussion.

2. A Nonlinear Variable Charge Equation for Dust Grains

We assume the existence of an unmagnetised plasma consisting of warm ions, hot and isothermal electrons and cold dust (impurity) particles, and consider one-dimensional propagation. We adopt the fluid equations for the ions and dust particles. The electron density follows the Boltzmann distribution

$$n_e = \exp(\phi)\,. \tag{1}$$

The continuity equation and the equation of motion for ions are described by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i\, v_i) = 0\,, \tag{2a}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} + \frac{3\sigma}{(1+aZ_d)^2}\, n_i\, \frac{\partial n_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0\,, \tag{2b}$$

where we express the pressure term in equation (2b) by the adiabatic thermodynamic equation of state (see the Appendix). Here $\sigma = T_i/T_e$ and $a = n_{d0}/n_0$, where T_i (T_e), n_0, Z_d are the ion (electron) temperature, the unperturbed background electron density and charge numbers of the dust particles, respectively.

We have the following two equations for the cold dust particles:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d\, v_d) = 0\,, \tag{3a}$$

$$\left(\frac{\partial}{\partial t} + v_d\frac{\partial}{\partial x}\right)v_d - \frac{Z_d}{\mu_d}\frac{\partial \phi}{\partial x} = 0\,, \tag{3b}$$

where the dust charge variable is determined to $Q_d = -eZ_d$, where e and Z_d are the magnitude of the electron charge and the dust charge number respectively.

Poisson's equation is given as

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + Z_d\, n_d\,. \tag{4}$$

In equation (3b) we have $\mu_d = m_d/m_i$ where $m_d\,(m_i)$ denotes the dust grain (ion) mass. The variables n_c, n_d, n_i, v_d, v and ϕ refer to the electron density, dust grain density, ion density, dust velocity, ion velocity and electrostatic potential respectively. The densities are normalised by the background electron density n_0. The space coordinate x, time t, velocities and electrostatic potential ϕ are normalised by the electron Debye length $\lambda_d = (\epsilon_0\,\kappa T_e/n_0\, e^2)^{1/2}$, the inverse ion plasma period $\omega_i^{-1} = (\epsilon_0\, m_i/n_0\, e^2)^{1/2}$, the ion sound velocity $C_s = (\kappa T_e/m_i)^{1/2}$, and $\kappa T_e/e$, respectively, where m_i and ϵ_0 are the ion mass and the permittivity of the vacuum respectively. We assume that the phase velocity of the ion-acoustic wave is low in comparison with the electron thermal velocity. Charge neutrality at equilibrium requires that $n_{i0} = n_0 + n_{d0}\, Z_d$, where n_{i0} (n_{d0}) denote the equilibrium ion (dust particle) density.

We assume that the charging of the dust grain particles arises from plasma currents due to the electrons and the ions reaching the grain surface. In this case, the dust grain charge variable Q_d is determined by the charge current balance equation (Melandso *et al.* 1993):

$$\left(\frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x}\right) Q_d = I_e + I_i \, . \tag{5}$$

Assuming that the streaming velocities of the electrons and ions are much smaller than their thermal velocities, we have the following expressions for the electron and ion currents for spherical grains of radius r:

$$I_e = \, - e\pi r^2 \sqrt{\frac{8T_e}{\pi m_e}} \, n_e \exp\left(\frac{e\varphi}{T_e}\right) , \tag{6}$$

$$I_i = e\pi r^2 \sqrt{\frac{8T_i}{\pi m_i}} \, n_i \left(1 - \frac{e\varphi}{T_i}\right) , \tag{7}$$

where $\varphi \equiv Q_d/r$ denotes the dust particle surface (floating) potential relative to the plasma electrostatic potential ϕ (Barnes *et al.* 1992; Varma *et al.* 1993). If the ion streaming velocity v_0 is much larger than the ion thermal velocity, the ion current is approximately expressed as $I_i \approx e\pi r^2 v_0 \, n_i (1 - 2e\varphi/m_i \, v_0^2)$. At equilibrium, equating $I_e + I_i$ to zero we obtain the stationary floating potential φ_0 and the equilibrium dust charge $Q_0 = C\varphi_0$, where C denotes the dust grain capacitance.

In the linear approximation and in the absence of charge perturbations, equations (1)–(4) give rise to the dispersion relation of the low frequency ion-acoustic waves in a dusty plasma with the ion temperature. We assume all the perturbations to be of the form $\exp[i(kx - \omega t)]$ and obtain the linear dispersion relation by carrying out the usual normal mode analysis. We obtain this as

$$\omega^2 = \left[1 + aZ_d\left(1 + \frac{Z_d}{\mu_d}\right) + 3\sigma\right] \frac{k^2}{1 + k^2} \, , \tag{8}$$

where ω and k are the frequency and the wave number respectively. The dispersion relation shows that the waves are of acoustic nature, whose phase velocity in the long wavelength limit is

$$\frac{\omega}{k} = \sqrt{1 + aZ_d\left(1 + \frac{Z_d}{\mu_d}\right) + 3\sigma} \, .$$

In order to solve equations (1)–(7), we introduce the variable $\xi = x - Mt$, which is the moving frame with velocity M. Then (2a)–(3b) and (4) become

$$-M\,\frac{\partial n_i}{\partial \xi} + \frac{\partial}{\partial \xi}(n_i\,v_i) = 0\,, \tag{9a}$$

$$-M\,\frac{\partial v_i}{\partial \xi} + v_i\,\frac{\partial v_i}{\partial \xi} + \frac{3\sigma}{(1+aZ_d)^2}\,n_i\,\frac{\partial n_i}{\partial \xi} + \frac{\partial \phi}{\partial \xi} = 0\,, \tag{9b}$$

$$-M\,\frac{\partial n_d}{\partial \xi} + \frac{\partial}{\partial \xi}(n_d\,v_d) = 0\,, \tag{10a}$$

$$-M\,\frac{\partial v_d}{\partial \xi} + v_d\frac{\partial v_d}{\partial \xi} - \frac{Z_d}{\mu_d}\frac{\partial \phi}{\partial \xi} = 0\,, \tag{10b}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_e - n + Z_d\,n_d\,. \tag{11}$$

Integrating (9a) and (9b) and using the boundary conditions $\phi \to 0$, $n_d \to a$, $n_i \to 1{+}aZ_d$, $v_i \to 0$, $v_d \to 0$ at $\xi \to \infty$, we obtain the ion density

$$n_i = 1 + aZ_d \Big/ \sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}}\,. \tag{12}$$

Integration of (10a) and (10b) gives rise to the dust density

$$n_d = a \Big/ \sqrt{1 + \frac{Z_d}{\mu_d}\frac{2\phi}{M^2}}\,, \tag{13}$$

after a brief calculation.

Using (1), (12) and (13), equation (11) reduces to the nonlinear Poisson equation

$$\begin{aligned}\frac{\partial^2 \phi}{\partial \xi^2} &= \exp(\phi) - 1 + aZ_d \Big/ \sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}} + aZ_d \Big/ \sqrt{1 + \frac{Z_d}{\mu_d}\frac{2\phi}{M^2}} \\ &\equiv -\,\frac{\partial V(\phi)}{\partial \phi}\,,\end{aligned} \tag{14}$$

where $V(\phi)$ denotes the pseudopotential. Integration of (14) gives the *Energy Law*,

$$\tfrac{1}{2}\left(\frac{\partial \phi}{\partial \xi}\right)^2 + V(\phi) = 0\,. \tag{15}$$

From (14), the pseudopotential becomes

$$-V(\phi) = \exp(\phi) - 1 + (1 + aZ_d)(M^2 - 3\sigma)\left\{\sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}} - 1\right\}$$
$$+ a\mu_d M^2\left\{\sqrt{1 + \frac{Z_d}{\mu_d}\frac{2\phi}{M^2}} - 1\right\}. \tag{16}$$

The oscillatory solution of the nonlinear ion-acoustic waves exists when the following two conditions are satisfied:

(i) The pseudopotential $V(\phi)$ has the maximum value if $d^2V(\phi)/d\phi^2 < 0$ at $\phi = 0$. This condition gives rise to

$$\frac{1 + aZ_d}{M^2 - 3\sigma} + \frac{Z_d}{\mu_d}\frac{aZ_d}{M^2} - 1 < 0. \tag{17}$$

It should be noted that $V(\phi)$ is real when

$$-\frac{\mu_d M^2}{2Z_d} < \phi < \frac{M^2 - 3\sigma}{2}.$$

The region for existence of ϕ depends on Z_d, μ_d, σ and M. Equation (17) implies that supersonic ion-acoustic waves can propagate in this system.

(ii) Nonlinear ion-acoustic waves exist only when $V(\phi_M) \geq 0$, where the maximum potential ϕ_M is determined by $\phi_M = (M^2 - 3\sigma)/2$. This implies that the following inequality holds:

$$1 + (1 + aZ_d)(M^2 - 3\sigma) \geq \exp\left(\frac{M^2 - 3\sigma}{2}\right) + a\mu_d M^2\left\{\sqrt{1 + \frac{Z_d}{\mu_d}\left(1 - \frac{3\sigma}{M^2}\right)} - 1\right\}. \tag{18}$$

We show the maximum Mach number as a function of the ion temperature σ in Fig. 1, where $a = 10^{-3}$, $\mu_d = 10^6$ and $Z_d = 10^3$. The lower and upper curves correspond to equations (17) and (18) respectively. The allowable Mach number lies in the region bounded by the two curves. The maximum Mach number and, correspondingly, the maximum amplitude of the ion-acoustic wave significantly depends on the parameter σ.

Next, we investigate dust charge fluctuations as a new effect. At equilibrium, equation (5) reduces to

$$e\pi r^2\sqrt{\frac{8T_e}{\pi m_e}}\left[-n_e(\phi)\exp\left(\frac{e\varphi}{T_e}\right) + \sqrt{\frac{\sigma}{\mu_i}}\,n_i(Z_d, \phi, \sigma)\left(1 - \frac{e\varphi}{T_i}\right)\right] = 0. \tag{19}$$

Since $\varphi = Q_d/r$, (19) is a strongly nonlinear equation. In order to solve (19) exactly, we normalise it and derive a nonlinear equation for the dust grain surface potential φ as

$$\phi + \varphi = -\tfrac{1}{2}\ln(\mu_i\,\sigma)\left(1 - \frac{2\phi}{M^2 - 3\sigma}\right) + \ln(1 + \alpha\varphi)(\sigma - \varphi), \tag{20}$$

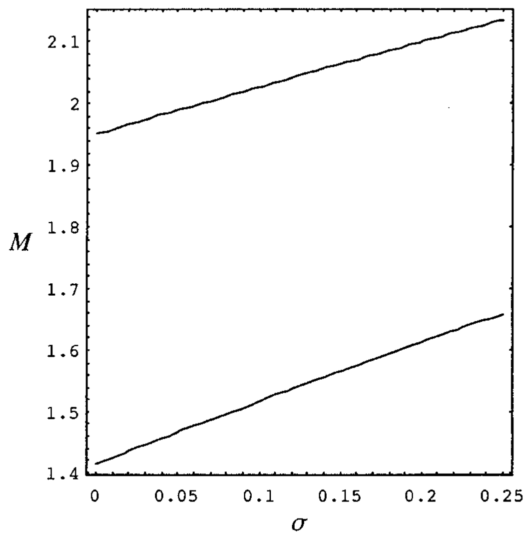

Fig. 1. Mach number M depending on the ion temperature σ, in the case of $a = 10^{-3}$, $\mu_d = 10^6$ and $Z_d = 10^3$. The lower and upper curves denote the profile of equations (13) and (14) respectively. The allowable Mach number lies in the region bounded by the two curves.

where $\alpha = ar/e$ and $\mu_i = 1836$. We regard (20) as a new equation for the dust charge fluctuation. Then, we can obtain the solution of (20) by numerical calculation in the next section.

3. Dust Charging Effect and the Nonlinear Structure of Large Amplitude Ion-acoustic Waves

(3a) Dependence of Large Amplitude Ion-acoustic Waves on the Dust Charge Fluctuation

We consider the nonlinear structures of large amplitude ion-acoustic waves in the case where the dust charging effect and ion temperature are important. We carry out a numerical analysis of equations obtained in the preceding section. For illustration purposes, we consider a dusty plasma in which most of the background electrons are collected by the negatively-charged dust grains. Such a situation, for example, is common to the F-ring of Saturn. Thus, without loss of generality, we investigate the effect of the dust charge fluctuation and calculate typical forms of the pseudopotential and the associated ion-acoustic wave structure.

First, in order to investigate the relationship between the dust grain surface potential Ψ and the electrostatic potential ϕ, we show the dependence of the dust charging effect on the electrostatic potential by numerical calculation. We show a φ–ϕ plane in Fig. 2, where $a = 10^{-3}$, $\mu_i = 1836$, $\mu_d = 10^6$, $M = 1{\cdot}5$ and $\sigma = 0{\cdot}1$. We understand that the dust grain surface potential increases as the negative electrostatic potential approaches zero, has a maximum at the specific positive potential and decreases as the small positive potential increases. We also illustrate a φ–ϕ plane in the vicinity of $\varphi = 0$ in Fig. 3. Fig. 4 illustrates

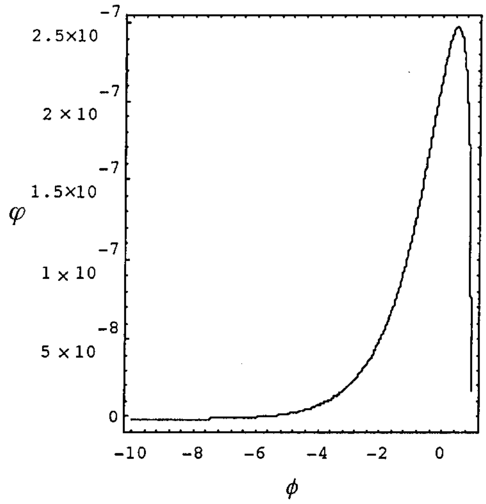

Fig. 2. Dependence of the dust grain surface potential on the electrostatic potential, where $a = 10^{-3}$, $\mu_i = 1836$, $\mu_d = 10^6$, $M = 1{\cdot}5$ and $\sigma = 0{\cdot}1$.

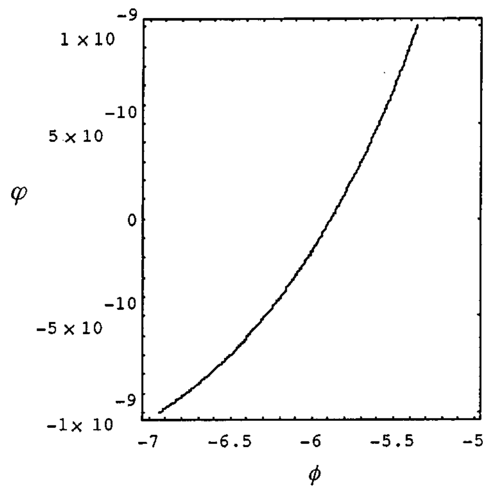

Fig. 3. A φ–ϕ plane in the vicinity of $\varphi = 0$, where $a = 10^{-3}$, $\mu_i = 1836$, $\mu_d = 10^6$, $M = 1{\cdot}5$ and $\sigma = 0{\cdot}1$.

the dependence of the dust grain potential on the effect of the ion temperature σ, where $a = 10^{-3}$, $\mu_i = 1836$, $\mu_d = 10^6$, $M = 1{\cdot}5$ and $\phi = -3{\cdot}0$. It turns out that the dust charge numbers decrease as the ion temperature increases. We note that the dust charge number Z_d is obtained by multiplying φ by $6{\cdot}25\times 10^{12}$, in the case of $r = 10^{-6}$ m.

Secondly, we study the process of the formation of the potential well by varying parameters such as the charge number, ion temperature and Mach number, using equation (16). As a result of this, it turns out that the formation of the well

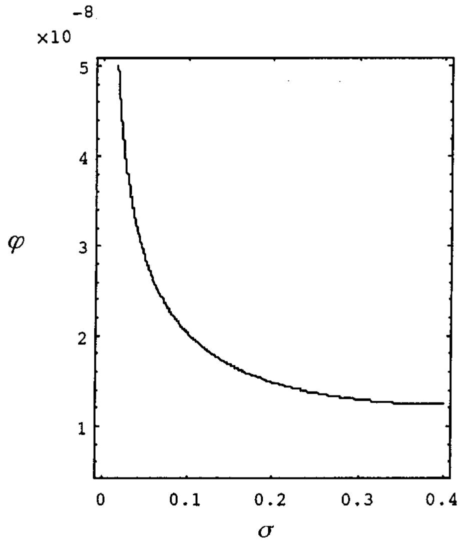

Fig. 4. Dependence of the dust grain surface potential on the ion temperature, where $a = 10^{-3}$, $\mu_i = 1836$, $\mu_d = 10^6$, $M = 1{\cdot}5$ and $\phi = -3{\cdot}0$.

sensitively depends on Z_d and σ, and that there are no nonlinear waves under these particular conditions. We show the region for existence of solitons and illustrate the dependence of the dust charge Z_d on the ion temperature σ and on the propagation speed in Figs 5 and 6 respectively. Fig. 5 shows the dust charge Z_d versus the ion temperature σ $(=T_i/T_e)$, for $a = 10^{-3}$, $\mu_d = 10^6$ and $M = 1{\cdot}6$. In Fig. 5, the compressive and rarefactive ion-acoustic solitons propagate in the domain A, no solitons exist in the domain B and only rarefactive solitons propagate in the restricted domain C. It turns out that the properties of the large amplitude nonlinear ion-acoustic waves change due to the dust charge. Fig. 6 illustrates the region for existence of ion-acoustic waves when the Mach number increases, that is $a = 10^{-3}$, $\mu_d = 10^6$ and $M = 1{\cdot}8$. When the velocity of the wave increases, the four domains newly appear due to the dust charge. Those are the domains where only rarefactive solitons exist in the domain A, where compressive and rarefactive solitons propagate in the domain B, where no solitons exist in the domain C, and where only compressive solitons propagate in the restricted domain D. We find that Figs 5 and 6 show the criteria for the formation of the large amplitude compressive and rarefactive solitary waves. Thus the dust charging effect is of crucial importance in the sense that the properties of the large amplitude ion-acoustic wave drastically change due to the dust charge.

Thirdly, we show a bird's eye view of the pseudopotential $V(\phi)$ in Fig. 7, in the case of $M = 1{\cdot}6$, $a = 10^{-3}$, $\mu_d = 10^6$ and $Z_d = 10^3$, by numerical calculation. Fig. 8 illustrates the dependence of $V(\phi)$ on the electrostatic potential ϕ when the ion temperature is fixed to be $\sigma = 0{\cdot}05$ in the case of Fig. 7. When the ion temperature increases, we show a bird's eye view for $M = 1{\cdot}8$, $a = 10^{-3}$, $\mu_d = 10^6$ and $Z_d = 10^3$ in Fig. 9. The pseudopotential $V(\phi)$ versus ϕ in this case is also illustrated in Fig. 10 for $\sigma = 0{\cdot}05$. In the case of Fig. 10, we can

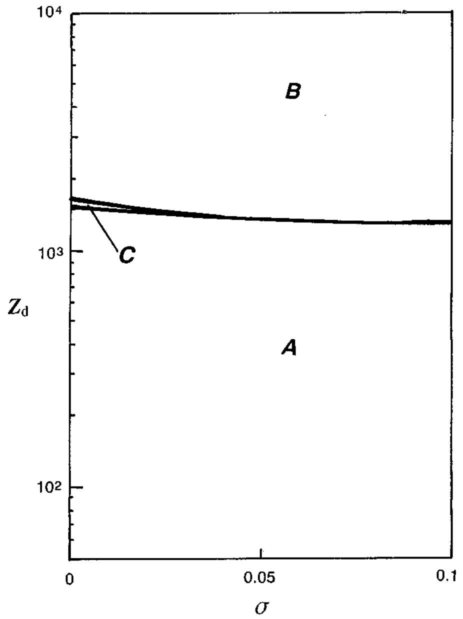

Fig. 5. Dependence of the domain of existence of ion-acoustic waves on the dust charge Z_d and the ion temperature σ, where $a = 10^{-3}$, $\mu_d = 10^6$ and $M = 1{\cdot}6$. The compressive and rarefactive solitons propagate in the domain A, no solitons exist in the domain B, and only rarefactive solitons propagate in the domain C.

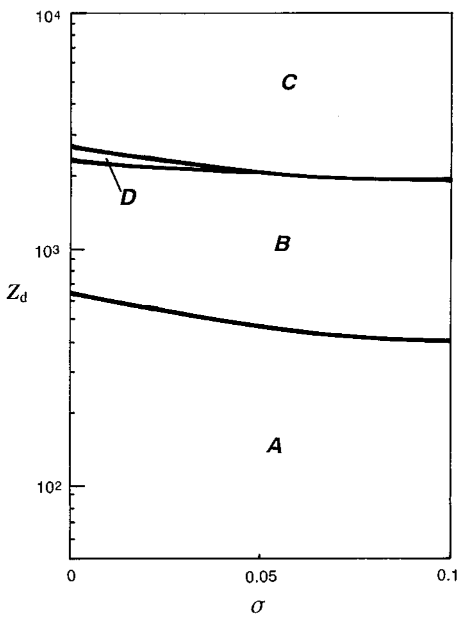

Fig. 6. Dependence of the domain of the existence of ion-acoustic waves on the dust charge Z_d and the ion temperature σ when the Mach number increases, where $a = 10^{-3}$, $\mu_d = 10^6$ and $M = 1{\cdot}8$. Only rarefactive solitons propagate in the domain A, compressive and rarefactive solitons exist in the domain B, no solitons propagate in the domain C, and only compressive solitons exist in the domain D.

show the formation of the large amplitude compressive and rarefactive solitons, by numerical calculation of the nonlinear differential equation (15) with (16). Clearly, Figs 11 and 12 represent the large amplitude compressive and rarefactive solitons respectively. We show the maximum amplitude of the compressive and rarefactive ion-acoustic waves in Figs 13 and 14 respectively, where a corresponds to $M = 1{\cdot}6$, and b corresponds to $M = 1{\cdot}8$. Here ϕ_M and ϕ_m denote the maximum amplitude of the compressive and rarefactive ion-acoustic wave, respectively.

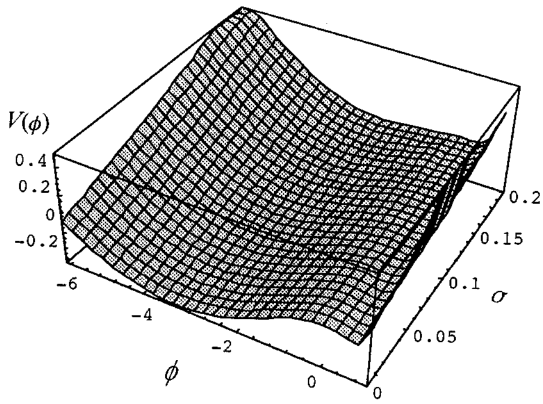

Fig. 7. Bird's eye view of the pseudopotential under the conditions of $a = 10^{-3}$, $\mu_d = 10^6$, $Z_d = 10^3$ and $M = 1{\cdot}6$.

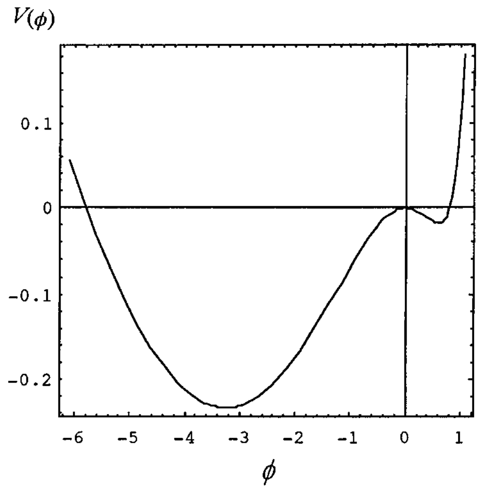

Fig. 8. A pseudopotential curve of ion-acoustic waves for $a = 10^{-3}$, $\mu_d = 10^6$, $Z = 10^3$, $M = 1{\cdot}6$ and $\sigma = 0{\cdot}05$.

From Figs 1–14, we understand the following:

(1) Supersonic ion-acoustic waves propagate in this system. In Fig. 1, the allowable Mach number shifts to the higher value as the ion temperature increases.

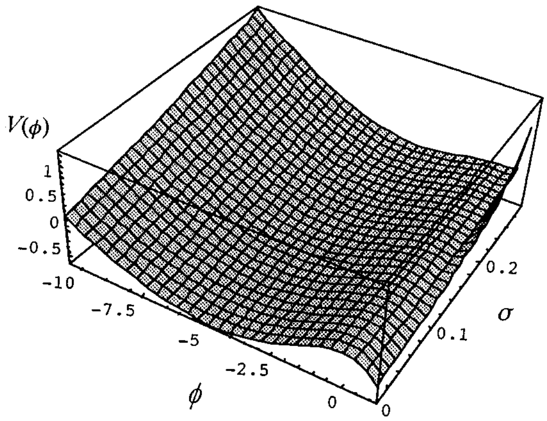

Fig. 9. Bird's eye view of the pseudopotential under the conditions of $a = 10^{-3}$, $\mu_d = 10^6$, $Z_d = 10^3$ and $M = 1{\cdot}8$.

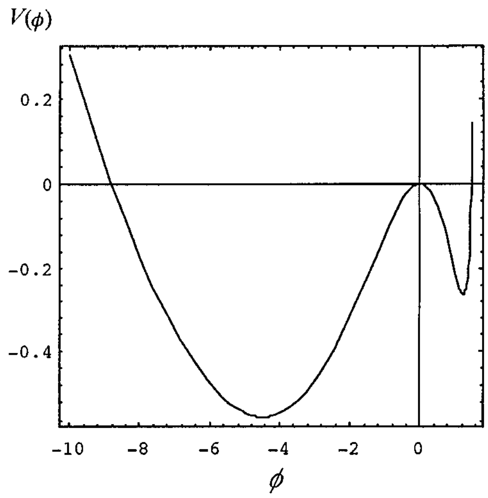

Fig. 10. A pseudopotential curve of ion-acoustic waves for $a = 10^{-3}$, $\mu_d = 10^6$, $Z_d = 10^3$, $M = 1{\cdot}8$ and $\sigma = 0{\cdot}05$.

(2) The variable charge of dust grains drastically changes due to the flow of the plasma potential to the surface of dust grain particles. When the ion temperature increases the dust charge quickly decreases. The dust grain surface potential sensitively depends on the negative and positive electrostatic potential and the Mach number. The dust charge number gradually increases as the negative electrostatic potential approaches zero, has a maximum at the specific positive potential and finally decreases as the small positive potential increases. The speed of the ion-acoustic wave increases as the dust charge increases.

(3) For low frequency ion-acoustic waves, the dust charging effect is of crucial importance in the sense that the region for existence of large amplitude

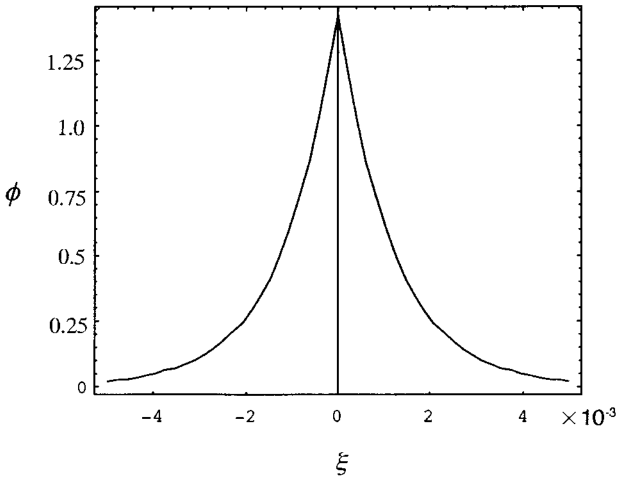

Fig. 11. A large amplitude compressive soliton corresponding to the positive potential of the pseudopotential seen in Fig. 10.

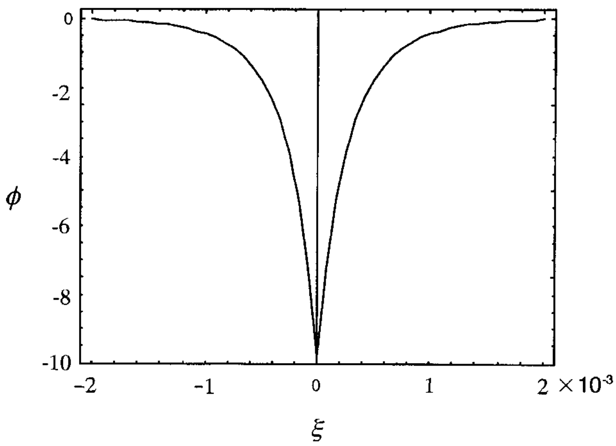

Fig. 12. A large amplitude rarefactive soliton corresponding to the negative potential of the pseudopotential seen in Fig. 10.

nonlinear ion-acoustic waves sensitively depends on the dust charge, as is seen in Figs 5 and 6.

(4) In Figs 7–10, when the ion temperature increases, the potential well becomes narrower. The potential well becomes wider as the Mach number increases.

(5) The profile of the pseudopotential sensitively depends on the Mach number, the parameters a, Z_d and μ_d. As is seen in Figs 11 and 12, large amplitude compressive and rarefactive solitons can propagate in a dusty plasma considered here.

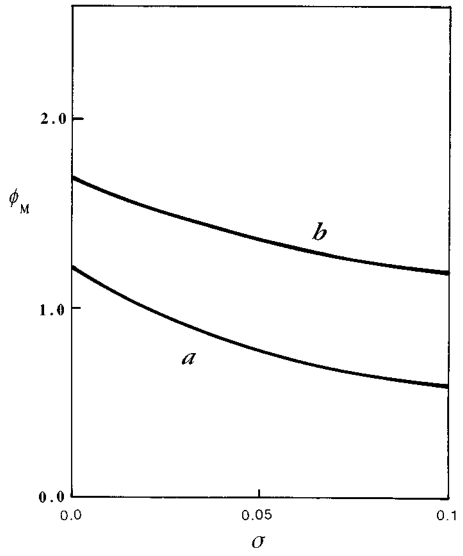

Fig. 13. Maximum amplitude ϕ_M of the compressive ion-acoustic solitary wave depending on the ion temperature σ, in the case of (a) $M = 1{\cdot}6$ and (b) $M = 1{\cdot}8$.

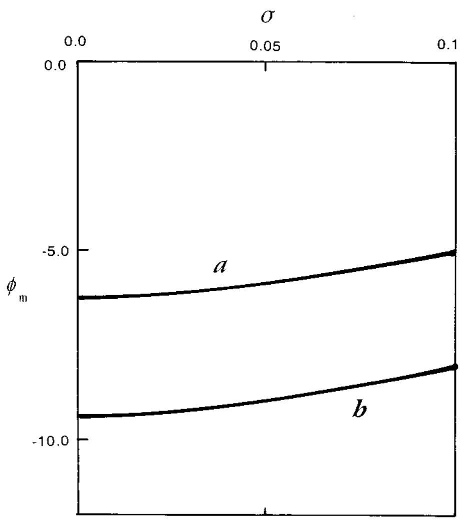

Fig. 14. Maximum amplitude ϕ_m of the rarefactive ion-acoustic solitary wave depending on the ion temperature σ, in the case of (a) $M = 1{\cdot}6$ and (b) $M = 1{\cdot}8$.

(6) In Figs 13 and 14, the maximum amplitude ϕ_M of the compressive ion-acoustic wave decreases as the ion temperature σ increases. The maximum amplitude increases as the Mach number increases. The maximum amplitude ϕ_m of the rarefactive ion-acoustic wave behaves in the same manner as that of the compressive one, whereas the amplitude of the rarefactive ion-acoustic wave is much larger than that of the compressive one.

Large amplitude nonlinear ion-acoustic waves can exist under the proper conditions mentioned above.

(3b) Small Amplitude Limit

In order to confirm the possibility of an ion-acoustic soliton in a dusty plasma, we study the pseudopotential in the small amplitude limit ($\phi < 1$). The specific results can be obtained by expanding $V(\phi)$ in powers of ϕ and keeping up to terms in ϕ^3. Accordingly, (16) takes the form

$$V(\phi) \approx \tfrac{1}{2}\left[-1 + \frac{1+aZ_d}{M^2-3\sigma} + \left(\frac{Z_d}{\mu_d}\right)^2 \frac{a\mu_d}{M^2}\right]\phi^2 + \tfrac{1}{6}\left[-1 + 3\frac{1+aZ_d}{(M^2-3\sigma)^2} - 3\left(\frac{Z_d}{\mu_d}\right)^3 \frac{a\mu_d}{M^4}\right]\phi^3 \,. \tag{21}$$

Then, integrating (15) with (19), we obtain a soliton solution

$$\phi = 3\frac{-1 + \dfrac{1+aZ_d}{M^2-3\sigma} + \left(\dfrac{Z_d}{\mu_d}\right)^2 \dfrac{a\mu_d}{M^2}}{-1 + \dfrac{1+aZ_d}{(M^2-3\sigma)^2} - 3\left(\dfrac{Z_d}{\mu_d}\right)^3 \dfrac{a\mu_d}{M^4}} \times \mathrm{sech}^2\left\{\tfrac{1}{2}\sqrt{-1 + \frac{1+aZ_d}{M^2-3\sigma} + \left(\frac{Z_d}{\mu_d}\right)^2 \frac{a\mu_d}{M^2}}(\xi-\xi_0)\right\} . \tag{22}$$

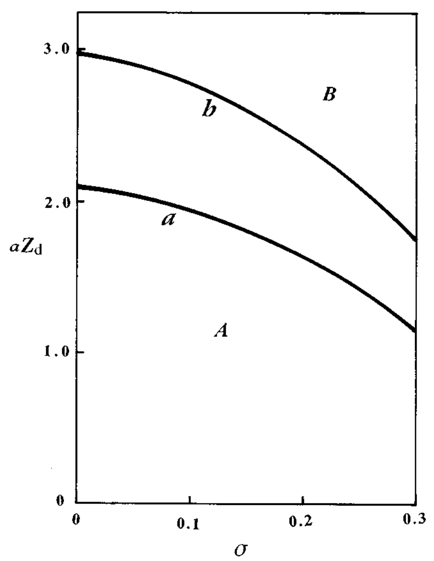

Fig. 15. Profile of the criterion of the small amplitude compressive and rarefactive ion-acoustic solitons. Shown is the dependence of the critical number of aZ_d on the ion temperature σ in the case of (a) $M = 1{\cdot}8$ and (b) $M = 2{\cdot}0$. The compressive soliton can propagate in the lower region A and the rarefactive soliton exists in the upper region B.

It turns out that the ion-acoustic soliton exists in the small amplitude limit. We note that the solutions have a wave structure with positive electrostatic potential (compressive soliton) and negative electrostatic potential (rarefactive soliton). The profile of the soliton is characterised by the signs of the coefficient of the sech^2 function. The criterion of the sign of (22) shows that the soliton is compressive when the amplitude of (22) is positive, and is rarefactive when the amplitude is negative. Since the coefficient of (22) depends on the ion temperature, we must seek the value of aZ_d where the coefficient undergoes a change in sign. This procedure determines the critical number of aZ_d depending on the effect of the ion temperature. For this purpose, we illustrate the critical number of aZ_d as a function of the ion temperature σ in Fig. 15. The curves a and b correspond to the Mach number $M = 1\cdot 8$ and $2\cdot 0$ respectively. As is seen in Fig. 15, we understand that the compressive soliton lies in the lower region (region A) bounded by the curve and the rarefactive soliton in the upper region (region B) bounded by the curve. It turns out that the critical number of aZ_d decreases as the ion temperature increases. It is also shown that only rarefactive solitons are possible in the case of $aZ_d > 2\cdot 1$ when the Mach number is $M = 1\cdot 8$, and $aZ_d > 3\cdot 0$ when $M = 2\cdot 0$.

4. Concluding Discussion

We found a nonlinear equation for the dust charging effect and studied large amplitude ion-acoustic waves in a dusty plasma with the ion temperature. We investigated the dependence of the pseudopotential on the several parameters and presented the region for existence of ion-acoustic waves on the basis of the fluid equations for the plasma. We showed the dependence of the amplitude of compressive and rarefactive supersonic ion-acoustic waves on the ion temperature by analysing the nonlinear structure of the pseudopotential.

We also presented the possible existence of large amplitude ion-acoustic waves in a dusty plasma whose constituents are Boltzmann distributed electrons, ions with the finite ion temperature and a cold dust fluid consisting of negatively-charged, micrometre-sized dust particles. Such plasmas may exist in both space environments and the laboratory. Typical results are illustrated in Figs 1–15. It is worth noting the following:

(1) The dust charge fluctuation drastically changes due to the plasma potential to the surface of dust grain particles. It turns out that the dust charge numbers rapidly decrease as the ion temperature increases. The dust grain surface potential sensitively depends on the negative and positive electrostatic potential and the Mach number. The dust charge number gradually increases as the negative electrostatic potential approaches zero, has a maximum at the specific positive potential and finally decreases as the small positive potential increases.

(2) Only supersonic ion-acoustic waves can propagate in this system. The effect of the ion temperature increases the propagation speed of the ion-acoustic wave.

(3) The existence of ion-acoustic waves depends sensitively on the ion temperature, Mach number, the ratio of the dust grain density to the background electron density, the ratio of the dust mass to the ion mass and charge numbers of the dust grains.

(4) Large amplitude compressive and rarefactive ion-acoustic solitons can propagate in this system and the region for existence of both solitons depends on the ion temperature and Mach number. The amplitude of both solitons decreases as the ion temperature increases, and increases as the Mach number increases.
(5) It turns out that the dust charging effect is of crucial importance for low frequency ion-acoustic waves in the sense that the region for existence of the nonlinear ion-acoustic waves sensitively depends on the dust charge.
(6) In the small amplitude limit, both compressive and rarefactive solitons can propagate and the criterion for both solitons depends on the ion temperature. It is also found that only rarefactive solitons exist under proper conditions.

The present investigation predicts new findings on the effects of the dust grain charging and finite ion temperature on large amplitude ion-acoustic waves in a dusty plasma. Hence, referring to the present study, we can understand the nonlinear properties of large amplitude ion-acoustic waves in plasmas where dust grains (impurity particles) exist. Although we have not referred to any specific observations, the present theory is applicable to analysing large amplitude ion-acoustic waves, such as shock and solitary waves, which may occur in space plasmas where micrometre-sized charged dust grains are present. It can also be useful in the diagnostics of laboratory plasmas.

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Appendix: Derivation of the Third Term in Equation (2b)

We consider the equation of motion with the ion pressure gradient force under the assumption of adiabatic ions and one-dimensional motion as

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)v_i+\frac{1}{n_i}\frac{\partial p}{\partial x}+\frac{\partial\phi}{\partial x}=0\,. \tag{23}$$

In order to close the system of equations, we use the thermodynamic equation of state relating the pressure p to the density n_i:

$$p=Cn_i^{\gamma}\,, \tag{24}$$

where C is a constant and $\gamma=C_p/C_v$ is the specific heat ratio. Differentiating (24) once and using $\gamma=3$ when the degrees of freedom equal one, we obtain

$$\frac{1}{n_i}\frac{\partial p}{\partial x}=3Cn_i\frac{\partial n_i}{\partial x}\,. \tag{25}$$

Here, the coefficient C becomes $\sigma/(1+aZ_d)^2$ by considering the normalisation and the boundary conditions. Equation (25) is the third term of equation (2b).

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