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A Nonlocal Potential Form for s-wave $\alpha - \alpha$ Scattering

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Abstract

Low energy s-wave $\alpha - \alpha$ phase shifts that agree well with the measured set have been extracted using a nonlocal interaction formed by folding (local real energy-dependent) nucleon- α -particle interactions with density matrix elements of the (projectile) α -particle. The resultant s-wave $\alpha - \alpha$ interaction is energy dependent and nonlocal.

1. Introduction

Analyses of low energy $\alpha-\alpha$ scattering in the past have been based upon local equivalent potentials found from folding model calculations such as those of the resonating group method (RGM) (Tanabe *et al.* 1975) or upon phenomenologically chosen interactions between the two α -particles, such as the local Gaussian function form of Buck *et al.* (1977) or of Woods–Saxon (WS) type as used by Marquez (1983). The phenomenological (real local) interactions are very similar in shape (Friedrich 1984) and they are angular momentum and energy independent. As noted by Friedrich (1984), in nuclear physics, interactions which give good fits not only to the scattering phase shifts but also to the bound state properties are rare. However, and as also noted by Friedrich (1984), there must be nonlocality in a proper description of the $\alpha-\alpha$ interaction and that nonlocality should have a range of about 2 fm.

Nonlocality of the α - α interaction is assured by RGM or similar calculations and as such they are based upon a specific form for the two-nucleon (NN)interaction, allow for explicit consideration of the internal structures of the colliding pair, and take care of antisymmetrisation between all nucleon pairs. But in most calculations of nucleus-nucleus scattering, local equivalent forms of the folded interactions are required. Nonlocality has been considered in few cases, and usually as a Frahn-Lemmer form (Perey and Buck 1962). In a first order expansion, the Frahn-Lemmer nonlocal interaction has a simple but energy dependent local equivalent form (Apagyi *et al.* 1990).

We have calculated nonlocal interactions to describe $\alpha - \alpha$ low energy (s-wave) scattering, seeking first to identify constraints in that specification needed to fit observed scattering data and second, if the first aim is successful, to assess if the utilitarian Frahn–Lemmer form of nonlocality can be a good approximant.

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2. Outline of the Calculations

In a representation with \mathbf{r}, \mathbf{r}' denoting relative coordinates between a colliding pair of particles, the Schrödinger equation describing their scattering by a nonlocal potential takes the form

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 - V_C(r) + E\right]\Psi(\mathbf{r}) = \int U(\mathbf{r}, \mathbf{r}') \,\Psi(\mathbf{r}') \,d\mathbf{r}'.$$
(1)

This reduces by using the partial wave expansion

$$\Psi(\mathbf{r}) = \sum_{lm} \frac{u_l(r)}{r} i^l Y_{lm}(\Omega_r) , \qquad (2)$$

and

$$U(\mathbf{r}, \mathbf{r}') = \sum_{LM} \frac{W_L(r, r')}{rr'} i^L Y_{LM}(\Omega_r) i^{-L} Y^*_{LM}(\Omega_{r'}), \qquad (3)$$

to a set of differentio-integral equations

$$\left[\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) - V_C(r) + E\right] u_L(r) = \int_0^\infty W_L(r,r') u_L(r') dr'.$$
(4)

Here $V_C(r)$ is the (local) Coulomb potential between the colliding nuclei and $W_L(r,r')$ are multipoles of the nonlocal interaction. In this study, we seek solutions for $\alpha - \alpha$ scattering and determine the nonlocal interactions, $U_{\alpha\alpha}(\mathbf{r},\mathbf{r}')$, by folding $N-\alpha$ interactions with a structure model for the α . The associated multipoles we designate as $W_0^{(1,2)}$, as we consider only *s*-wave scattering and as there can be two identifiable contributions to the nonlocal interaction from the folding procedure.

(2a) The Folding Process

In a single folding model to define the α - α nonlocal interaction, the nucleon in a nucleon- α -particle $(N-\alpha)$ interaction is presumed to be one of the four nucleons comprising the projectile α -particle. The task then is to fold the chosen $N-\alpha$ interaction with the wave function of that nucleon as defined by the appropriate internal nuclear wave function. That requires one to evaluate the multiparticle matrix element

$$U_{\alpha\alpha} = \langle \Psi(1\dots4) \Big| \sum_{N=1}^{4} V_{N\alpha} \Big| \Psi(1\dots4) \rangle, \qquad (5)$$

with $\langle \mathbf{R} | \Psi(1...4) \rangle$ being the four-body wave function for the ground state of the α -particle. As all four nucleons in the α are equivalent, it is useful to choose

a specific entry ('1') and write

$$U_{\alpha\alpha} = 4 \langle \Psi(1\dots4) | V_{1\alpha} | \Psi(1\dots4) \rangle, \qquad (6)$$

as, with the four-body state expanded in cofactors,

$$|\Psi(1\dots4)\rangle = \sqrt{\frac{1}{4}} \sum_{jm\zeta} |\phi_{jm\zeta}(1)\rangle \Xi(2,3,4) ,$$

$$\Xi(2,3,4) = a_{jm\zeta} |\Psi(1\dots4)\rangle , \qquad (7)$$

with j and m being the total angular momentum quantum number and its projection while ζ is the isospin, Equation (6) becomes

$$U_{\alpha\alpha} = \sum_{\substack{jm\zeta\\j'm'\zeta'}} \langle \Psi | a^{\dagger}_{j'm'\zeta'} a_{jm\zeta} | \Psi \rangle \left\langle \phi_{j'm'\zeta'}(1) | V_{1\alpha} | \phi_{jm\zeta}(1) \right\rangle.$$
(8)

Then, as the ground state spin is zero and we ignore any breaking of the Hartree condition in the wave function for the α , the transition density matrix operator, $a_{j'm'\zeta'}^{\dagger}a_{jm\zeta}$, reduces to the number operator for which

$$\langle \Psi | a_{j'm'\zeta'}^{\dagger} a_{jm\zeta} | \Psi \rangle = \delta_{jj'} \delta_{mm'} \delta_{\zeta\zeta'} \ \eta_{jm\zeta} , \qquad (9)$$

where $\eta_{jm\zeta} = 1$ if a nucleon in the state $|jm\zeta\rangle$ exists in $|\Psi\rangle$, and zero otherwise. Studies of low energy $N-\alpha$ scattering (Alexander *et al.* 1996) indicate that the central interaction for this system is 5 to 10 times stronger than the spin–orbit component and so in the first instance, and for simplicity, we neglect the spin dependence of the $N-\alpha$ interaction. In that case the $\alpha-\alpha$ interaction has the form

$$U_{\alpha\alpha} = \sum_{nl} \frac{2}{2l+1} [n_{n,l}^{(p)} + n_{n,l}^{(n)}] \{ U_{\alpha\alpha}^{(1)}(n,l) + U_{\alpha\alpha}^{(2)}(n,l) \} , \qquad (10)$$

with the summations over the principal n and orbital angular momentum l quantum numbers running over states in the structure model in which the ground state of the projectile α -particle has (single nucleon) occupancies of $n_{n,l}^{(p,n)}$ for protons and neutrons respectively. For the α they are dominated by the 0s state values. Even large basis shell model calculations (Dortmans *et al.* 1997) give 90% 0s occupancy for the α -particle. The superscripts 1 and 2 designate terms originating from local and nonlocal attributes of the chosen nucleon- α interaction respectively, as will be specified shortly. The shell contributions to these α - α potentials are given by

$$U_{\alpha\alpha}^{(1,2)}(n,l) \equiv U_{\alpha\alpha}^{(1,2)}(\mathbf{r},\mathbf{r}') = \langle \phi_{nlm}(\mathbf{r_1}) || V_{1\alpha}(\boldsymbol{\rho},\boldsymbol{\rho}') || \phi_{nlm}(\mathbf{r_1'}) \rangle, \qquad (11)$$

in which the integration coordinates $\mathbf{r_1}$ and $\mathbf{r'_1}$ are easily transformed to specify integrations over relative $N-\alpha$ coordinates, $\boldsymbol{\rho}$ and $\boldsymbol{\rho'}$ as the Jacobian of such a coordinate transformation is unity.

(2b) Specification of the $N-\alpha$ Interactions

We have generated the nonlocal interaction multipoles by folding $N-\alpha$ interactions with the ground state density profile of the nucleon within the projectile α -particle. Two forms for that (real) $N-\alpha$ interaction have been used.

First we have used the results of Lassaut and Vinh-Mau (1977) that were based upon a model form for NN interactions specified by Brink and Boeker (1967). The associated $N-\alpha$ interactions have the form

$$V_{1\alpha}(\boldsymbol{\rho}, \boldsymbol{\rho}') = U_{\rm D}(\boldsymbol{\rho}) \,\delta(\boldsymbol{\rho} - \boldsymbol{\rho}' + U_{\rm NL}(\boldsymbol{\rho}, \boldsymbol{\rho}')\,, \tag{12}$$

in which the local and nonlocal terms are given by

$$U_{\rm D}(\rho) = \sum_{i=1}^{2} U_{{\rm D},i}^{(0)} \exp\left(-\rho^2/\lambda_i^2\right) \,, \tag{13}$$

$$U_{\rm NL}(\boldsymbol{\rho}, \boldsymbol{\rho}') = \frac{1}{\pi^{\frac{3}{2}}} \sum_{i=1}^{2} U_{{\rm NL},i}^{(0)} \exp(-s^2/\beta_i^2) \exp(-R^2/b_0^2), \qquad (14)$$

repectively. The centre of mass and relative coordinates ${\bf R}$ and ${\bf s}$ are defined as usual by

$$\mathbf{R} = \frac{1}{2}(\boldsymbol{\rho} + \boldsymbol{\rho}'), \qquad \mathbf{s} = \boldsymbol{\rho} - \boldsymbol{\rho}'.$$
(15)

For completeness, the potential strengths U, ranges (λ, β) and nucleon bound state harmonic oscillator length b_0 used by Lassaut and Vinh-Mau (1977) are listed in Table 1 for their B1 and C1 parameter sets. The $N-\alpha$ cross sections that are obtained with this folded interaction agree reasonably with the data at low energies. But they are not as good as the results found using an optical model potential approach. The optical potential of Satchler *et al.* (1968) is the

Table 1. Parameters of the $N-\alpha$ potentials for different NN forces

All lengths are in fm and the local (D) and nonlocal potential (NL) strengths are in MeV and MeV/fm³ respectively. The link between the $N-\alpha$ and N-N interactions is given by Lassaut and Vinh-Mau (1977)

N–N	$U_{\mathrm{D},1}$	$U_{\mathrm{D},2}$	λ_1	λ_2	$U_{\rm NL,1}$	$U_{\rm NL,2}$	β_1	β_2	b_0
B1 C1	$\begin{array}{c} -77 \cdot 7 \\ -94 \cdot 85 \end{array}$	$230 \cdot 35 \\ 340 \cdot 63$	$\begin{array}{c} 1 \cdot 98 \\ 1 \cdot 92 \end{array}$	$1 \cdot 57 \\ 1 \cdot 49$	$\begin{array}{c} -72 \cdot 92 \\ -46 \cdot 56 \end{array}$	$-514 \cdot 2 \\ -1086 \cdot 5$	$\begin{array}{c} 1 \cdot 25 \\ 1 \cdot 24 \end{array}$	$\begin{array}{c} 0\cdot 68 \\ 0\cdot 68 \end{array}$	$1 \cdot 40$ $1 \cdot 32$

second $N{-}\alpha$ interaction we have used in the folding procedure. It is purely local and real and has the form

$$V_{1\alpha}(\rho, \rho', \epsilon) = \left[-V_{n,p} f(\rho, R_0, a_0) + \left(\frac{\hbar}{m_{\pi}c}\right)^2 V_s \vec{L} \cdot \vec{\sigma} \frac{1}{\rho} \frac{d}{d\rho} f(\rho, R_s, a_s) \right] \delta(\rho - \rho') ,$$

$$f(\rho, R, a) = \left[1 + \exp\left(\frac{\rho - R}{a}\right) \right]^{-1}, \qquad (16)$$

where the relevant parameter values (we ignore the spin-orbit attribute again as it is relatively weak) are $V_{\rm n} = 41 \cdot 8 \,\mathrm{MeV}$, $V_{\rm p} = 43 \cdot 0 \,\mathrm{MeV}$, $R_0 = (M_{\alpha}/M_{\rm p})^{\frac{1}{3}}(1\cdot5-0\cdot01\epsilon)\,\mathrm{fm}$, $(M_{\alpha}/M_{\rm p}) = 3\cdot973$ and $a_0 = 0\cdot25\,\mathrm{fm}$, for an $N-\alpha$ laboratory energy ϵ . Note that it is the radial parameter of the central potential rather than the potential strength that is energy dependent. Using the above parameter values ('set 8'), Satchler *et al.* (1968) found good fits to neutron and proton elastic scattering data.

(2c) The s-wave Nonlocal Multipoles for the α - α System

For simplicity we have taken a packed 0s-shell model for the α -particle. Better representations exist, but until such extra details are shown to be essential in analysis of the α - α scattering properties, they are not required. Furthermore we have used harmonic oscillator functions with the C1 oscillator length (see Table 1) for the (n = l = 0) bound states of the nucleons, i.e.

$$\phi_{nlm}(\mathbf{r}_N) \to \phi_{000}(\mathbf{r}_N) = \mathcal{N}_{00} \exp\left(-\frac{1}{2b_0^2} r_N^2\right). \tag{17}$$

Standard algebra yields for the first component of the α - α s-wave (L = 0) nonlocal interaction

$$W_0^{(1)}(r,r') = 4\pi \mathcal{N}_{00}^2 rr' \exp\left(-\frac{1}{2b_0^2}r^2\right) \exp\left(-\frac{1}{2b_0^2}r'^2\right)$$
$$\times \int d\rho \,\rho^2 \,\exp\left(-\frac{1}{b_0^2}\rho^2\right) \,i_0\!\!\left(i\frac{1}{b_0^2}r\rho\right) \,i_0\!\!\left(i\frac{1}{b_0^2}r'\rho\right) \,V_{1\alpha}(\rho) \,. \tag{18}$$

The $i_0(ix)$ are modified spherical Bessel functions of imaginary argument and $V_{1\alpha}(\rho)$ is given either by the Lassaut and Vinh-Mau (1977) interaction or by the central part of the optical potential of Satchler *et al.* (1968). While the decaying exponentials ensure that these integrands are bounded at all radii, the Bessel functions rapidly increase with the moduli of their arguments so that any energy variation, even the small amounts associated with the Satchler *et al.* potential can have a dramatic effect upon the calculated nonlocal multipole functions.

Only with use of the Lassaut and Vinh-Mau (1977) interaction does the nonlocal α - α interaction have a contribution from the second term in equation (10). For the simple structure model it reduces to

$$W_{0}^{(2)}(r,r') = 4\pi^{2} \mathcal{N}_{00}^{2} rr' \exp\left(-\frac{1}{2b_{0}^{2}}r^{2}\right) \exp\left(-\frac{1}{2b_{0}^{2}}r'^{2}\right) \sum_{k} U_{\mathrm{NL},k}^{(0)}$$

$$\times \int d\rho \,\rho^{2} \exp\left[-\frac{1}{2}\left(\frac{1}{b_{0}^{2}} + X_{k}^{(+)}\right)\rho^{2}\right] \, i_{0}\!\left(i\frac{1}{b_{0}^{2}}r\rho\right)$$

$$\times \int d\rho' \,\rho'^{2} \exp\left[-\frac{1}{2}\left(\frac{1}{b_{0}^{2}} + X_{k}^{(+)}\right)\rho'^{2}\right]$$

$$\times i_{0}\!\left(i\frac{1}{b_{0}^{2}}r'\rho'\right) i_{0}\!\left(iX_{k}^{(-)}\rho\rho'\right), \qquad(19)$$

where $X_k^{(\pm)}$ are given by

$$X_k^{(\pm)} = \frac{2}{\beta_k^2} \pm \frac{1}{2b_0^2} \,. \tag{20}$$



Fig. 1. The s-wave $\alpha - \alpha$ scattering phase shifts as a function of the laboratory energy. The experimental data were taken from Darriulat (1965) and the dashed and solid curves depict the results found with the $\alpha - \alpha$ interaction given by folding the Lassaut and Vinh-Mau (1977) force (Brink and Boeker C1 force parameters) and by folding the central part of the energy dependent $N-\alpha$ phenomenological optical model potential of Satchler *et al.* (1968).

3. Results of Calculations

By folding the Lassaut and Vinh-Mau (1977) force we obtain a nonlocal α - α interaction that is real but independent of the energy of the collision. The



Fig. 2. The s-wave multipole of the α - α interaction derived by folding the $N-\alpha$ optical potential of Satchler *et al.* (1968) for a (laboratory) scattering energy of 0.4 MeV. The vertical axis gives the multipole strength in MeV fm² while the planar variations are the radii r and r' in units of fm. (b) As for (a), but at an energy of 10.88 MeV. (c) As for (a), but at an energy of 22.9 MeV.

potentials obtained starting with either the B1 or C1 parameter sets are very similar. With those potentials, solutions of equation (4) were found for incident energies up to 22.9 MeV. The α - α s-wave scattering phase shifts obtained are compared with experimental data in Fig. 1; the results are portrayed by the dashed curve. Clearly the result found using this energy independent force does not describe the data well except at very low α -particle energies. On the other hand, upon folding the N- α optical potential of Satchler *et al.* (1968), we obtain a nonlocal α - α interaction that is again real but is now energy dependent. In so doing we used the approximation $E_{\alpha\alpha}^{(lab)} \approx 4E_{\alpha N}^{(lab)}$ to select the specific N- α potential to fold at each energy. The slight variation between the neutron and proton potential strengths had little effect on the calculated α - α phase shifts and the results, displayed by the solid curve in Fig. 1, compare well with the data.

The s-wave multipoles of the $\alpha-\alpha$ interaction that we have found by folding the potential of Satchler *et al.* (1968) are shown in Fig. 2 for three incident energies. The radial variations are smooth and attractive having Gaussian form, reflecting that the exponential terms in equation (18) dominate at large radii. The nonlocality is substantial as the interactions are not negligible even several fm off the (r = r') diagonal. As a first approximation, the diagonal values of these multipoles can be considered as the local part of the fields. Those diagonal values are given in Fig. 3 for five incident energies to 22.9 MeV. This figure stresses that the multipoles are strongly energy dependent. Hence they do not reflect the nonlocality of the Frahn-Lemmer type as used by Perey and Buck (1962), nor is there a simple prescription we can give to estimate any equivalent local form. The success of the phenomenological studies (Buck *et al.* 1977; Marquez 1983) and the similarity of the potential shapes may lead one to consider these potentials to be the equivalent local form, though the lack of energy dependence of those phenomenological forms does not encourage that belief.



Fig. 3. Diagonal (r = r') variation of the s-wave multipoles at diverse (laboratory) scattering energies. The results shown from bottom to top coincide with scattering energies of 0.4, 5.26, 10.88, 15.2 and 22.9MeV respectively.

4. Conclusions

We have calculated nonlocal interactions to describe $\alpha - \alpha$ low energy (s-wave) scattering by folding candidate $N-\alpha$ interactions with the density matrix elements of the ground state of the α -particle. Two candidate $N-\alpha$ interactions have been

used. The first was the (analytic) form found by Lassaut and Vinh-Mau (1977) by their folding of a simple form for the NN interaction with the same α -particle density matrix elements. An energy independent, real, nonlocal $\alpha - \alpha$ interaction results. The second candidate $N-\alpha$ interaction was the central real part of the optical model potential found by Satchler et al. (1968) to fit $N-\alpha$ scattering data. It gave a real, energy dependent form for the nonlocal $\alpha - \alpha$ interaction. When the energy of the $N-\alpha$ interaction to be folded was chosen to be a quarter of the energy of the α - α scattering system, the s-wave nonlocal multipoles led to a very good fit to the energy variation of the $\alpha - \alpha$ s-wave scattering phase shifts. The simple (energy independent) form for the underlying NN interaction for this folding process we have used is typical of many others in the literature and especially those used in double folding studies of heavy ion collisions. The one we have used is inadequate to analyse the α - α scattering data and we surmise that so also would be the use of others. That is all the more likely given the successful results obtained at higher energies and with microscopic model N-nucleus optical potentials based upon nuclear matter q matrices (Karataglidis et al. 1995). The associated effective NN interactions are complex and energy and density dependent. With decreasing energy, the NN effective interaction will remain dependent on energy and on the nuclear medium at least.

A second aim of this study was to assess if the utilitarian Frahn-Lemmer form of nonlocality can be a good approximant. Our results indicate that such is not the case given the strong energy dependence found in the more appropriate of the set of s-wave multipoles that we obtained, i.e. those formed by folding the Satchler *et al.* (1968) $N-\alpha$ optical model potential.

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