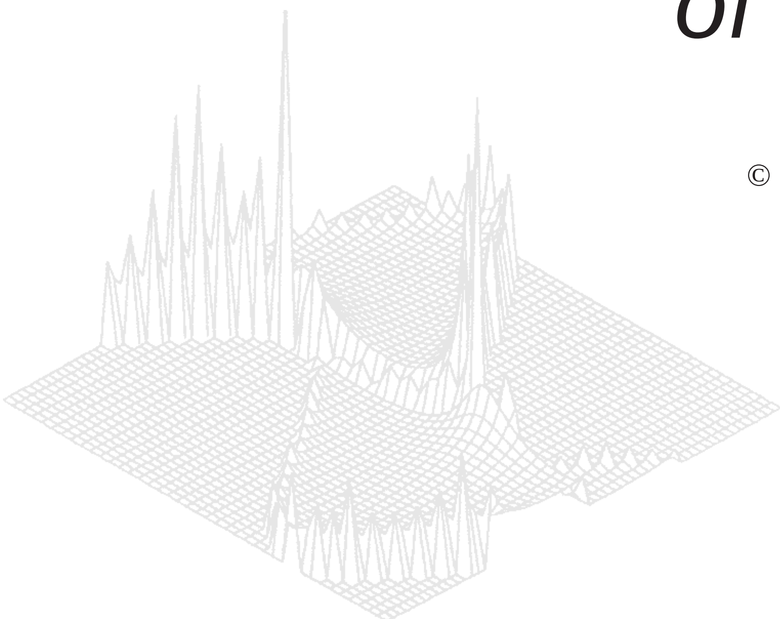
CSIRO PUBLISHING

Australian Journal of Physics

Volume 50, 1997



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia

Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: peter.robertson@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO Australia and
the Australian Academy of Science

CSIRO AUSTRALIA

Academy of Science

Aust. J. Phys., 1998, **51**, 9–19

Some Physical Implications of the Gravitoelectromagnetic Field in Fractal Space–Time Theory

M. Agop,[A,B] *V. Griga,*[B] *C. Gh. Buzea,*[C] *I. Petreus,*[B] *C. Buzea,*[D] *C. Marin*[B] *and N. Rezlescu*[C]

[A] Mailing address: Ph.D. Agop Maricel, Str. Moara de Foc nr. 10, bl. 406, sc. B, et VIII, ap. 35, Iasi 6600, Romania.
email: picardout@phys-iasi.ro
[B] Department of Physics, Technical 'Gh. Asachi' University, Iasi 6600, Romania.
[C] Superconductivity Research Laboratory, Institute of Technical Physics, B-dul D. Mangeron, Iasi 6600, Romania.
[D] Research Institute of Electrical Communication, 2-1-1 Katahira, Aoba-ku, Sendai 980–70, Japan.

Abstract

It is shown that in terms of the fractal space–time theory the gravitoelectric potential is responsible for the quantisation of the planetary and binary galaxy motions. On a cosmic scale a homogeneous gravitomagnetic field allows not only an ordering of the Universe, but a 'global' redshift quantisation of galaxies as well.

1. Introduction

In a previous paper (Der Sarkissian 1984) it was suggested that a cosmic version of ordinary quantum mechanics may be responsible for the observed physical properties of galaxies [this intriguing possibility has emerged because recession velocities for single and double galaxies appear to be quantised (Tifft and Cocke 1984)]. In this earlier work the kinetic energy is quantised by using a plausible form for the well-established kinetic-energy operator. Both physical quantities are closely related and both may be considered observable for an equivalent point galaxy. The redshift just happens to be easier to measure directly at this time. The kinetic energy is measured indirectly at this time, but the possibility of direct measurement does not appear to be ruled out.

An alternative form of cosmic quantum mechanics was suggested independently by Cocke (1983). This model differs by requiring a two component spinor formalism. Ordinary quantum mechanics has no need for such complexity at the nonrelativistic level. The need for spinors presumably arises as an appropriate relativistic generalisation. The model quantises the redshift by inventing a 'redshift' operator $\sim\hat{Z}$, assumed to be proportional to the linear momentum operator for an equivalent point galaxy.

The work of DerSarkissian (1984) predicts that $\Delta v = v_{m+1} - v_m$ (the velocity spacing between adjacent quantum states of recession) is proportional to $m^{-1/2}$ for large m. In Cocke's (1983) paper the eigenvalue problem for the kinetic-energy

 10.1071/PH97022 0004-9506/98/010009$05.00

operator is essentially equivalent to the eigenvalue problem for the operator $\hat{Z}^2$. The resulting eigenvalues are $(z_m)^2$, proportional to m. The large m behaviour of Δv is, therefore, also proportional to $m^{-1/2}$. However, the model then invokes the 'redshift combination principle' which does not flow from the spinor formalism for equivalent point galaxies, but is a subsidiary assumption designed to exclude certain redshift states and to achieve the result Δv = constant.

A new and unitary way of studying the cosmic phenomena may be achieved by using fractal space–time theory. The idea that the space–time of microphysics is fractal, rather than flat and Minkowskian as assumed up to now, was suggested over ten years ago (Nottale and Schneider 1984). This proposal was itself based on earlier results (Feynman and Hibbs 1965; Allen 1983; see in particular Schweber 1986) concerning the geometrical structure of quantum paths. These studies have shown that the typical trajectories of quantum mechanical particles are continuous but nondifferentiable, and can be characterised by a fractal dimension which jumps from $\mathcal{D} = 1$ at large length-scales to $\mathcal{D} = 2$ at small length-scales, the transition occurring on about the de Broglie scale (see Nottale 1989).

Now, such a fractal dimension $\mathcal{D} = 2$ plays a particular role in physics. It is well known that this is the fractal dimension of Brownian motion (Mandelbrot 1982), i.e. from the mathematical viewpoint, of a Markov–Wiener process. This observation leads us to recall a related attempt at understanding quantum behaviour, namely Nelson's (1985) stochastic quantum mechanics. In this approach, it is assumed that any particle is subjected to an underlying Brownian motion of unknown origin, which is described by two (forward and backward) Wiener processes: when combined they yield the complex nature of the wave function Ψ and they transform Newton's equation of dynamics into the Schrödinger equation. A generalisation of Newton's equation in terms of the Ψ function is (Nottale 1985)

$$\nabla U = 2\mathrm{i}\ Dm\ \frac{\delta}{\mathrm{d}t}(\Delta\ \ln\ \Psi)\,. \tag{1}$$

Nottale (1985) generalised Schrödinger's equation to

$$D^2\Delta\Psi + \mathrm{i}\ D\ \frac{\partial\Psi}{\partial t} - \frac{U}{2m}\Psi = 0\,. \tag{2}$$

In these relations

$$\frac{\delta}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathcal{V}.\nabla - \mathrm{i}\ D\Delta \tag{3}$$

will play the role of a new kind of 'quantum-covariant derivative' with the complex speed $\mathcal{V}$:

$$\mathcal{V} = -2\mathrm{i}\ D\nabla\ \ln\ \Psi\,, \tag{4}$$

U being the generalised potential and D a diffusion coefficient, depending on the fractal dimension. Thus, the fundamentals of the fractal space–time theory are established (Nottale 1985).

Both conceptual (complex nature of the wave function, probabilistic nature of quantum theory, principle of correspondence, quantum–classical transition, divergence of masses and charges, nature of Planck scale, nature and quantisation of the electric charge, origin of mass discretisation of elementary particles, nature of the cosmological constant, etc.) and quantised results (mass–charge relations, electroweak scale, electron scale, elementary fermion mass spectrum, etc.) are obtained using this theory (Nottale 1996).

In this paper, by extending the fractal space–time theory to a cosmic scale, it is shown that a gravitoelectric potential allows the quantisation of planetary and binary galaxy motions, and a homogeneous gravitomagnetic field allows an ordering of the Universe and 'global' redshift quantisation of galaxies.

2. Solar System Quantisation

Let us consider a two-body system of comparable masses interacting by means of a gravitoelectric potential (Peng 1993):

$$U(r) = \frac{Gm_1\ m_2}{r}\,, \tag{5}$$

where G is Newton's constant, m_1, m_2 are the masses of the two bodies and r is the distance between them. Since the gravitoelectric potential (5) depends only on the relative coordinate, one can use the results of Titeica (1984) to separate the centre of mass motion. Acting this way in the centre of mass system, the Hamiltonian takes the form

$$E = \frac{p^2}{2\mu} - \frac{GM\mu}{r}\,, \tag{6}$$

where p is the relative momentum, $\mu = m_1\ m_2/(m_1{+}m_2)$ the reduced mass and $M = m_1{+}m_2$ the total mass.

In light of the fractal space–time and taking into account the Hamiltonian (6) and the operators correspondence $\hat{p} \rightarrow 2i\ \mu D\nabla$ (Nottale 1996), the double Wiener function Ψ attached to the system considered will satisfy the equation

$$\left[-2\mu D^2\nabla^2 - \frac{GM\mu}{r}\right]\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r})\,. \tag{7}$$

[The same result (7) is obtained using equation (2) and the correspondences $\hat{E} \rightarrow 2i\ \mu D\ \partial/\partial t$ (Nottale 1996), $m\ \rightarrow\ \mu$, respectively.]

Since the gravitoelectric potential is a central potential, this equation, due to Titeica's (1984) method, admits the eigenvalues

$$E_n = -\tfrac{1}{2}\ \frac{GM\mu}{a_0}\ \frac{1}{n^2}\,, \tag{8}$$

with a_0 a fundamental length

$$a_0 = \frac{4D^2}{GM}\,, \tag{9}$$

and the eigenfunctions

$$\Psi_{nlm}(r,\ \theta,\ \varphi) = \left(\frac{1}{4\pi n}\right)^{1/2} \left(\frac{2}{na_0}\right)^{3/2} \left[\frac{2l+1}{2}\ \frac{(n-l-1)!(l-|m|)!}{[(n+l)!]^3(l+|m|)!}\right]$$
$$\times\ P_l^m(\cos\theta)\ \mathrm{e}^{\mathrm{i}m\varphi}\ \mathrm{e}^{-r/na_0}\left(\frac{2r}{na_0}\right)^l L_{n+l}^{2l+1}\left(\frac{2r}{na_0}\right), \quad (10)$$

where P_l^m are the Legendre polynomials, L_{n+l}^{2l+1} are the generalised Laguerre polynomials, and n, l, m are the principal quantum number, the orbital quantum number and the magnetic quantum number respectively (Titeica 1984).

The average distance in terms of two quantum numbers n and l, a_{nl}, is given by the relation (Titeica 1984)

$$a_{nl} = [\tfrac{3}{2}n^2 - \tfrac{1}{2}l(l+1)]a_0\,. \quad (11)$$

The condition for the model given to fit the Solar System experimental data implies a choice between two diffusion coefficients: one corresponding to the inner planets (Mercury, Venus, Earth and Mars) having the value $D_1 \sim 0{\cdot}43{\times}10^{15}$ m^2 s^{-1} and the second for the outer planets (Jupiter, Saturn, Uranus, Neptune and Pluto) having the value $D_2 \sim 2{\cdot}24{\times}10^{15}$ m^2 s^{-1}. Consequently, from the relation

$$a_i = \frac{4D_i^2}{GM}, \qquad i = 1,\ 2\,, \quad (12)$$

deduced from (9) in the limit $m_2 = M_\mathrm{S} \gg m_1$, with M_S the mass of the Sun, for the inner planets' system there is a fundamental length $a_1 \sim 0{\cdot}038$ a.u. and for the outer planets' system $a_2 \sim 1{\cdot}028$ a.u.

The Sun–planet distance is derived by substituting in (11) $l = n - 1$, which could correspond to circular orbits. It results in

$$a_n = (n^2 + \tfrac{1}{2}n)a_i, \qquad i = 1, 2\,. \quad (13)$$

Equation (13) has already been given by Nottale (1996*b*).

In Table 1 we compare experimental data [the large semi-axis of planetary orbits (Anuarul Astronomic 1996)] with data calculated using relation (13). Distances for the inner planetary system are obtained from relation (13) with $i = 1$ and $n = 3, 4, 5, 6$. For $n = 1, 2$ the model predicts the existence of two intra-Mercury planets.

The region between Mars and Jupiter is where the two systems overlap. The emptiness of the orbits $n = 7$ and $n = 10$ is easily understood, since they coincide with resonances 1 : 4 and 2 : 3 with Jupiter, where small time-scale dynamic chaos is expected to occur (Wisdom 1987). Values for $n = 8$, 9 may belong to some asteroids [besides 97% of all asteroids have a large semi-axis value ranging between $a = 2{\cdot}17$ a.u. and $a = 3{\cdot}64$ a.u., with an average of $\langle a\rangle = 2{\cdot}75$ a.u., corresponding to the Ceres asteroid (Anuarul Astronomic 1996)].

Table 1. Planetary motion quantisation

Planet	n (a.u.)	a_i ($m^2\ s^{-1}$)	D_i (a.u.)	a_n (a.u.)	a_{exp}	Observations
	1			0·057	—	
	2			0·190	—	
Mercury	3	0·038	$0{\cdot}43\times10^{15}$	0·399	0·387	Inner
Venus	4			0·684	0·723	planetary
Earth	5			1·045	1·000	system
Mars	6			1·482	1·523	
	7			1·995	—	
	8			2·584	—	Resonances
	9			3·249	—	and asteroid
	10			3·990	—	chains
	1			1·542	—	
Jupiter	2			5·140	5·202	
Saturn	3			10·796	9·536	Outer
Uranus	4	1·028	$2{\cdot}24\times10^{15}$	18·210	19·210	planetary
Neptune	5			28·275	30·138	system
Pluto	6			40·099	39·390	

The distances for the outer planetary system are computed with relation (13) taking $i = 2$ and $n = 2, 3, 4, 5, 6$. The average distance of the inner Solar System ($\langle a_1 \rangle \sim 1{\cdot}567$ a.u.) is in very good agreement with $n = 1$ for the outer system. Note also the agreement of Neptune and especially Pluto with the outer relation (recall that they did not fit the original Titus–Bode law).

The very same structure of the Solar System is revealed by its planetary experimental mass distribution. Thus the following may be concluded (see Fig. 1):

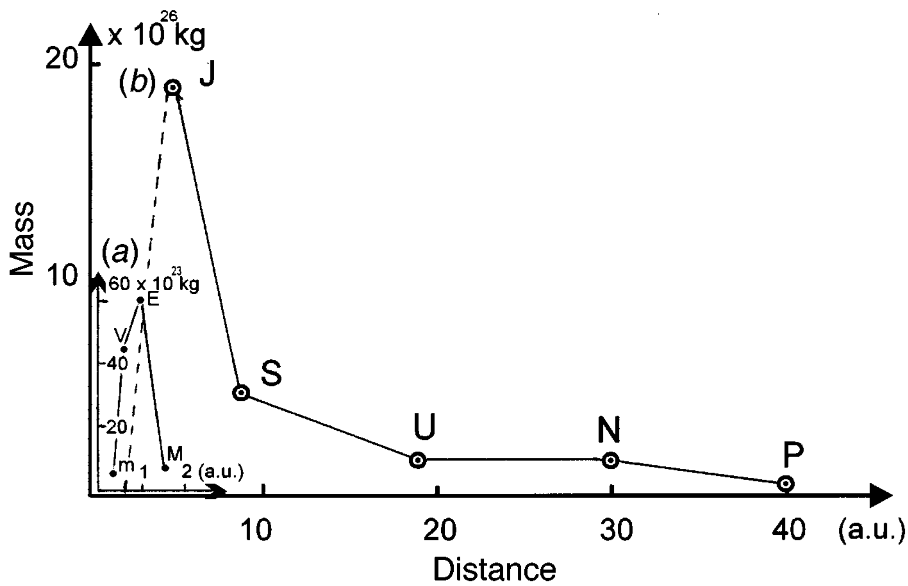


Fig. 1. Experimental mass distribution in the Solar System.

(i) Curve (a) gives the mass distribution in the inner planetary system. The distribution maximum is obtained for $r \sim 1$ a.u. which is the Earth orbit. The astronomical data indicate an increase of mass distribution from Mercury ($m_{\rm Me} \sim 3{\cdot}3{\times}10^{23}$ kg) towards Earth ($m_{\rm E} \sim 59{\cdot}7{\times}10^{23}$ kg), the maximum being focused on it, and a decrease of this distribution towards the asteroids chain.

(ii) Curve (b) gives the mass distribution in the outer planetary system. The maximum of the distribution is obtained for $r \sim 5$ a.u., and corresponds to the Jupiter orbit. The astronomic data show an increase of the mass distribution from the asteroid chain towards Jupiter ($M_{\rm J} \sim 18{\cdot}9{\times}10^{26}$ kg), the maximum being focused on this planet, and a decrease of this distribution towards Pluto.

It can be noticed that the experimental curves in Fig. 1 may be described by the radial function $r^2\ R_{20}^2\ (r)$ (Titeica 1984).

Since the third Kepler law is verified, it follows that the specific kinetic momentum ($L/m_1 = rv$) is given by relation

$$(rv)_n = \sqrt{n^2 + \tfrac{1}{2}n}\ (2D_i), \qquad i = 1,\ 2\,. \tag{14}$$

In Table 2 we compare the specific kinetic moment calculated with relation (14) with the experimental data (Anuarul Astronomic 1996).

Table 2. Quantisation of the specific kinetic moment

Planet	n	$2D_i$ ($\rm m^2\,s^{-1}$)	$(rv)_n{\times}10^{15}$ ($\rm m^2\,s^{-1}$)	$(rv)_{\rm exp}{\times}10^{15}$ ($\rm m^2\,s^{-1}$)
Mercury	3	$2D_1 = 0{\cdot}86{\times}10^{15}$	2·78	2·76
Venus	4		3·64	3·78
Earth	5		4·50	4·45
Mars	6		5·37	5·49
Jupiter	2	$2D_2 = 4{\cdot}48{\times}10^{15}$	10·01	10·15
Saturn	3		14·51	13·75
Uranus	4		19·00	19·55
Neptune	5		23·49	24·55
Pluto	6		27·97	27·91

The fact that 98% of the total specific kinetic moment of the Solar System is assigned to the planets and only 2% to the Sun, explains the 'export' of solar kinetic moment by slowing down the central core spin of the solar vortex (Popescu 1980). Thus, one gets a result postulated by all contemporary cosmological theories: The values of the diffusion coefficients, D_1 and D_2, must not be postulated. Thus, $D_1 \sim (1/2{\cdot}3)(rv)_{\rm Mercury}$ and $D_2 \sim (1/2{\cdot}2)(rv)_{\rm Jupiter}$.

3. Quantisation of Galaxy Pairs

Let us apply the same quantisation procedure previously used to study galaxy pairs. We find that the pair energy $\epsilon_n = E_n$ [see relations (8) and (9)] is quantised as

$$\epsilon_n = -\tfrac{1}{2}\mu\left(\frac{GM}{2Dn}\right)^2, \tag{15}$$

and that the relative velocity in binary galaxies must take only preferential values given by

$$v_n = \frac{GM}{2Dn} = \frac{v_0}{n}, \qquad v_0 = \frac{GM}{2D}, \tag{16}$$

as in the hydrogen atom in the old quantum mechanics. Equation (16) has been given by Nottale (1996).

Table 3. Speed quantisation of galaxy pairs

Double galaxy	(m_1 and m_2)$\times 10^{10}$ ($M_\odot$)	v (km s^{-1})	n	$nDM\times 10^{67}$ (J s)
NGC-3958	12·0			
NGC-3963	9·0	72·03	∼1	8·00
NGC-4294	1·4			
NGC-4299	8·8	34·98	∼2	7·77
NGC-4085	1·3			
NGC-4088	6·0	25·04	∼3	8·34
NGC-3504	1·0			
NGC-3512	5·0	20·58	∼4	9·10
NGC-6542	2·7			
NGC-6528	1·7	15·09	∼5	8·38

The condition for the proposed model to fit the experimental data implies that $D \sim 0{\cdot}1925 \times 10^{27}$ m^2 s^{-1}. We give in Table 3 the speed quantisation for some typical pairs of galaxies (Van Moersel 1983; Dickel and Rood 1983). We note the following:

(a) Such a theoretical result seems to provide an explanation for Tifft's (1984) effect of redshift quantisation in binary galaxies. Indeed it has been claimed by Tifft that the velocity differences in isolated galaxy pairs was not distributed at random, but showed preferential values near 72, 36 and 24 km s^{-1}, i.e. $(72/n)$ with $n = 1, 2, 3, \ldots$. This result, in particular the 72 km s^{-1} periodicity was confirmed by several authors (Schneider and Salpeter 1992; Cocke 1992).

(b) A cosmological Planck constant, $\hbar_{\rm g} \sim 8 \times 10^{67}$ J s, exists. This value is close to the one given by DerSarkissian (1984) ($\hbar_{\rm g} \sim 7\times 10^{67}$ J s), but differs from another value given by Cocke (1983) ($\hbar \sim 2\times 10^{67}$ J s).

(c) The ground state energy of a typical pair can be calculated using the relation (15) with $m_1 = m_2 \sim 10^{41}$ kg (the mass of the Milky Way). One gets $\epsilon_n \sim 10^{49}$ J. Under these conditions the excited state lifetime can be estimated from the relation $\epsilon_n\, t = \hbar_{\rm g}/2$. If ϵ_n is the energy difference between the ground state and the first excited state, then $t \sim 2 \times 10^{11}$ yr. Therefore, if a double galaxy is formed in an excited state, it remains in that state for its entire existence. The 'quantum excitation' ($\omega = \epsilon_n/\hbar_{\rm g} \sim 8\times 10^{-18}$ Hz) is initially monochromatic gravitational radiation. It must somehow be converted to a spectrum of electromagnetic radiation, which is emitted during the lifetime of the double galaxy. The conversion mechanism may be related to the gravitational collapse in the galactic nuclei of the pair. Since $\hbar_{\rm g}$ is so large, it is plausible to expect radio wave emission to dominate. These thoughts lead to two further possibilities which have not been considered yet:

(c_1) Some radioactive quasars may be disguised double galaxies in highly excited quantum states and at an early stage of evolution. Preliminary evidence suggests there may be localised radio sources inside some quasars, but better telescope resolution may be required to test the idea. A rough estimate of the quasar's average power output in this model is of an acceptable order of magnitude, providing the mass of a typical component is taken as $m_1 = m_2 \sim 10^{43}$ kg and the average time of existence is $\tau \sim 10^9$ yr, i.e. $P_{\rm ave} \approx (\epsilon_n - \epsilon_1)/\tau \sim 10^{39}$ W. This result fits the experimental data (Ureche 1987).

(c_2) Double galaxies may be strong radio sources, compared to single, galaxies (Ureche 1987). It was shown for a sample of isolated galaxies and isolated doubles, virtually all of them being spirals, that compact radio sources occurred four times more frequently in doubles than in singles. There was also a higher occurrence of strong radio emission from double compared to single, with a greater power output in doubles associated with the more frequent occurrence of active galactic nuclei in their components. Although these results are encouraging, they are not definitive, because of the small sample size and of other possible selection biases.

4. Universe Quantisation

Let us study the motion of a particle with mass m in a homogeneous gravitomagnetic field with the vector potential (Agop *et al.* 1997)

$$A_x = -B_{\rm g}\, y, \qquad A_y = A_z = 0\,. \tag{17}$$

In these conditions, having in view the results given by Agop *et al.* (1996), the Hamiltonian becomes

$$\hat{H} = \frac{1}{2m}(\hat{p}_x + 4mB_{\rm g}\ y)^2 + \frac{\hat{p}_y^2}{2m} + \frac{\hat{p}_z^2}{2m}\,. \tag{18}$$

By considering fractal space–time, the double Wiener function Ψ associated with the system, taking into account the procedure in Section 2, will satisfy the equation

$$\hat{H}\Psi = E\Psi\,. \tag{19}$$

The operator (18) does not contain explicitly the coordinates x and z. Therefore the operators $\hat{p}_x = 2\mathrm{i}mD\ \partial/\partial x$ and $\hat{p}_y = 2\mathrm{i}mD\ \partial/\partial y$ (Nottale 1996) commute with $\hat{H}$, i.e. the x and z components of the generalised impulse are conservative. It turns out that one can choose Ψ of the form:

$$\Psi = \exp\left[\frac{\mathrm{i}}{2mD}(p_x x + p_y y)\right]\chi(y)\,. \tag{20}$$

Substituting (20) in (19) one gets the equation for the function $\chi(y)$:

$$\chi'' + \frac{1}{2mD^2}\left[E - \frac{p_z^2}{2m} - \frac{m}{2}(4B_\mathrm{g})^2(y-y_0)^2\right]\chi = 0\,, \tag{21}$$

with

$$y_0 = -\frac{p_x}{4mB_\mathrm{g}}\,. \tag{22}$$

Formally this equation coincides with the Schrödinger equation for a linear oscillator which oscillates with frequency $\omega_\mathrm{g} = 2B_\mathrm{g}$ (Agop *et al.* 1996) about the equilibrium position $y = y_0$. Therefore the constant $E - p_z^2/2m$, which plays the role of the oscillator energy, may take the values $(n + \frac{1}{2})2mD\omega_\mathrm{g}$ where n is an integer.

Thus, the eigenvalues of the energy have the form

$$E' = E - \frac{p_z^2}{2m} = (n + \tfrac{1}{2})4mD\omega_\mathrm{g}\,, \tag{23}$$

and the eigenfunctions have the form

$$\Psi \sim \exp\left[\frac{\mathrm{i}}{2mD}(p_x\, x + p_y\, y)\right]\ \exp\left[-\frac{(y-y_0)^2}{2a_0^2}\right] H_n\left[\frac{(y-y_0)}{a_0}\right], \tag{24}$$

where H_n are Hermite polynomials and a is the fundamental length

$$a_0 = (D/\omega_\mathrm{g})\,. \tag{25}$$

The first term in (23) corresponds to motion in the x, y plane. In classical mechanics this is circular motion around a fix point. The conservative parameter [the operator associated with this parameter commutes with the Hamiltonian (18)] $x_0 = p_y/4mB_\mathrm{g} + x$ corresponds to the classical x coordinate of the circle's centre, while the conservative parameter $y_0 = p_x/4mB_g\ + y$ corresponds to the classical y coordinates of the same circle's centre. Operators $\hat{x}_0$ and $\hat{y}_0$ do not commute, thus the coordinates x_0, y_0 cannot have simultaneously determined values.

Since (23) does not contain p_x which takes continuous values, the energy levels are degenerate with continuous multiplicity. The multiplicity, however, becomes finite if the motion in the x, y plane is bounded by a large but finite area $s = e_x\ e_y$. The number of discrete values of p_x in the interval Δp_x is equal to $(e_x/4\pi mD)\ \Delta p_x$. All the values of p_x for which the centre of the orbit is situated inside s are admissible. From $0 < y_0 < e_y$ one can get $\Delta p_x = 4mB_\mathrm{g}\, e_y$. Therefore, the number of states for given n and p_z will be $B_\mathrm{g}\ s/\pi D$. If the motion domain is bounded and placed along the z axis (by the length e_z), then the number of possible values in this interval is $(B_\mathrm{g}\ V/4\pi^2 mD^2)\Delta p_z$.

In this context, by considering a Universe filled with a cosmic fluid formed of identical particles with mass m, we predict that, in the presence of a homogeneous gravitomagnetic field, 'matter' will have the tendency to form structures according to the various modes of the quantised 1D harmonic oscillator, as given by

$$|\Psi|^2 \approx \exp\left[-\frac{(y-y_0)^2}{a_0^2}\right] H_n^2\left[\frac{(y-y_0)}{a_0}\right].$$

The zero mode is a Gaussian with the dispersion $\sigma_0 = a_0/\sqrt{2}$; the mode $n = 1$ is a binary structure whose peaks are situated at $x_{\rm p} = \pm a_0$; the mode $n = 2$ has three peaks at $x_{\rm p} = 0$; $\approx\pm 1\cdot 6\ a_0$; for $n = 3$ one finds $x_{\rm p} \approx \pm 0\cdot 6\ a_0$ and $\approx\pm 2\ a_0$. More generally, the position of the most extreme peak can be approximated by

$$x_{\max} \approx (n + \tfrac{3}{2})a_0/2\,. \tag{26}$$

If we now consider the momentum representation rather than the position one, predict a velocity distribution that is given by exactly the same functions, but with a replaced by the characteristic velocity

$$v_0 = \omega_{\rm g}\ a_0 = (D\omega_{\rm g})^{\frac{1}{2}}\,. \tag{27}$$

In this condition the difference between the extreme velocity peaks is of the order of $\approx 2v_0$, $\approx 3v_0$, $\approx 4v_0$ for the modes $n = 1,\ 2,\ 3$, respectively. Therefore the linear-like quantisation of the harmonic oscillator case yields a remarkable explanation of the 'global' quantisation in units of $36\ {\rm km\ s^{-1}}$ found by Guthrie and Napier (1991).

5. Conclusions

In this paper, by extending the fractal space–time theory to a cosmic scale, the following results are obtained:

(1) It was shown that in the presence of a gravitoelectric potential the planetary motion is quantised. Thus, one can explain not only the mass distribution in the Solar System, but also the solar kinetic-moment 'export' by slowing down the central core spin of the solar vortex.

(2) We have given an explanation for Tifft's effect concerning the redshift quantisation in binary galaxies.

(3) We have also shown that in the presence of a homogeneous gravitomagnetic field the various cosmological constituents of the Universe will be situated at preferential positions and move with preferential velocities, as described by the various structures implied by the quantisation of the harmonic oscillator. In other words, we expect the Universe to be locally structured, in position and velocity, according to the dynamic symmetry group of the 1D harmonic oscillator. In this way the 'global' redshift quantisation at $36\ {\rm km\ s^{-1}}$ is explained as well.

Acknowledgments

This paper was substantially improved by a referee's observations, to whom authors are indebted.

References

Agop, M., Buzea, C., and Buzea, C. Gh. (1997). Gravitomagnetic field, spontaneous symmetry breaking and periodic property of space. *Nuovo Cimento* (submitted).

Agop, M., Buzea, C. Gh., Griga, V., Ciubotariu, C., Buzea, C., Stan, C., and Tatomir, D. (1996). *Aust. J. Phys.* **49**, 1063.

Allen, A. D. (1983). *Speculations Sci. Tech.* **6**, 165.
Anuarul Astronomic (1996). (Eds M. Stravinschi *et al.*) (Ed. Acad. Bucuresti).
Cocke, W. J. (1983). *Astrophys. J. Lett.* **23**, 239.
Cocke, W. J. (1992). *Astrophys. J.* **393**, 59.
DerSarkissian, M. (1984). *Lett. Nuovo Cimento* **40**, 390.
Dickel, J., and Rood, H. (1980). *Astron. J.* **85**, 1003.
Feynman, R. P., and Hibbs, A. R. (1965). 'Quantum Mechanics and Path Integrals' (McGraw–Hill: New York).
Guthrie, B. N., and Napier, W. M. (1991). *Mon. Not. R. Astron. Soc.* **253**, 533.
Mandelbrot, B. (1982). 'The Fractals Geometry of Nature' (Freeman: San Francisco).
Nelson, E. (1985). 'Quantum Fluctuations' (Princeton Univ. Press).
Nottale, L. (1989). *Int. J. Mod. Phys.* A **4**, 5047.
Nottale, L. (1995). *In* 'Quantum Mechanics, Diffusion and Chaotic Fractals' (Eds M. S. El Naschie *et al.*), p. 51 (Pergamon: New York).
Nottale, L. (1996*a*). *Chaos, Solitons & Fractals* **7**, 877.
Nottale, L. (1996*b*). *Astron. Astrophys.* **315**, L9.
Nottale, L., and Schneider, J. (1984). *J. Math. Phys.* **25**, 1296.
Peng, H. (1983). *Gen. Rel. Gravit.* **15**, 725.
Popescu, J. N. (1980). 'Gravitatia' (Ed. Stiintifica si Enciclopedica: Bucuresti).
Schneider, S. E., and Salpeter, E. E. (1992). *Astrophys. J.* **385**, 492.
Schweber, S. S. (1986). *Rev. Mod. Phys.* **58**, 449.
Tifft, W., and Cocke, W. (1984). *Astrophys. J.* **287**, 492.
Titeica, S. (1984). 'Mecania Cuantia' (Ed. Acad. Bucuresti).
Ureche, V. (1987). 'Universul Astrofizia' (Ed. Dacia: Cluj Napoca).
Van Moersel, G. (1983). *Astron. Astrophys. Suppl. Ser.* Part I, **53**, 271; Part II, **53**, 287; Part III, **54**, 1; Part IV, **54**, 19.
Wisdom, J. (1987). *Icarus* **72**, 241.

Manuscript received 2 April, accepted 31 July 1997