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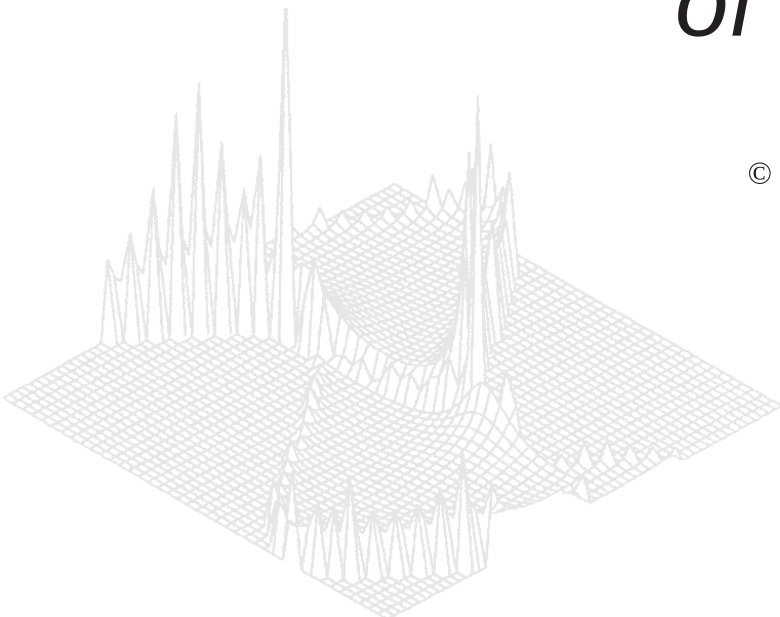
Australian Journal of Physics

Volume 50, 1997



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Aust. J. Phys., 1998, **51**, 21–33

Structure of the World Crystal

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Abstract

Identifying space with a superconducting cosmic dust, the structure of the 'world crystal' is built. Particles moving on open geodesics are assimilated with the 'electron gas' in a lattice, and those moving on closed geodesics with nodes of the same lattice. By considering that this kind of structure may be achieved by pinning on a background gravitomagnetic field, the properties of such fields are studied. From this analysis and computing the background energy we are lead to a Cantorian-fractal structure of space–time, which allows one to interpret the gravitational interaction in terms of a mechanism similar to the composite fermions mechanism.

1. Introduction

Recent work (Ciubotariu and Agop 1996; Agop *et al.* 1996) has shown that in a gravitomagnetic field, the space orders as a 'crystal'. In our view the existence of a background gravitomagnetic field 'grafts' such crystals on 'space', and holds the structure created by 'pinning.'

In this work, by identifying space with a superconducting cosmic dust, the structure of such a 'crystal' is conceived by studying the background gravitomagnetic field properties and computing the background energy. In this context we can show that space–time is Cantorian-fractal and we suggest a 'composite fermionising' mechanism to interpret the gravitational interaction.

2. Background Gravitomagnetic Field and Its Properties

If $\boldsymbol{A}_{\rm g}$ is the background gravitomagnetic vector potential, the expression for the mass current may be derived taking into account the work by Agop *et al.* (1966) and the correspondence $\mathcal{D}\rightarrow\hbar/2m$, where $\mathcal{D}$ is the diffusion coefficient

 10.1071/PH97023 0004-9506/98/010021$05.00

which depends on the fractal scale (Nottale 1995). It results in

$$\boldsymbol{j} = -4\mathrm{i}\, m\mathcal{D}(\psi^* \nabla\psi - \psi\nabla\psi^*) - 16m\boldsymbol{A}_{\mathrm{g}}\, |\psi|^2\,, \tag{1}$$

and by equating it to zero one gets

$$\boldsymbol{A}_{\mathrm{g}} = \frac{\mathrm{i}\,\mathcal{D}}{4}\left(\frac{\nabla\psi}{\psi} - \frac{\nabla\psi^*}{\psi^*}\right). \tag{2}$$

By materialising Ψ in (2) one can choose the type of 'crystal'. With this end in view let us 'identify' the space with cosmic dust. Since the latter is a 'huge gravitational superconductor' (Agop *et al.* 1996), Ψ will satisfy a generalised Ginzburg–Landau-like equation with the 'mass coefficient' $\alpha = E/2\mathcal{D}^2$, where E is the energy of a particle from the cosmic dust, and β the self-interaction constant.

For $E > 0$, the stationary solution of the equation in the unidimensional case is (Rezlescu *et al.* 1966)

$$\Psi_0 = a_0\,\mathrm{sn}[\Omega_0(x - x_0);\, k_0]; \quad \Omega_0^2 = a - a_0^2\,\beta/2; \quad x_0 = \mathrm{constant}\,, \tag{3}$$

where a_0 is an amplitude and sn the Jacobi elliptic function of module k_0, and where

$$k_0^2 = \frac{a_0^2\,\beta}{2}\left(\alpha - \frac{a_0^2\,\beta}{2}\right)^{-1} \tag{4}$$

corresponds to motion on open geodesics (Agop *et al.* 1996). In this case the cosmic dust behaves like an electron gas in a crystalline lattice.

For $E < 0$ and hard-nonlinearity ($\beta > 0$) the stationary solution of the same equation in the unidimensional case is (Jackson 1991)

$$\Psi = a\,\mathrm{cn}[\Omega(x - x_0);\, k]; \qquad \Omega^2 = -\alpha + \beta a^2\,, \tag{5}$$

where a is an amplitude and cn the Jacobi elliptic function of module k, and where

$$k^2 = \frac{\beta a^2}{2(-\alpha + \beta a^2)} \tag{6}$$

corresponds to motion on closed geodesics (Agop *et al.* 1996). In this case the cosmic dust 'condenses' on the 'crystal' nodes making various elements of the Universe (galactic nuclei, etc.). Thus one can introduce the 'world crystal'.

Afterwards, let us admit that space may be 'identified' with a plane lattice. Analytically, this situation may be achieved by extending the elliptic fuction cn from (6), to the complex plane, and thus one can find (all the underlined symbols denote complex variables)

$$\Psi = a\,\mathrm{cn}(\underline{u};\, k)\,, \tag{7}$$

$$\underline{u} = \frac{K}{a}\,\underline{z}; \quad \underline{z} = x + \mathrm{i}\,y; \quad \frac{K'}{K} = \frac{b}{a}\,, \tag{8}$$

where K, K' are the real and the imaginary period (Bowman 1953) and a, b the lattice constants. Under these circumstances the lattice will be characterised by the potential $\underline{\Omega} = \mathcal{D} \ln \Psi$ and is depicted in Fig. 1 (Morse and Feshbacj 1963).

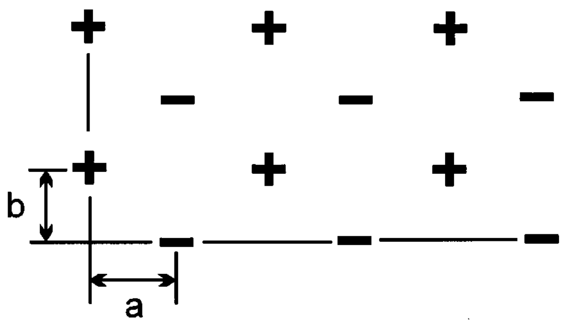

Fig. 1. 2D distribution of sources corresponding to the potential $\underline{\Omega} = \mathcal{D} \ln \Psi$.

Taking into account (7), the following relations may be written:

$$\frac{\nabla\psi}{\psi} = \frac{K(k)}{2\mathrm{i}a}\,\frac{\mathrm{sn}(\underline{u};\, k)\,\mathrm{dn}(\underline{u};\, k)}{\mathrm{cn}(\underline{u}; k)}\,,$$

$$\frac{\nabla\psi^*}{\psi^*} = -\,\frac{K(k)}{2\mathrm{i}a}\,\frac{\mathrm{sn}(\underline{u}^*; k)\,\mathrm{dn}(\underline{u}^*; k)}{\mathrm{cn}(\underline{u}^*; k)}\,. \tag{9}$$

We replace the above expressions in (2) and obtain the background gravitomagnetic vector potential:

$$\begin{aligned} \boldsymbol{A}_{\mathrm{g}} &= -\,\frac{\mathcal{D}K(k)}{8a}\left(\frac{\mathrm{sn}(\underline{u}; k)\,\mathrm{dn}(\underline{u}; k)}{\mathrm{cn}(\underline{u}; k)} + \frac{\mathrm{sn}(\underline{u}^*; k)\,\mathrm{dn}(\underline{u}^*; k)}{\mathrm{cn}(\underline{u}^*; k)}\right) \\ &= -\,\frac{\mathcal{D}K(k)}{4a}\,\mathrm{Re}\left[\frac{\mathrm{sn}(\underline{u}; k)\,\mathrm{dn}(\underline{u}; k)}{\mathrm{cn}(\underline{u}; k}\right]. \end{aligned} \tag{10}$$

Using the relations of transformation for the elliptic functions of complex argument into elliptic functions of real argument (Bowman 1953) and introducing the notation

$$\begin{aligned}
&s = \mathrm{sn}(\alpha, k), \qquad s_1 = \mathrm{sn}(\beta, k'),\\
&c = \mathrm{cn}(\alpha, k), \qquad c_1 = \mathrm{cn}(\beta, k'),\\
&d = \mathrm{dn}(\alpha, k), \qquad d_1 = \mathrm{dn}(\beta, k'),\\
&\alpha = \frac{K}{a}\,x, \qquad \beta = \frac{K}{a}\,y\,,
\end{aligned} \tag{11}$$

where k' is the complementary modulus $(k^2 + k'^2 = 1)$, relation (10) becomes

$$\boldsymbol{A}_{\mathrm{g}} = -\frac{\mathcal{D}K(k)}{4a}\,\frac{scd[c_1^2(d_1^2 + k^2c^2s_1^2) - s_1^2d_1^2(d^2c_1^2 - k^2s^2)]}{(1 - d^2s_1^2)(c\,c_1^2 + s^2d^2s_1^2\,d_1^2)}\,. \tag{12}$$

The two components (along the $0x$ and $0y$ axes) of the background gravitomagnetic vector potential field may be obtained if we allow the degeneracies (Rezlescu *et al.* 1997): (i) $k = 1$, $k' = 0$, $K = \infty$, $K' = \pi/2$ for $A_{\mathrm{g}x}$ and (ii) $k = 0$, $k' = 1$, $K = \pi/2$, $K' = \infty$ for $A_{\mathrm{g}y}$. This results in

$$\begin{aligned}
A_{\mathrm{g}x} &= -\,\frac{\mathcal{D}}{16b}\,\frac{\mathrm{sh}\alpha\,\mathrm{ch}\alpha}{\mathrm{ch}^2\alpha - \sin^2\beta}\,,\\
A_{\mathrm{g}y} &= -\,\frac{\mathcal{D}}{16a}\,\frac{\sin\gamma\,\cos\gamma}{\cos^2\gamma\,\mathrm{ch}^2\delta + \sin^2\gamma\,\mathrm{sh}^2\delta}\,,
\end{aligned} \tag{13}$$

where

$$\alpha = \frac{\pi x}{2b}, \quad \beta = \frac{\pi y}{2a} \quad \gamma = \frac{\pi x}{2a}, \quad \delta = \frac{\pi y}{2b}\,. \tag{14}$$

Since only the background gravitomagnetic field has a direct physical meaning, we derive this parameter using the relation

$$\boldsymbol{B}_{\mathrm{g}} = \nabla \times \boldsymbol{A}_{\mathrm{g}}\,. \tag{15}$$

One gets

$$\begin{aligned}
B_{\mathrm{g}z} = -\,\frac{\pi\mathcal{D}}{64}\Bigg[&\frac{1}{a}\,\frac{2\cos2\gamma(\cos^2\gamma\,\mathrm{ch}^2\delta + \sin^2\gamma\,\mathrm{sh}^2\delta) + \sin^2 2\gamma}{(\cos^2\gamma\,\mathrm{ch}^2\delta + \sin^2\gamma\,\mathrm{sh}^2\delta)^2}\\
&- \frac{1}{b}\,\frac{\mathrm{sh}2\alpha\,\sin2\beta}{(\mathrm{ch}^2\alpha - \sin^2\beta)^2}\Bigg]\,.
\end{aligned} \tag{16}$$

The space structure (background gravitomagnetic field) dependence on the reduced coordinates $x_{\mathrm{r}} = x/a$ and $y_{\mathrm{r}} = y/b$ is plotted in Fig. 2. In our view the ‘space discontinuities’ correspond to the ‘lattice’ nodes. Thus, if such a node is 'identified' with a galaxy, then the 'space discontinuity' is produced by the high spinning velocity of the galactic nucleus.

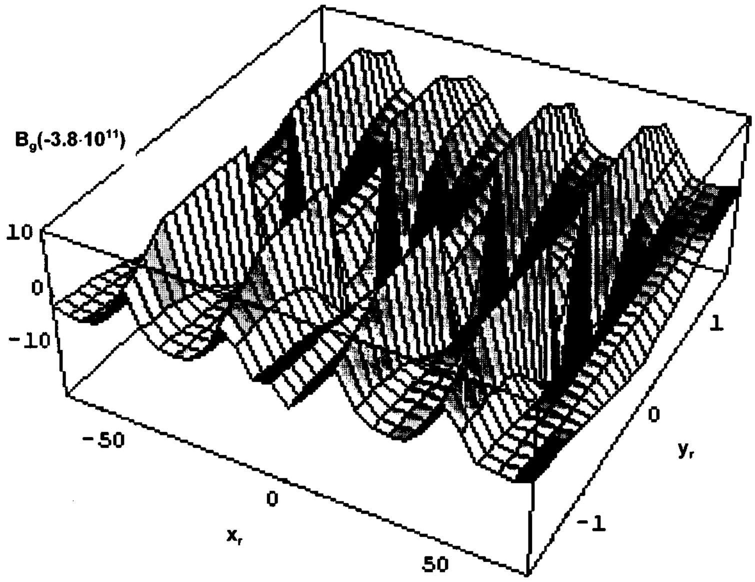

Fig. 2. Space structure (background gravitomagnetic field) dependence on the reduced coordinates $x_{\mathrm{r}} = x/a$ and $y_{\mathrm{r}} = y/b$.

Now, one can calculate the average background gravitomagnetic field [we mediate (16) over a unit cell]:

$$\langle B\rangle = \iint B\,\mathrm{d}S \Big/ \iint \mathrm{d}S = \frac{1}{ab}\iint B\,\mathrm{d}S\,, \tag{17}$$

or explicitly

$$\langle B\rangle = -\frac{\mathcal{D}}{16ab}\left\{\frac{b}{a} - \frac{1}{\pi}\ln\left[\frac{\mathrm{ch}(\pi a/b) + \cos(\pi b/a)}{2\cos^2(\pi b/2a)\,\mathrm{ch}^2(\pi a/2b)}\right]\right\}. \tag{18}$$

A graphic representation of (17) is given in Fig. 3. One can see that for $a = b$ this field diverges. It means that space features a certain anisotropy in terms of a remnant rotation of the Universe. Besides, an anisotropy of radiation which can be related to a rotation of the Universe with angular velocity 10^{-13} rad per year has been observed indirectly by Birch (1982) for 132 strong radio sources. For $b > a$ the graph has discontinuities. The obvious significance of these discontinuities results from Fig. 4 where we have plotted the dependence $\langle B\rangle/|B|$ on the ratio b/a, with $|B| = \mathcal{D}/16S$, $S = a\,b$. One notices the divergences of the background gravitomagnetic field occuring in the 3D representation for $a/b = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$. The presence of such sequences reflects the Cantorian-fractal structure of space–time (El Naschie *et al.* 1995).

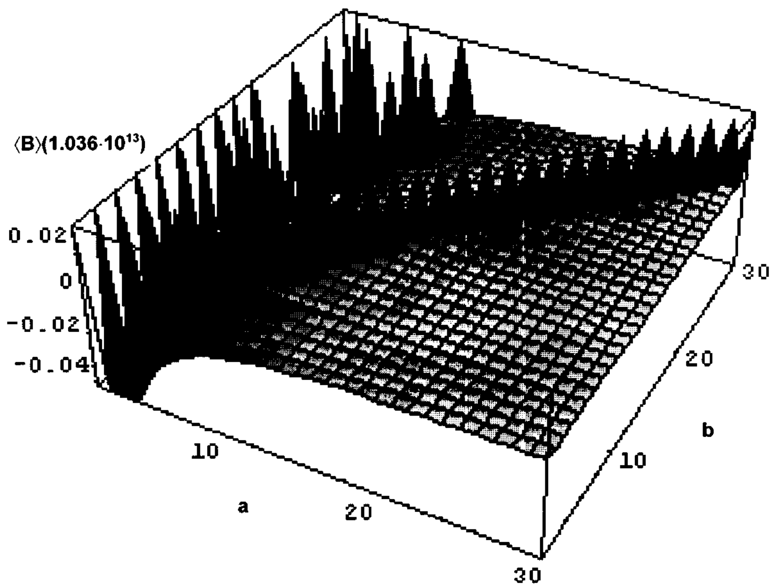

Fig. 3. Average background gravitomagnetic field dependence on the lattice constants a and b.

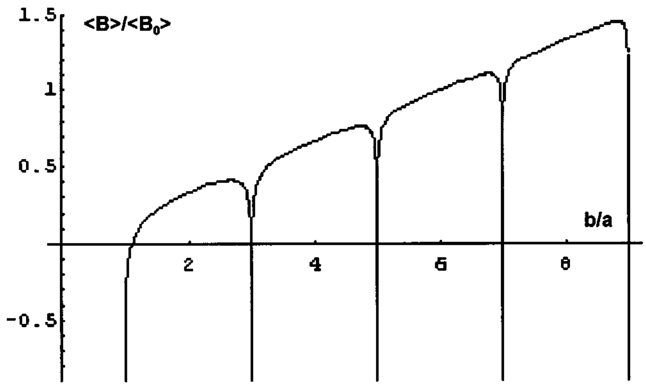

Fig. 4. Dependence of $\langle B\rangle/|B|$ on the ratio b/a.

3. Background Energy

The Cantorian-fractal structure of space–time assumes for the speed $\underline{V}$ of a particle the expression (Nottale 1996)

$$\underline{V} = -2i\mathcal{D}\nabla \ln \Psi \,. \tag{19}$$

By taking into account (7), one obtains

$$\underline{V} = \frac{\mathcal{D}K}{a}\,\frac{\mathrm{sn}\,\underline{u}\,\mathrm{dn}\,\underline{u}}{\mathrm{cn}\,\underline{u}}\,. \tag{20}$$

The speed components along the $0x$ $(\underline{V})_x$ and $0y$ $(\underline{V})_y$ axes respectively are obtained on the following degeneracies (Rezlescu *et al.* 1997): (i) $k = 1$, $k' = 0$, $K = \infty$, $K' = \pi/2$ for $(\underline{V})_x = V_x - \mathrm{i}\, U_x$ and (ii) $k = 0$, $k' = 1$, $K = \pi/2$, $K' = \infty$ for $(\underline{V})_y = V_y - \mathrm{i}\, U_y$.

Using again the relations of transformation for elliptic functions of complex argument into elliptic functions of real argument (Bowman 1953) and the notation (11), relation (20) becomes

$$\underline{V} = -\frac{\mathcal{D}K}{a}\left\{\frac{scd[c_1^2(d_1^2 + k^2c^2s_1^2) - s_1^2d_1^2(d^2c_1^2 - k^2s^2)]}{(1 - d^2s_1^2)(c^2c_1^2 + s^2d^2s_c^2\, d_1^2)}\right.$$
$$\left. + \mathrm{i}\,\frac{s_1\, c_1\, d_1[c^2(d^2c_1^2 - k^2s^2) + s^2d^2(d_1^2 + k^2c^2s_1^2)]}{(1 - d^2s_1^2)(c^2c_1^2 + s^2d^2s_1^2\, d_1^2)}\right\}, \tag{21}$$

from which, taking into account the degeneracies (i) and (ii), we get

$$V_x - \mathrm{i}\, U_x = -\frac{\pi\mathcal{D}}{2b}\left\{\frac{\mathrm{sh}\alpha\,\mathrm{ch}\alpha}{\mathrm{ch}^2\alpha - \sin^2\beta} + \mathrm{i}\,\frac{\sin\beta\cos\beta}{\mathrm{ch}^2\alpha - \sin^2\beta}\right\}, \tag{22}$$

$$V_y - \mathrm{i}\, U_y = \frac{\pi\mathcal{D}}{2a}\left\{\frac{\sin\alpha\cos\alpha}{\cos^2\alpha\,\mathrm{ch}^2\beta + \sin^2\alpha\,\mathrm{sh}^2\beta} + \mathrm{i}\,\frac{\mathrm{sh}\beta\,\mathrm{ch}\beta}{\cos^2\alpha\,\mathrm{ch}^2\beta + \sin^2\alpha\,\mathrm{sh}^2\beta}\right\}. \tag{23}$$

In relations (22) and (23) the expressions for α and β are different because they are chosen in order to eliminate the infinities, having in view the ratio K'/K and the cases (i) and (ii). Consequently, these relations become

$$V_x = -\frac{\pi\mathcal{D}}{2b}\frac{\mathrm{sh}(\pi x/2b)\,\mathrm{ch}(\pi x/2b)}{\mathrm{ch}^2(\pi x/2b) - \sin^2(\pi y/2a)},$$

$$U_x = \frac{\pi\mathcal{D}}{2b}\,\frac{\sin(\pi y/2a)\cos(\pi y/2a)}{\mathrm{ch}^2(\pi x/2b) - \sin^2(\pi y/2a)}, \tag{24}$$

and

$$V_y = -\frac{\pi\mathcal{D}}{2a}\,\frac{\sin(\pi x/2a)\cos(\pi x/2a)}{\cos^2(\pi x/2a)\mathrm{ch}^2(\pi y/2b) + \sin^2(\pi x/2a)\,\mathrm{sh}^2(\pi y/2b)},$$

$$U_y = \frac{\pi\mathcal{D}}{2a}\,\frac{\mathrm{sh}(\pi y/2b)\,\mathrm{ch}(\pi y/2b)}{\cos^2(\pi x/2a)\,\mathrm{ch}^2(\pi y/2b) + \sin^2(\pi y/2a)\,\mathrm{sh}^2(\pi y/2b)}. \tag{25}$$

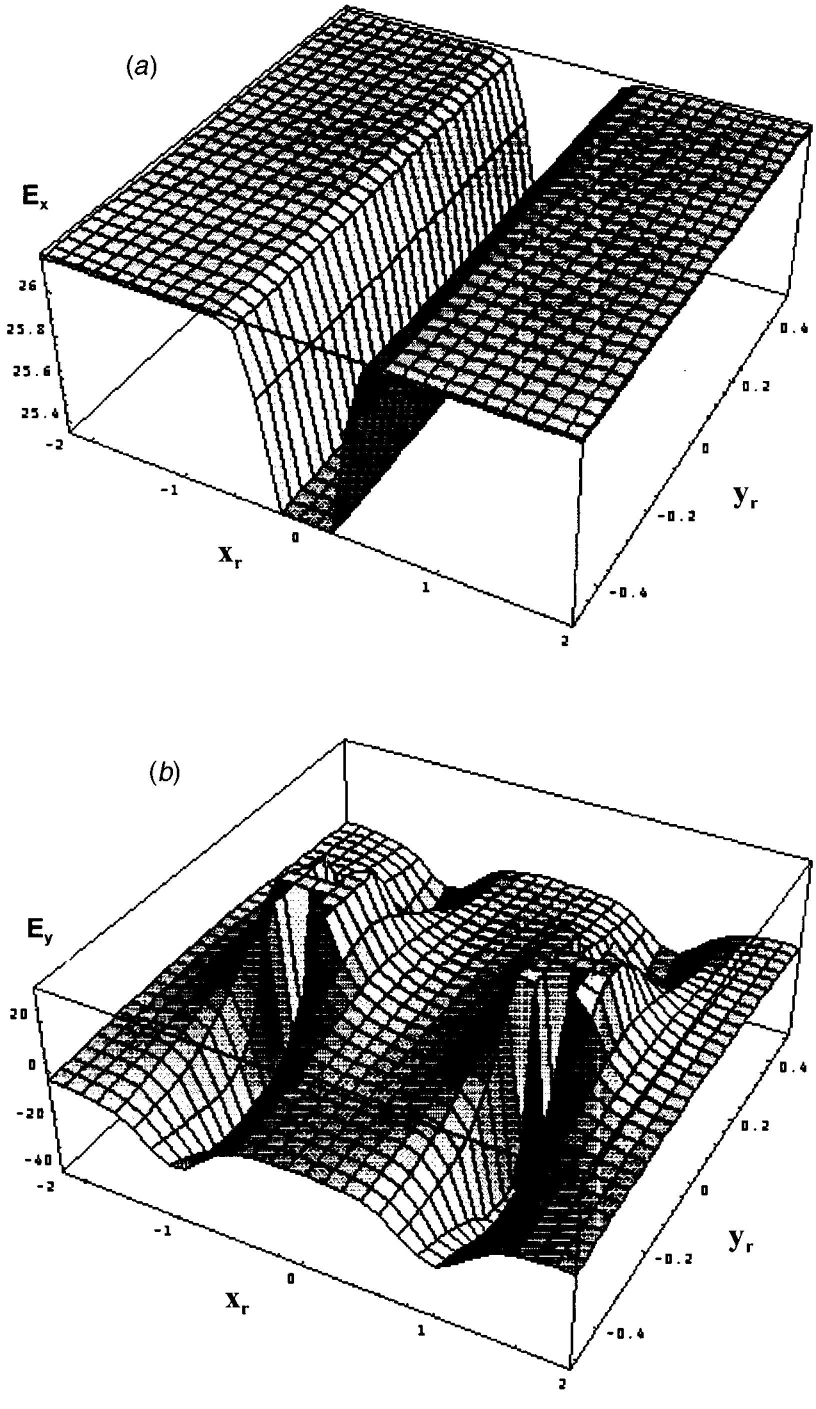

Fig. 5. 3D plot of (*a*) E_x and (*b*) E_y versus the distance along the x and y axes.

At this point, with the help of $\underline{V}$, one builds the energy (Nottale 1996):

$$E = \tfrac{1}{2}m\underline{V}^2 = \tfrac{1}{2}m(\boldsymbol{V} - \mathrm{i}\,\boldsymbol{U})^2 = \tfrac{1}{2}m(\boldsymbol{V}^2 - \boldsymbol{U}^2) - \mathrm{i}\,m\boldsymbol{U}\,\boldsymbol{.}\,\boldsymbol{V}\,. \tag{26}$$

In the following we consider only the real part of this energy, since the imaginary part is related, in our view, to the dissipative processes (see Ciubotaru 1991 concerning the gravitational conductivity). Thus, we recognise the following components of the energy:

$$E_x = \tfrac{1}{2}m(V_x^2 - U_x^2)\,, \tag{27}$$

$$E_y = \tfrac{1}{2}m(V_y^2 - U_y^2)\,, \tag{28}$$

or more explicitly, having in view relations (24) and (25),

$$E_x = \frac{m\pi^2\mathcal{D}^2}{8b^2}\,\frac{\mathrm{sh}^2(\pi x/2b)\,\mathrm{ch}^2(\pi x/2b) - \sin^2(\pi y/2a)\cos^2(\pi y/2a)}{[\mathrm{ch}^2(\pi x/2b) - \sin^2(\pi y/2a)]^2}\,, \tag{29}$$

$$E_y = \frac{m\pi^2\mathcal{D}^2}{8a^2}\,\frac{\sin^2(\pi x/2a)\cos^2(\pi\xi/2a) - \mathrm{sh}^2(\pi y/2b)\,\mathrm{ch}^2(\pi y/2b)}{[\cos^2(\pi x/2a)\,\mathrm{ch}^2(\pi y/2b) + \sin^2(\pi x/2a)\,\mathrm{sh}^2(\pi y/2b)]^2}\,. \tag{30}$$

In Fig. 5 we have plotted the energies versus the distance along the x and y axes. The oscillatory behaviour of E_y as a function of the spatial coordinates x, y is determined by the space 'crystalline' structure (Fig. 5*b*). The plot of E_y is much more interesting since it shows clearly the existence of an energy gap along the $0x$ direction.

In order to express the dependence of the energy only on the system parameters (lattice constants a and b), we have to mediate over a period ($K = a$ and $K' = b$), i.e.

$$\langle E_x\rangle = \int_0^a \int_0^b E_x\,\mathrm{d}x\,\mathrm{d}y/ab\,, \tag{31}$$

$$\langle E_y\rangle = \int_0^a \int_0^b E_y\,\mathrm{d}x\,\mathrm{d}y/ab\,. \tag{32}$$

Substituting (29) and (30) in (31) and (32) respectively, we get

$$\langle E_x\rangle = \frac{m\pi^2\mathcal{D}^2}{8b^2}\left\{1 - \frac{4}{\pi^2}\arctan\left[\frac{\mathrm{e}^{\pi a/b} - 1}{\mathrm{e}^{\pi a/b} + 1}\tan\left(\frac{\pi b}{2a}\right)\right]\right\}, \tag{33}$$

$$\langle E_y\rangle = -\frac{m\pi^2\mathcal{D}^2}{8a^2}\left(1 - \frac{2}{\pi}\right), \tag{34}$$

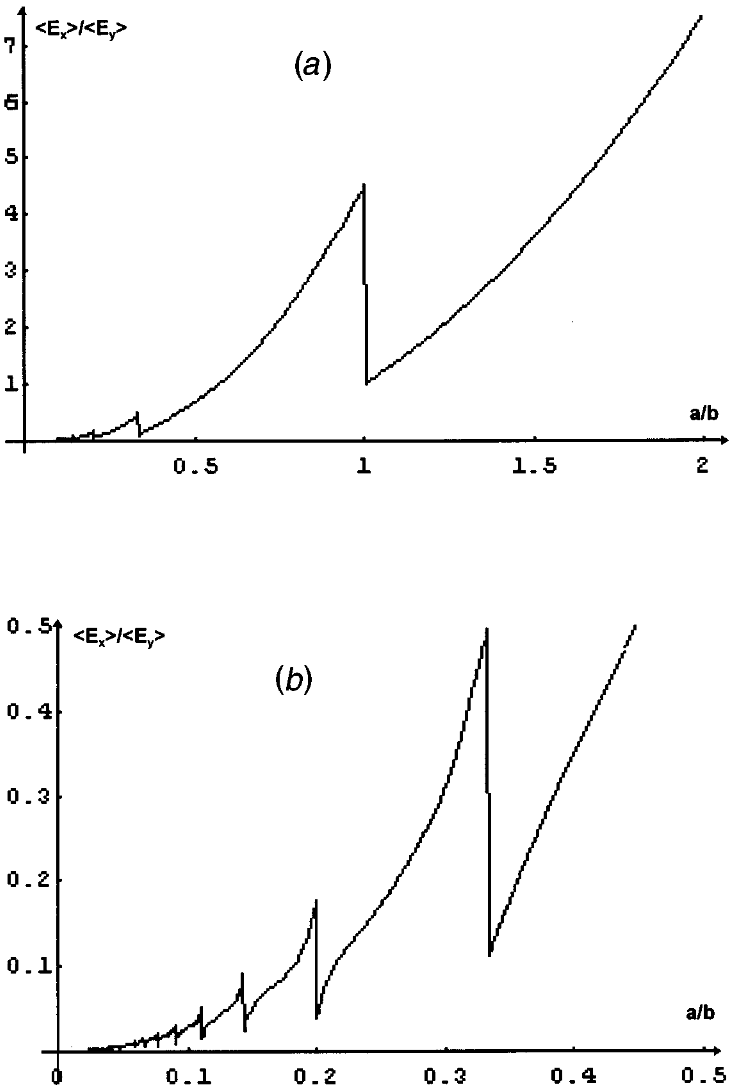

Fig. 6. (a) Dependence of the ratio $\langle E_x\rangle/\langle E_y\rangle$ on a/b; (b) the same plot for $a/b<0{\cdot}5$.

and the ratio $\langle E_x\rangle/\langle E_y\rangle$ (making an abstraction of the minus sign) is

$$\frac{\langle E_x\rangle}{\langle E_y\rangle}=\frac{a^2}{b^2}\frac{1-\dfrac{4}{\pi^2}\arctan\left[\dfrac{\mathrm{e}^{\pi a/b}-1}{\mathrm{e}^{\pi a/b}+1}\tan\left(\dfrac{\pi b}{a}\right)\right]}{1-\dfrac{2}{\pi}}\,. \tag{35}$$

In Fig. 6a we plot the dependence of the ratio $\langle E_x\rangle/\langle E_y\rangle$ on the ratio a/b and in Fig. 6b the same dependence for $a/b<0{\cdot}5$. One can notice a prominent discontinuity for $a = b$, and other discontinuities occurring for $a/b = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$. We understand this result as being a consequence of the Cantorian-fractal structure of space–time. [It is in agreement with the results obtained by Agop *et al.* (1996).]

4. Conclusions

The main conclusions may be summarised as follows:

(i) The equivalence between the Cantonian-fractal structure of space–time and the composite fermions mechanism (Rezlescu *et al.* 1997) advances an interesting perspective on gravitation: the strong repulsive interaction between the galactic nuclei responsible for the Hubble effect (Agop *et al.* 1995) may be replaced by a weak 'gauge' interaction between the 'gravitational anyons' (see Fig. 7) (gravitoholons and gravitospinons). Besides, the available experimental evidence also appears to support the idea that S0-type galaxies and spirals possess similar amounts of spin (Bertola and Capaccioli 1978), suggesting that all spiral galaxies (treated as equivalent point particles) may be spin-$\frac{1}{2}$ fermions, in the same sense as elementary particles [i.e. the spin of the Milky Way and Andromeda are consistent with the assignment $S_{\mathrm{g}} = \hbar_{\mathrm{g}}/2$ (Harrison 1973) for $\hbar_{\mathrm{g}} \sim 8 \times 10^{67}$ J s]. Consequently, a new type of gravitational superconductivity with mass and fractional spin, may be 'generated' (see Samuel 1993), which is completely different from 'classical gravitational superconductivity' (Agop *et al.* 1996).

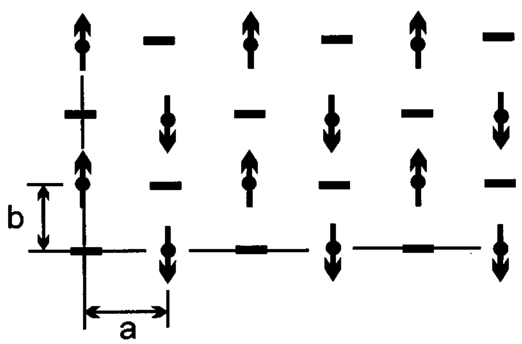

Fig. 7. Strong repulsive interaction between the galactic nuclei (horizontal bars) may be replaced by a weak 'gauge' interaction between the 'gravitational anyons' (up and down arrows).

If such a mechanism works for gravitation, one expects that the average background gravitomagnetic field of our galaxy, with the gas of 'gravitational anyons' moving in this field, to be close to the value obtained for the Solar System. Indeed for M (mass of Milky Way) $\sim 10^{41}$ kg, R (radius of Milky Way disk) $\sim 0{\cdot}5\times 10^{21}$ m and $\hbar_{\mathrm{g}} \sim 8\times 10^{67}$ J s (Agop *et al.* 1997), the average background gravitomagnetic field, $B_{0\mathrm{G}}$ has the value

$$B_{0\mathrm{G}} \sim \frac{\hbar_{\mathrm{g}}}{MS} = \frac{\hbar_{\mathrm{g}}}{\pi M R^2} \sim 10^{-15}\ \mathrm{s}^{-1}\,. \tag{36}$$

On a planetary scale, for R_S (Sun's radius) $\sim 7\times10^8$ m, V_S (equatorial rotation speed) $\sim 2\times10^3\ \mathrm{m\,s^{-1}}$ and R (radius of the Solar System) $\sim 10^{13}$ m, the average background gravitomagnetic field is

$$B_{0S} \sim \frac{D}{S} = \frac{R_S V_S}{\pi R^2} \sim 10^{-15}\ \mathrm{s^{-1}}\,. \tag{37}$$

(ii) Taking into account the results from Section 3, the background energy of our galaxy is

$$E \sim \frac{\hbar_g^2\,\pi^2}{8M\pi R^2} \sim 10^{51}\ \mathrm{J}\,, \tag{38}$$

and the actual power is

$$P = E/\tau \sim 10^{33}\ \mathrm{W}\,, \tag{39}$$

where the approximations $b^2 \sim \pi R^2$ and $\tau \sim 10^{10}$ yr are allowed. The value is close to the experimental observations (Ureche 1987).

The presence of discontinuities in the expression of the 'background energy' due to the Cantorian-fractal structure of space–time may be evidenced by the lack of some sequences from the radiation spectrum emitted by the galaxy.

(iii) The dual (wave–particle) properties of matter are dictated by space (i.e. by the background gravitomagnetic field). Thus, for cosmic dust free particles with a 'characteristic diffusion length' $\lambda_D = \mathcal{D}/2a \gg$ lattice constant $\lambda = \mathcal{D}\,(\beta/\alpha)^{1/2}$, the wave character prevails. In this case (3) reduces to the 'wave'

$$\Psi = a\sin[\Omega(x - x_0)]\,. \tag{40}$$

For cosmic dust free particles with a 'characteristic diffusion length' $\lambda_D \ll \lambda$, the particle character prevails. In this situation (3) reduces to the 'kink'

$$\Psi = a\,\mathrm{th}[\Omega(x - x_0)]\,. \tag{41}$$

Acknowledgments

This paper was substantially improved by a referee's observations, to whom the authors are indebted.

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Manuscript received 2 April, accepted 31 July 1997