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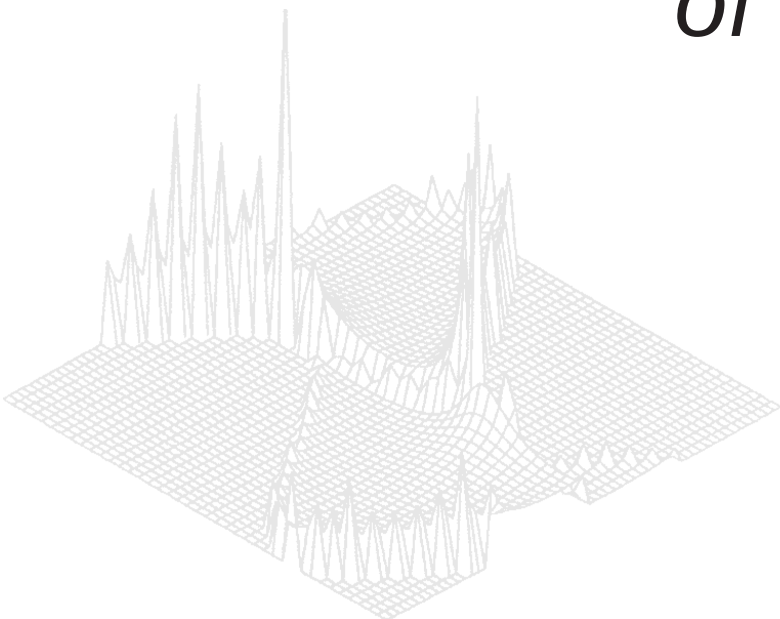

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Experimental and Theoretical Investigations of Electron Dynamics in a Semiconductor Sinai Billiard

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Abstract

We describe a surface gate-defined mesoscopic semiconductor billiard with a square geometry which we can evolve continuously to a Sinai geometry by adjusting the bias on a circular gate located at the centre of the square. We concentrate on clusters of magneto-conductance structure formed on two magnetic field scales, which emerge during the transition from the square to the Sinai geometry. This change in shape is accompanied by a transition from non-chaotic to chaotic electron dynamics, which has been a topic of considerable interest for many years. The observed structure is due to quantum chaotic processes induced by the presence of the circle. Our experimental results agree with classical dynamics simulations which suggest the two field scales are determined by two families of electron trajectories which sample the full geometry and corner sub-geometry. Finally, we report the observation of striking similarities in the structures observed on the two different field scales. This indicates the emergence of self-similarity with the introduction of the circle to the Sinai billiard geometry.

1. Introduction

Since the discovery of chaotic behaviour in physical systems (see for example Ott 1994), it has been of considerable interest to manufacture a system where a transition from non-chaotic to chaotic behaviour can be induced by an easily controllable experimental parameter. Investigations of the classical dynamics of particles interacting with combinations of straight and curved walls were carried out by Sinai in 1970 (Sinai 1970). In particular, he demonstrated that certain combinations lead to particle trajectories becoming chaotic, whilst for other combinations, trajectories are non-chaotic. However, until the development of mesoscopic semiconductor systems, experimental analogues of Sinai's work have only been possible in microwave cavities (Doron *et al*. 1990). Advances in semiconductor fabrication have led to the ability to deposit intricate gate patterns on the surface of AlGaAs/GaAs heterostructures. Under negative bias, the regions under the gate become depleted, allowing sub-micron patterns such as cavities to be formed in the two dimensional electron gas (2DEG) located at the AlGaAs/GaAs interface. In addition, molecular beam epitaxy allows such heterostructures to be created with an electron mean free path much greater than

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1 μm. Combining these two fabrication and growth techniques we create a system where the trajectories of electrons injected into the cavity are primarily determined by the cavity geometry rather than material-related scattering processes (i.e. ballistic transport) (Marcus *et al.* 1992; Chang *et al.* 1994). The gate pattern used in our experiments consists of a square cavity with a circle at the centre, as shown in Figs 1*a* and 1*b*. Billiards with this geometry are known as Sinai billiards. By independently adjusting the voltage on the inner, circular gate it is possible to introduce and remove the circular feature in the billiard geometry allowing not only for investigation of the concepts proposed by Sinai, but the transition between chaotic and non-chaotic dynamics in a single device. Such

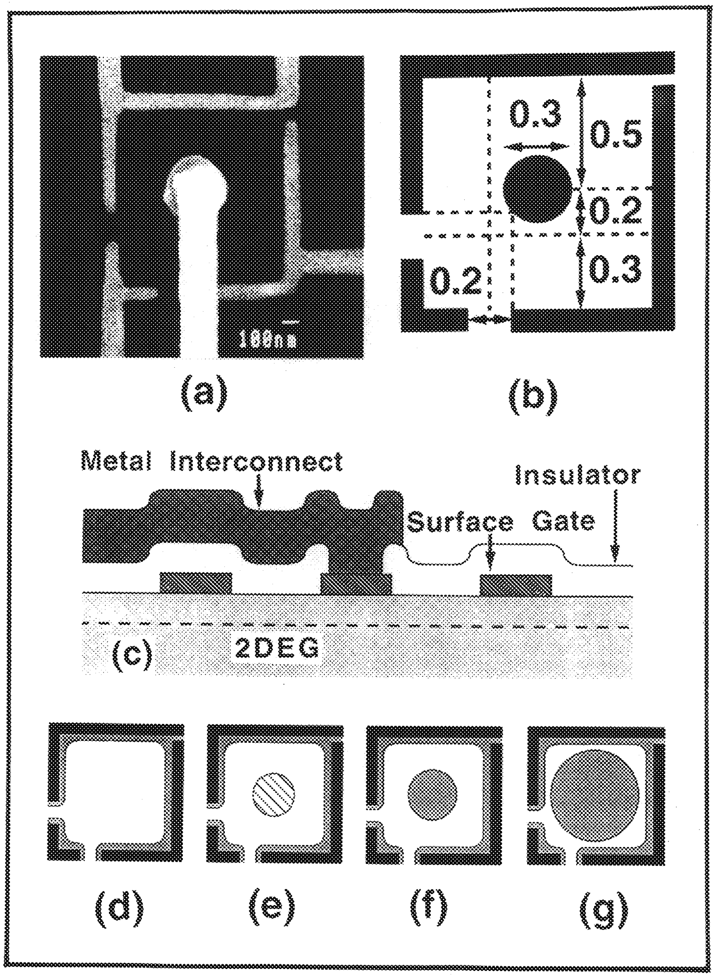

Fig. 1. (*a*) Scanning electron micrograph of the Sinai billiard gate pattern. (*b*) Schematic diagram of gate pattern (all dimensions in μm). (*c*) Schematic cross section of the bridging technique used to provide electrical connection to the Sinai diffuser. Parts (*d*) to (*g*) demonstrate the changing diffuser radius as the transition from a square (*d*) at $V_{\rm I} = +0{\cdot}7$ V to a Sinai billiard is made. At (*e*) $V_{\rm I} = 0$ V and the diffuser radius is close to the lithographic radius of the circular gate. Parts (*f*) and (*g*) are at more negative $V_{\rm I}$ values where the depletion region increases in size.

a device acts as a chaotic transistor. Whereas a standard transitor modulates the electron flow somewhat like a tap, our device fine tunes the electron dynamics, switching between the two (chaotic/non-chaotic) dynamical regimes. In this paper, we examine clusters of magneto-conductance structure obtained from the billiard as the biases applied to both the inner circular and outer square gates are changed. The clusters are observed at millikelvin temperatures in order to preserve phase coherence of the electron waves and the data serve as a probe of quantum chaos—the quantum behaviour of a classically chaotic system.

2. Classical Simulations of the Electron Dynamics in the Sinai Billiard

Traditional descriptions of the depletion patterns created by the surface gates have considered such a confining potential to be an infinite square well in nature. Under such a 'hard-walled' description the circle acts as a 'Sinai diffuser' which leads to an exponential divergence of electron trajectories with small differences in initial condition—a characteristic of chaotic behaviour (Ott 1994; Sinai 1970). This is in contrast to the non-chaotic dynamics of trajectories shaped by straight walls where such exponential divergence does not occur—in the empty square, for example. To date a number of geometries topologically equivalent to a Sinai billiard have been investigated using mesoscopic semiconductor devices (Clarke *et al*. 1995; Budantsev *et al*. 1996; Taylor *et al*. 1997*a*). However, the billiard geometry we have used is the most flexible because it is the only device to date where the Sinai diffuser can be removed. To experimentally sample the dynamics one needs an 'open' system, hence our device has entrance and exit ports which inject and collect electrons allowing us to measure the transmission of trajectories through the billiard. The configuration and lithographic dimensions of our device are shown in Figs 1*a* and 1*b*. Simulations where 10^6 classical trajectories were injected through the entrance port with an angular spread of $\pm 30°$ about the port axis were carried out for a number of Sinai diffuser radii. The results are displayed in Fig. 2 as plots of the number of trajectories $N(L)$ versus their length L. For the empty square ($R = 0\ \mu$m) the length distribution is determined by the escape of trajectories from a purely non-chaotic system and is bounded by a power law ($N \propto L^{-3}$) (Jensen 1991). The introduction of a very small ($R = 0\cdot 01\ \mu$m) Sinai diffuser leads to a radical change in the length distribution which clearly begins to condense onto an exponential distribution associated with chaotic behaviour (Jensen 1991). This is due to some of the trajectories managing to 'find' the diffuser. As R is increased further, more and more trajectories strike the diffuser, and the exponential distribution characteristic of chaotic scattering becomes considerably more dominant. At $R = 0\cdot 15\ \mu$m, points remaining above the exponential are due to trajectories injected slightly off the port axis towards the side wall, which bounce alternately between the top and bottom walls before leaving through one of the ports. These trajectories do not strike the diffuser and hence do not condense onto the exponential distribution, remaining instead on the power law distribution mentioned earlier. By $R = 0\cdot 3\ \mu$m this path is blocked and the last power law remnant disappears. In the radius range $0\cdot 3$–$0\cdot 4\ \mu$m the emergence of a second exponential distribution occurs for small L. This second exponential is due to trajectories restricted to a sub-geometry defined by the bottom left corner of the billiard shown in Fig. 1. This is confirmed at $R = 0\cdot 45\ \mu$m where the diffuser is sufficiently large that it touches the square walls, pinching off the channel and preventing trajectories from circulating around the diffuser. Only the corner sub-geometry still supports trajectories in this pinched off state. Here we see that only the distribution associated earlier with the sub-geometry remains. In Sections 3 and 4 we investigate electron transmission through a semiconductor Sinai billiard for diffuser radii where the existence of both families of chaotic trajectories is predicted.

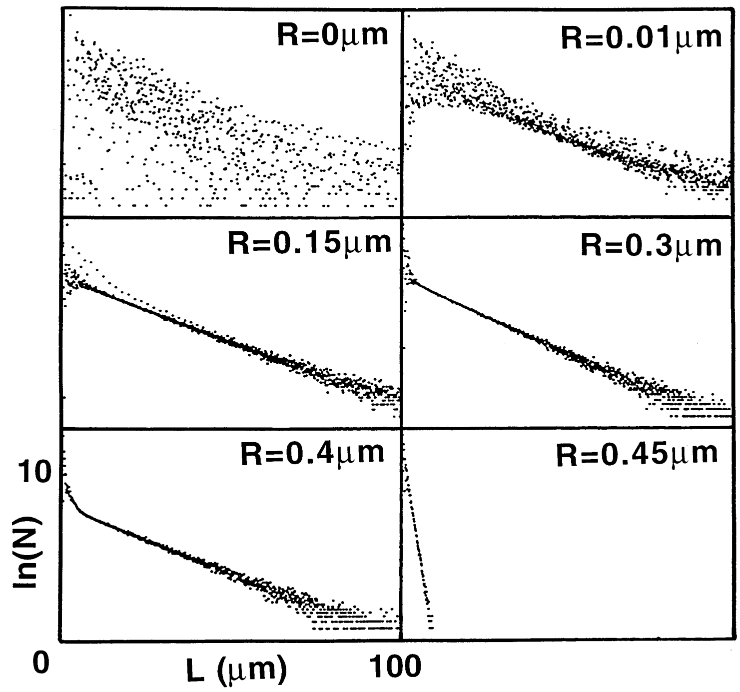

Fig. 2. Classical simulation length distributions $\ln[N(L)]$ versus L for six values of the diffuser radius R.

3. Semiconductor Sinai Billiards

Details of the device and its fabrication are discussed elsewhere (Taylor 1994; Taylor *et al.* 1997*a*, 1997*b*). As shown in Fig. 1 we have an AlGaAs/GaAs heterostructure with a 2DEG 161 nm below the surface onto which are deposited metal gates using an EBL/liftoff technique (Taylor 1994). The significant new technology in this device is the bridging interconnect which allows the circle to be biased independently of the square, as shown in Fig. 1*c*. Previous billiards which were topologically equivalent to the Sinai geometry lacked the ability to accurately and independently control the diffuser's presence and its curvature in a single device. The square formed by the outer gates is 1 μm wide which is significantly less than the electronic mean free path of 25 μm ensuring that transport in the cavity is indeed ballistic. The 50 nm gap in the upper right corner is depleted under typical outer gate voltages. At the inner gate voltage $V_{\rm I} = +0\cdot7$ V, the presence of the circular gate is minimised (Taylor *et al.* 1997*a*; Taylor 1994). As the positive bias is reduced the 2DEG becomes depleted in the region below the gate, with full depletion occurring by $V_{\mathrm{I}} = 0$ V. The radius of the central depletion region continues to increase as V_{I} becomes more negative (as demonstrated in Figs 1*d*–1*g*). At the maximum bias of $-3\cdot 0$ V, we determine the central radius to be $0\cdot 37\ \mu$m. This value is obtained from magneto-conductance oscillations assuming a flux period of $h/2e$. This radius value is consistent with other characterisations showing R to increase linearly by $\Delta R \approx +40$ nm for $\Delta V_{\mathrm{I}} \approx -0\cdot 5$ V above its lithographic radius of $0\cdot 15\ \mu$m at $V_{\mathrm{I}} = 0$ V (Taylor *et al.* 1997*a*, 1997*b*). This leads to a value of $R = 0\cdot 39\ \mu$m for $V_{\mathrm{I}} = -3$ V. A comparison of these calculations with those obtained from the potential profile generated by the self-consistent solution of the Schrödinger and Poisson equations will be presented elsewhere (Fromhold *et al.* 1997). Leakage of current from the gate to the 2DEG prevents the application of more negative biases. Measurements of the low field magneto-conductance provide a 'magneto-fingerprint' of electron trajectories through the billiard. At low temperatures (lattice temperature of 30 mK), electrons maintain phase coherence for times significant compared to the traversal time of the geometry. Quantum interference effects can be assessed semiclassically by monitoring the phase accumulated by electrons as they traverse classical trajectories in the presence of a magnetic vector potential (Marcus *et al.* 1992; Chang *et al.* 1994; Aharonov and Bohm 1959). Such an approach is successful in explaining the observed magneto-conductance structure.

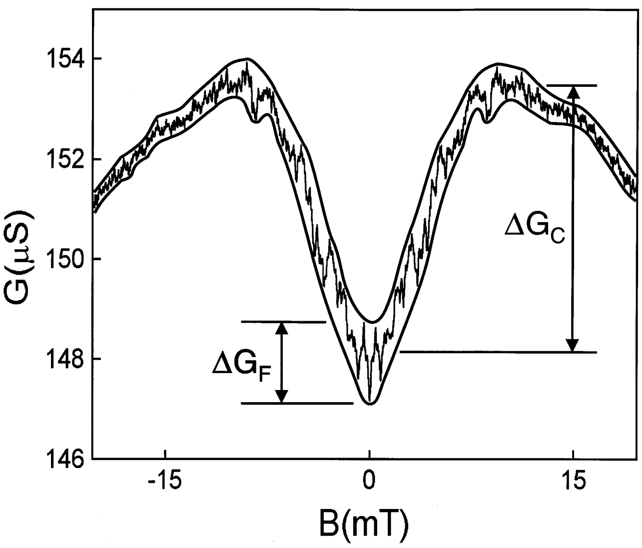

Fig. 3. Fine and coarse scale magneto-conductance structure for $V_{\mathrm{O}} = -0\cdot 51$ V and $V_{\mathrm{I}} = -2\cdot 8$ V.

As the diffuser is switched on and its radius increased, we observe the emergence of clusters of magneto-conductance structure forming around two distinct field scales. The characteristic field scale ΔB_{C} of the coarse structure is $\sim$20 times larger than the characteristic field scale ΔB_{F} of the fine structure. Structure on both scales appears as a central trough (of characteristic height ΔG_{C} and ΔG_{F} for the coarse and fine structure respectively) with quasi-periodic structure, reminiscent of Al'tshuler–Aronov–Spivak (AAS) oscillations in disordered conductors (Al'tshuler *et al.* 1981). As shown in Fig. 3 for $R \approx 0\cdot37$ μm, the fine scale structure is superimposed on the central coarse-scale 'host' trough. The amplitude of the fine structure is visibly damped to either side of the central coarse trough as demonstrated by the envelope displayed in Fig. 3. Assuming that the Aharonov–Bohm (1959) flux–area relation holds, we expect

$$\frac{\Delta B_{\mathrm{F}}}{\Delta B_{\mathrm{C}}} = \frac{A_{\mathrm{C}}}{A_{\mathrm{F}}}\,, \tag{1}$$

where A_{C} and A_{F} are the characteristic areas enclosed by the coarse and fine trajectories respectively. The resulting $A_{\mathrm{C}}/A_{\mathrm{F}}$ is consistent with the coarse structure being associated with electron trajectories restricted to the corner sub-geometry and fine structure with trajectories sampling the full geometry (i.e. circulating around the diffuser). This picture is also confirmed by the fine structure evolution observed in Fig. 4. As V_{I} is increased from $+0\cdot7$ V (top trace) to $-3\cdot1$ V (bottom trace) in Fig. 4*a*, the geometry changes from that of an empty square to a Sinai billiard with $R \approx 0\cdot37$ μm. The fine structure evolves markedly with increasing diffuser radius. Higher inner gate voltages cannot be used to achieve pinch-off due to problems with electrons leaking from the gates into the 2DEG. Instead, the outer gate voltage is increased, extending the region depleted below it inwards (see inset to Fig. 4*b*). The undepleted region between

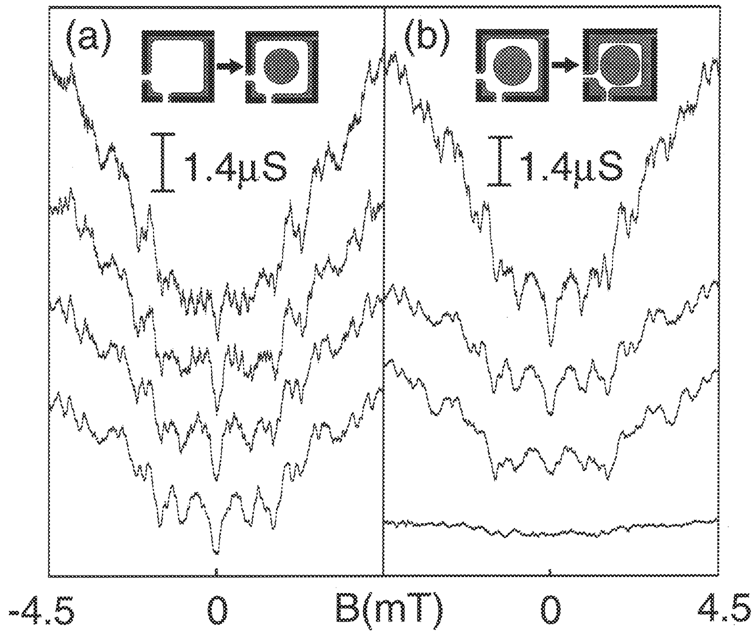

Fig. 4. (*a*) Fine structure for $V_{\mathrm{O}} = -0\cdot51$ V. Evolution from square to Sinai: V_{I} and $G(B=0)$ values are (from top to bottom) ($+0\cdot7$ V, 265 μS), ($-0\cdot5$ V, 209 μS), ($-1\cdot7$ V, 173 μS) and ($-3\cdot1$ V, 151 μS). (*b*) Fine structure for $V_{\mathrm{I}} = -3\cdot1$ V. Evolution from Sinai geometry to pinch-off: V_{O} and $G(B=0)$ values are (from top to bottom) ($-0\cdot50$ V, 231 μS), ($-0\cdot51$ V, 151 μS), ($-0\cdot52$ V, 108 μS) and ($-0\cdot55$ V, 26 μS).

the square and circular gates contracts under this process until the depletion regions merge and the conducting channel is pinched off. The result of this is shown in Fig. 4*b* where the fine structure is removed as the outer gate voltage is taken from $-0\cdot 50$ V (top trace) to $-0\cdot 54$ V (bottom trace) to pinch off the channel. This is consistent with the fine structure being associated with trajectories which circulate the diffuser, as these trajectories will be blocked in the pinched off state. It should be noted that this is not due to changing port widths as the outer gate voltage is increased, indeed this should have the opposite effect, since increased trough amplitude is expected with narrowing ports under a semiclassical approach (Baranger and Mello 1995). Clearly the presence of the Sinai diffuser plays a major role in the electron dynamics through the device. A transition from non-chaotic to chaotic dynamics occurs as the diffuser is activated, followed by an interplay of two families of trajectories, generating magneto-conductance structure on two distinct magnetic field scales, as the diffuser radius is increased.

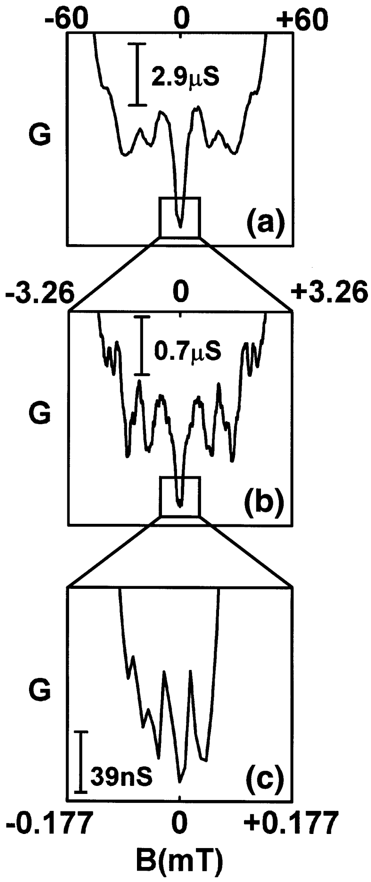

Fig. 5. Magneto-conductance structure at three different magnetic field scales for $V_{\rm O} = -0\cdot 512$ V and $V_{\rm I} = -2\cdot 7$ V demonstrating similarity between scales: (*a*) coarse scale, (*b*) fine scale and (*c*) ultrafine scale.

4. Self-similarity in the Sinai Billiard

The experimental results suggest that the origin of the coarse and fine field scales of magneto-conductance structure may be more fundamental. Fig. 5*a* displays structure observed on the coarse scale for an outer gate voltage $V_O = -0 \cdot 51$ V and inner gate voltage $V_I = -2 \cdot 7$ V. A closer look at the central trough reveals fine structure, as mentioned earlier, with a characteristic field scale ~20 times smaller than that of the background coarse structure (Fig. 5*b*). Zooming in a second time, at a much finer magnetic field scale, ultrafine structure is observed residing on the fine structure trough (Fig. 5*c*). Of most interest here is the similarity observed between the structure obtained on the three different magnetic field scales, in particular between the coarse and fine levels. The ultrafine structure is quite close to the magnetic field resolution limit of the measuring electronics and although the similarity between it and the fine structure is not as striking as the coarse to fine, it is apparent nonetheless. Fig. 6 provides another example of the similarity between fine and ultrafine levels for $V_O = -0 \cdot 524$ V and $V_I = -1 \cdot 5$ V. Such 'exact' self-similarity is a characteristic of uniform fractal behaviour (Mandelbrot 1982) which is generally associated with chaotic systems. Hence, it is not surprising that the self-similarity is only observed whilst the Sinai diffuser is activated and the associated electron dynamics are chaotic. Indeed for the empty square $V_I = +0 \cdot 7$ V, which is a non-chaotic system, exact self-similarity is not observed on any field scale.

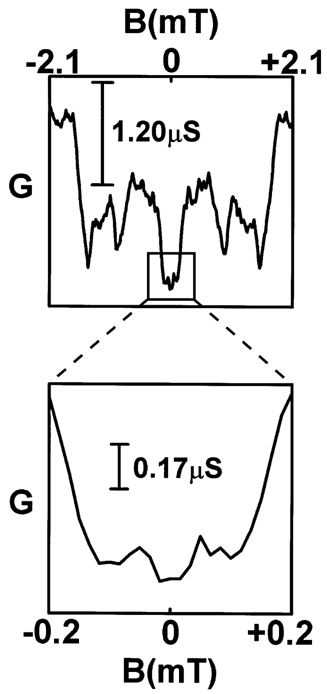

Fig. 6. Fine and ultrafine magneto-conductance structure for $V_O = -0 \cdot 524$ V and $V_I = -1 \cdot 5$ V.

5. Conclusions

Physical realisation of a Sinai billiard using a surface gated AlGaAs/GaAs heterostructure involving new device technologies has resulted in a system where the nature of electron dynamics (chaotic/non-chaotic) can be easily changed via a simple gate voltage adjustment which controls the presence and radius of a circular Sinai diffuser in the billiard geometry. Classical dynamics simulations have demonstrated a striking change in the trajectory length distribution, associated with a change in electron dynamics as the Sinai diffuser is activated, and the emergence of two distribution components at higher diffuser radius. One of these distributions is associated with coarse scale magneto-conductance structure due to electron trajectories confined to the corner sub-geometry, with the other associated with fine scale structure due to trajectories circulating the Sinai diffuser. Experiments on the billiard have demonstrated the evolution of structure on two field scales (coarse and fine) as the diffuser is introduced, with a striking similarity apparent between them. The observation of self-similarity in the magneto-conductance structure of the billiard is of considerable interest to the further understanding of uniform fractal behaviour and its relation to particle dynamics in such a system. The fine structure is removed as the channel between the diffuser and the square wall is pinched off, consistent with the association between fine structure and trajectories sampling the full billiard geometry. In turn, coarse structure is associated with the corner sub-geometry.

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