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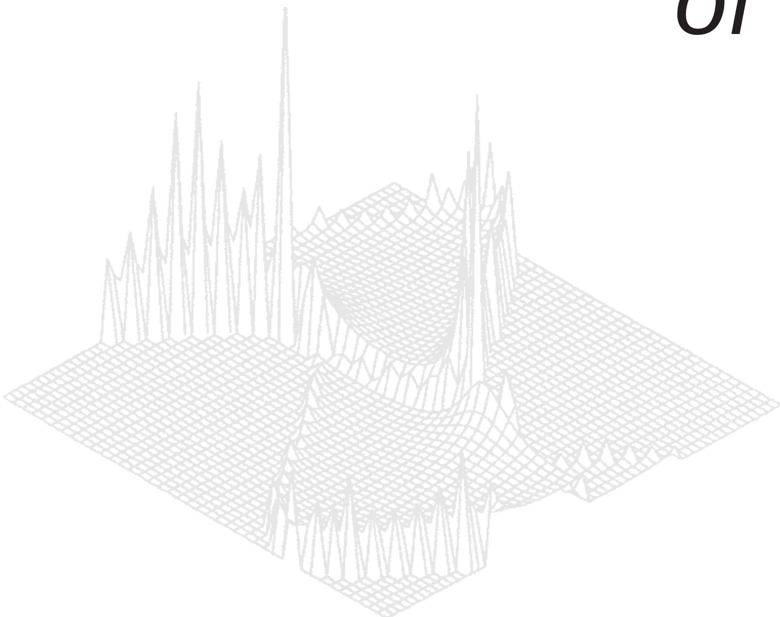

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Wigner's Variant of Bell's Inequality

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Abstract

It is shown that Wigner's variant of Bell's inequality does not exclude all local hidden variable explanations of the Einstein–Podolsky–Rosen problem.

1. Introduction

In the foundation of quantum mechanics, Bell's (1964) inequalities play an important role. With these inequalities it became plausible that quantum mechanics did not need to be supplemented with extra hidden variables, in order to restore locality and causality to the theory, such as was argued by Einstein *et al*. (1935).

Bell's inequalities are related to Bohm's reformulation of Einstein's problem. Bohm and Aharonov (1957) reformulated Einstein's arguments into a 'paradoxical' correlation between spin states of spatially separated particles. Their argument went as follows: Suppose $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the spin vectors of two particles prepared somehow in the singlet state. At a later stage, the particles are separated and their respective spins are measured with Stern–Gerlach magnets. If $\vec{a}$ is a normalised three-dimensional parameter vector of the magnet used to measure the spin of particle-1 and the measurement of $\vec{\sigma}_1.\vec{a}$ yields $+1$, then according to quantum mechanics, for particle-2 the result of measuring $\vec{\sigma}_2.\vec{a}$ will be -1 and vice versa. In the Bell inequalities, the parameter vector for measuring the spin of particle-2 may be different from the parameter vector for particle-1. In view of arguments on the inconsistency of local hidden variable models put forward by Greenberger *et al*. (1989), Hardy (1993) and Jordan (1994), without making use of the Bell inequalities, it is interesting to take a closer look at these inequalities.

The inequalities are a well studied subject to which many authors have contributed. In the present paper only a few of these contributions are mentioned and there is no intention of presenting a complete list of all relevant work on this subject. This incompleteness is also unavoidable because the topic here is Wigner's version of Bell's inequality, one which should not be burdened with, for instance, discussions on the detector efficiency in an actual experiment. In the present paper we concentrate on the algebraic-probabilistic side of the problem.

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For recent studies in quantum inequalities the reader is, for instance, referred to papers presented at a congress in honour of J. A. Wheeler (Greenberger and Zeilinger 1995).

In the present paper the probability of finding equal spin in both wings of the experiment is studied. It is demonstrated that the proposed model does not contradict standard quantum mechanics and that it, at the same time, can violate Wigner's version of Bell's inequality. The starting point of the analysis is the description of the equal spin probability in terms of a hidden state. This enables us to redefine the relation between the probability of equal spin in both wings and the associated local hidden variable weighted probability integrals. Instead of the linear relationship between the probability and the local hidden variable weighted probability integrals, such as proposed by Wigner, nonlinear relationships arise. The question then occurs whether or not such equations represent a local model. This question is referred back to the possibility of creating, in an analogous way, Wigner's local model from the postulated hidden states and the weighted probability integrals.

2. The Model

In a series of lectures Wigner (1983) derived the Bell inequality as follows. The spin part of the two particle state vector in the singlet state is

$$\psi = \sigma_+(1)\sigma_-(2) - \sigma_-(2)\sigma_+(2)\,. \tag{1}$$

Here $\sigma_+(1)$ indicates a positive spin for particle-1 in the z direction, etc. According to quantum mechanics, a correlation exists between measurements of spin components along the directions $\vec{e}_1$ and $\vec{e}_2$, which enclose the angle $\theta_{1,2}$.

The probability of equal spins reads

$$P_{+,+}(\theta_{1,2}) = \tfrac{1}{2}\sin^2(\theta_{1,2}/2) = P_{-,-}(\theta_{1,2})\,, \tag{2}$$

while for opposite spins

$$P_{+,-}(\theta_{1,2}) = \tfrac{1}{2}\cos^2(\theta_{1,2}/2) = P_{-,+}(\theta_{1,2})\,. \tag{3}$$

The possibility that local hidden variables (LHVs) uniquely determine the spins was contradicted by Wigner as follows: First let us examine spin components along the directions $\vec{e}_1$, $\vec{e}_2$ and $\vec{e}_3$. Secondly, let $(++-;\ --+)$, for instance, denote the LHV probability-weighted integral, whereby for $(++-;\ --+)$ we may deduce for particle-1 (the particle in the left-wing of the experiment) positive spin components (spin-up = +) along $\vec{e}_1$ and $\vec{e}_2$ and negative (spin-down = $-$) along $\vec{e}_3$, while for particle-2 (the particle in the right-wing of the experiment), the mirror image of particle-1 is obtained. In addition, because of the singlet state, integrals with equal spin components in the same direction vanish, for instance $(++-;\ +-+) = 0$.

From the previous, a relation between quantum and LHV probabilities is obtained for positive spins along $\vec{e}_k$, $k = 1, 2, 3$, enclosing the angles $\theta_{k,m}$ $k < m = 1, 2, 3$. Hence, according to Wigner (1983),

$$\begin{aligned}
\tfrac{1}{2}\sin^2(\theta_{1,2}/2) &= (+-+;-+-) + (+--;-++), \\
\tfrac{1}{2}\sin^2(\theta_{2,3}/2) &= (-+-;+-+) + (++-;--+)\,, \\
\tfrac{1}{2}\sin^2(\theta_{1,3}/2) &= (++-;--+) + (+--;-++)\,.
\end{aligned} \tag{4}$$

Observe that for identical particles probabilities like $(+-+;\ -+-)$ and $(-+-;\ +-+)$ are most likely equal.

The previous equations lead to

$$D \equiv \sin^2(\theta_{1,2}/2) + \sin^2(\theta_{2,3}/2) - \sin^2(\theta_{1,3}/2) = 4(+-+;-+-)\,. \tag{5}$$

Because $(+-+;\ -+-)$ is, by definition, positive it follows that D must be positive definite. Clearly, this is contradictory (Wigner 1983) and, hence, the existence of LHVs is refuted.

At first instance it might appear as if the previous analysis covers all possible LHV models. This will be questioned however. Observe e.g. how the previous conclusion depends on the way the probabilities $P_{+,+}(\theta_{k,m})$, $k < m = 1, 2, 3$, are associated with the LHV weighted probability integrals like, for instance, $(++-;\ --+)$. There is this nagging possibility that, perhaps, things were presented too simple.

An indication of this possible state of affairs is the fact that in Wigner's model the probabilities are considered additive. This implies that the occurrence of a configuration like, e.g., +,−,+, in one wing of the experiment is independent of the possible occurrence of another configuration, e.g. +,−,−, in the same wing. The assumed additivity appears self-evident, but what will happen if the possible occurrence of, for instance +,−,+, is not independent of the possible occurrence of, for instance +,−,−, in the same wing of the experiment? In other words the present paper investigates what will happen if the additivity of probability is not introduced as a hidden restriction on LHV models.

Suppose that there exist hidden states $|\vec{s}\rangle$, with $\vec{s} \in S^3 = \{+,-\}^3$, and that the inner product of hidden states equals the probability weighted integral like, for instance, $(++-;--+) = \langle ++-|--+\rangle$, etc. Because of the relation with the probability integrals we have, for instance, $\langle ++-|++-\rangle = 0$. This is in agreement with the quantum description of the singlet state which eliminates the probability that both particles have the same spin along any of the axes, $\vec{e}_k$, $k = 1, 2, 3$. We then see that the first relation in equation (4) can be reproduced with $\{\langle -+-| + \langle -++|\}\{|+-+\rangle + |+--\rangle\}$. Generally, we suppose that $(\vec{s}_1;\vec{s}_2) = (\vec{s}_2;\vec{s}_1)$, which is different from zero, only when 'the mirror image condition', $\vec{s}_1 = -\vec{s}_2$, related to the singlet state of both particles, for $\vec{s}_1$ and $\vec{s}_2$, applies.

In the second place, let us for brevity define $x = (+-+;\ -+-) = (-+-;\ +-+)$, $y = (+--;\ -++) = (-++;\ +--)$ and $z = (++-;\ --+) = (--+;\ ++-)$. The symmetry in the LHV integrals is not an essential restriction.

In the third place, let us assume that the probabilities $P_{+,+}(\theta_{k,m})$ arise from the application of projection operators $\Pi_{+,+}(\theta_{k,m})$ on the compound state vector,

$$|\Phi\rangle = \sum_{\vec{s}\in S^3} |\vec{s}\rangle \,. \tag{6}$$

Here, in accordance with the mirror image condition, we assume that the 'bra'-type state vector refers to particle-1, while the 'ket'-type vector refers to particle-2.

The operators are defined by

$$\begin{aligned}
\Phi_{+,+}(\theta_{1,2}) &= \{|-++\rangle\langle+-+|\} - \{t_{1,2}|---\rangle\langle+++|\}\,,\\
\Phi_{+,+}(\theta_{2,3}) &= \{|+-+\rangle\langle++-|\} - \{t_{2,3}|---\rangle\langle+++|\}\,,\\
\Phi_{+,+}(\theta_{1,3}) &= \{|-++\rangle\langle++-|\} - \{t_{1,3}|---\rangle\langle+++|\}\,.
\end{aligned} \tag{7}$$

Here the $t_{k,m}$ are real positive numbers, while the braces are employed to distinguish between operators. For instance, the operator $\Pi_{+,+}(\theta_{1,2})$ is the difference between the operator $|-++\rangle\langle+-+|$ and the operator $t_{1,2}|---\rangle\langle+++|$, etc. The operators do not violate locality because there is no indication how the state of particle-1 would depend on the state of particle-2 and vice versa.

In the fourth place, we define the relation between the hidden states and the probability $P_{+,+}(\theta_{k,m})$. We have

$$P_{+,+}(\theta_{k,m}) = \langle\Phi|\Pi_{+,+}(\theta_{k,m})|\Phi\rangle, \quad \forall_{k<m\in\{1,2,3\}}\,, \tag{8}$$

This gives

$$P_{+,+}(\theta_{1,2}) = xy - p_{1,2}, \quad P_{+,+}(\theta_{2,3}) = xz - p_{2,3}, \quad P_{+,+}(\theta_{1,3}) = yz - p_{1,3}\,. \tag{9}$$

Here we suppose that $p_{k,m} = (---;+++)^2 t_{k,m}$, with $t_{k,m}$ real and positive.

Suppose we define the outcome of the probability integrals, thereby using the x, y, z notation given previously, with for completeness $x = (+-+;\ -+-) = (-+-;\ +-+)$, $y = (+--;\ -++) = (-++;\ +--)$ and $z = (++-;\ --+) = (--+;\ ++-)$:

$$\begin{aligned}
x &= \sqrt{\tfrac{1}{2}}\sqrt{\frac{[\sin^2(\theta_{1,2}/2) + 2p_{1,2}][\sin^2(\theta_{2,3}/2) + 2p_{2,3}]}{\sin^2(\theta_{1,3}/2) + 2p_{1,3}}}\,,\\
y &= \sqrt{\tfrac{1}{2}}\sqrt{\frac{[\sin^2(\theta_{1,3}/2) + 2p_{1,3}][\sin^2(\theta_{1,2}/2) + 2p_{1,2}]}{\sin^2(\theta_{2,3}/2) + 2p_{2,3}}}\,,\\
z &= \sqrt{\tfrac{1}{2}}\sqrt{\frac{[\sin^2(\theta_{2,3}/2) + 2p_{2,3}][\sin^2(\theta_{1,3}/2) + 2p_{1,3}]}{\sin^2(\theta_{1,2}/2) + 2p_{1,2}}}\,,
\end{aligned} \tag{10}$$

The probability weighted integrals must be within the interval [0, 1]. Hence the condition

$$\tfrac{1}{2} \leq \sin^2(\theta_{k,m}/2) + 2p_{k,m} \leq 1, \quad \forall_{k<m\in\{1,2,3\}}\,, \tag{11}$$

must apply. This implies that for $\theta_{k,m} \in [0,\, 2\pi]$ we have

$$P_{+,+}(\theta_{k,m}) = \tfrac{1}{2}\sin^2(\theta_{k,m}/2), \quad \forall_{k<m\in\{1,2,3\}}\,. \tag{12}$$

Table 1. Numerical simulation of instances violating Bell's inequality $\boldsymbol{D = xy - p_{1,2} + xz - p_{2,3} - yz + p_{1,3} = \{\sin^2(\theta_{1,2}/2) + \sin^2(\theta_{2,3}/2) - \sin^2(\theta_{1,3}/2)\}/2}$
The x, y, z are as given in equation (10), while $p_{k,m} = p(\theta_{k,m})$ are as in (13) ($c = 1{\cdot}35/4$)

D	$\theta_{1,2}$ (deg.)	$\theta_{2,3}$ (deg.)	$\theta_{1,3}$ (deg.)	x	y	z	$p_{1,2}$	$p_{2,3}$	$p_{1,3}$
−0·15	351·2	46·3	308·3	0·552	0·559	0·477	0·003	0·077	0·095
−0·27	56·2	11·3	123·0	0·441	0·614	0·684	0·111	0·005	0·386
−0·02	103·6	68·9	194·2	0·465	0·796	0·621	0·309	0·160	0·492

In Table 1 the results of the numerical simulation of instances where Bell's inequality is violated are presented, whereby $p_{k,m} = p(\theta_{k,m})$ is taken with

$$p(\theta) = \frac{c}{\pi^2}\theta^2 - \frac{2c}{\pi}\theta + c \quad (\theta \in [0, 2\pi])\,, \tag{13}$$

and $c = 1{\cdot}35/4$. This entails that quantum probabilities can be reproduced with the present LHV scheme. Any argument on the nonlocality of the Π-operators of equation (7) also has to explain why operators like

$$\Omega_{+,+}(\theta_{1,2}) = \frac{|-+-\rangle\langle+--|}{(+--;-++)} + \frac{|-++\rangle\langle+-+|}{(+-+;-+-)} \tag{14}$$

can be employed such as in (8) to produce an accepted local model (equation 4) (Wigner 1983). Alternatively, calling $|\Phi\rangle$ non-local leads us to the same question. Hence, if $|\Phi\rangle$ and/or $\Omega_{+,+}(\theta_{1,2})$ are non-local, how can the LHV measure $P_{+,+}(\theta_{1,2})$ be construed out of non-local entities, such as in $P_{+,+}(\theta_{1,2}) = \langle\Phi|\Omega_{+,+}(\theta_{1,2})|\Phi\rangle$?

Concerning the relation between the hidden states $|\vec{s}\rangle$, with $\vec{s} \in S^3 = \{+,-\}^3$, and standard quantum mechanics, the following remarks are made. In the paper it is claimed that

$$\exists_{|\vec{s}_1\rangle,|\vec{s}_2\rangle}\langle\vec{s}_1|\vec{s}_2\rangle = (\vec{s}_1;\vec{s}_2)\,. \tag{15}$$

A possible objection to the existence of such states is that, because $\langle\vec{s}|\vec{s}\rangle = 0$, they appear to contradict standard quantum mechanics. However, it is not strictly necessary that $|\vec{s}\rangle$ is a standard quantum state (i.e. $\langle\vec{s}|\vec{s}\rangle \neq 0$). There is nothing in the orthodox interpretation of quantum mechanics which implies that $|\vec{s}\rangle$ *must* be a part of standard quantum mechanics, otherwise the alternative is flawed.

Recall that we claim that there is an alternative explanation without *contradicting* standard quantum mechanics. The only thing that is necessary here is, hence, that no contradiction arises between postulating the existence of local hidden states and standard quantum mechanics.

Moreover, we claim that the mirror image condition, which reflects the singlet state, is insufficiently described by standard quantum mechanics. This is the bare essence of Einstein's criticism of standard quantum mechanics. Because the, experimentally controllable, argument to rule out local hidden states is based on the denial that a local model may reproduce the quantum probabilities, while here we have demonstrated that such a model actually may do this, an important reason to demand that $|\vec{s}\rangle$ is a part of standard quantum mechanics has been eliminated.

Finally, it will be shown below that $|\vec{s}\rangle$ can, without conflict, be related to standard quantum mechanics, despite the fact that $\langle\vec{s}|\vec{s}\rangle = 0$. Here we assume that the $|\vec{s}\rangle$ are not elementary, but are built out of standard quantum states. Suppose that

$$\langle\vec{s}_1|\vec{s}_2\rangle = \prod_{k=1}^{3} \langle s_1^k|s_2^k\rangle \,. \tag{16}$$

The superscripts denote the kth component ($k = 1,2,3$) of the vector. Moreover, suppose there are quantum states

$$|\psi(\vec{s})\rangle = |\psi_1(s^1)\rangle|\psi_2(s^2)\rangle|\psi_1(s^3)\rangle, \quad |\varphi(\vec{s})\rangle = |\varphi_1(s^1)\rangle|\varphi_2(s^2)\rangle|\varphi_3(s^3)\rangle,$$

such that, from $|s_\alpha^k\rangle = |\psi_k(s_\alpha^k)\rangle + |\varphi_k(s_\alpha^k)\rangle$ $(\alpha = 1,2, k = 1,2,3)$, we have

$$\begin{aligned}\langle\vec{s}_1|\vec{s}_2\rangle = \prod_{k=1}^{3} [\psi_k(s_1^k)|\psi_k(s_2^k)\rangle + \langle\varphi(s_1^k)|\varphi_k(s_2^k)\rangle \\ + \langle\psi_k(s_1^k)|\varphi_k(s_2^k)\rangle + \langle\varphi_k(s_1^k)|\psi_k(s_2^k)\rangle] \,.\end{aligned} \tag{17}$$

In standard quantum mechanics we must have that

$$\langle\psi(\vec{s}_1)|\psi(\vec{s}_2)\rangle = \langle\psi_1(s_1^1)|\psi_1(s_2^1)\rangle\langle\psi_2(s_1^2)|\psi_2(s_2^2)\rangle\langle\psi_1(s_1^3)|\psi_1(s_2^3)\rangle,$$

and similarly $\langle\varphi(\vec{s}_1)|\varphi(\vec{s}_2)\rangle$, are both not equal to zero when $\vec{s}_1 = \vec{s}_2$. Let us assume that $\langle\psi_k(s_1^k)|\psi_k(s_2^k)\rangle = \langle\varphi_k(s_1^k)|\varphi_k(s_2^k)\rangle = 1$ when $s_1^k = s_2^k$ $(k = 1,2,3)$. Moreover, observe that there is nothing in standard quantum mechanics to prevent $\langle\psi_k(s_1^k)|\varphi_k(s_2^k)\rangle = \langle\varphi_k(s_1^k)|\psi_k(s_2^k)\rangle = -1$, when $s_1^k = s_2^k$ $(k = 1,2,3)$. Recall that the quantum correlation $-\cos(\theta)$ itself can take values between -1 and $+1$. Hence, it can be acknowledged that the postulation of $|\psi_k(s_\alpha^k)\rangle$ and $|\varphi_k(s_\alpha^k)\rangle$ $(\alpha = 1,2,\ k = 1,2,3)$ does not contradict standard quantum mechanics. Therefore, the postulation of hidden states $|\vec{s}\rangle$ does not contradict standard quantum mechanics.

Concerning the interesting question of whether or not possible phase factors in the hidden states will change the quantum probability, the following must be noted. In the first place, we must observe that the probability integral only depends on the spin configuration in both wings of the experiment. In the second place, the local hidden states only take care of the probabilistic connection between the two wings. The probabilistic connection will only change when the spin configurations change. Hence, possible phase transformations are, by necessity, restricted by the conservation of probability $(\vec{s}_1;\ \vec{s}_2)$. A transformation with a phase ω then gives

$$|\vec{s}\rangle_{\text{transformed}} = e^{i\omega}|\vec{s}\rangle\,. \tag{18}$$

This implies that the inner product of the transformed hidden states, e.g., ${}_{\text{transformed}}\langle\vec{s}_1|\vec{s}_2\rangle_{\text{transformed}}$ must be equal to the original inner product $\langle\vec{s}_1|\vec{s}_2\rangle = (\vec{s}_1;\vec{s}_2)$.

Hence, no conflict arises between postulating local hidden states $|\vec{s}\rangle$ and standard quantum mechanics, because $\langle\vec{s}|\vec{s}\rangle = 0$ does not entail $|\vec{s}\rangle = 0$. The case where $\langle\vec{s}_1|\vec{s}_2\rangle = (\vec{s}_1;\vec{s}_2) \geq 0$, with $\vec{s}_1 = -\vec{s}_2$, has been dealt with previously and enables us to reproduce quantum mechanical results. This easily follows from the association of the probability integral $(\vec{s}_1;\vec{s}_2)$ with the spin configuration in the left-wing $\langle\vec{s}_1|$, and the spin configuration in the right-wing $|\vec{s}_2\rangle$. Moreover, the phase transformations in the hidden state $|\vec{s}\rangle$ are restricted by the conservation of probability.

In conclusion, there are no reasons left to rule out the present local hidden variables model. The demand that $|\vec{s}\rangle$ *must* be a standard quantum mechanical state is unwarranted, while calling the model nonlocal immediately questions Wigner's accepted local model.

3. Summary

It has been demonstrated here that a local hidden state exists which may reproduce the quantum probability $P_{+,+}(\theta_{k,m}) = \frac{1}{2}\sin^2(\theta_{k,m}/2)$. This probability refers to the situation where equal spin occurs along the unitary vector $\vec{e}_k$ in one wing, and the unitary vector $\vec{e}_m$ in the other wing of the experiment, where $\theta_{k,m}$ $(k < m)$ is the angle between the two unitary vectors, $\theta_{k,m} = \cos^{-1}(\vec{e}_k \cdot \vec{e}_m)$. It was demonstrated that the proposed model does not contradict quantum mechanics and that it, at the same time, is able to violate Wigner's version of Bell's inequality. The author believes that this analysis questions the elimination of local hidden variables by verifying Bell's inequalities in actual experiments. As examples of those experiments the reader is, in the first instance, referred to polarisation of photon pairs (Aspect *et al.* 1982) or spin correlation with pairs of protons (Lamehi-Rachti and Mittig 1976). Of course, more experiments have been conducted in this field. For more recent developments in experimentation, the reader is referred to Greenberger and Zeilinger (1995).

The key point of the analysis is the redefinition of the relation between the quantum probability; for instance, the probability of finding equal spins in both wings and the local hidden variable weighted probability integrals. It should be noted that when one opposes such a redefinition by claiming that it is nonlocal, one also has to explain why Wigner's model, which can be construed in an similar fashion, can be called local. Hence, given the extra condition that additivity and non-additivity can be obtained from similar operators and if explicit reasons are there to call $|\Phi\rangle$ and/or the operators $\Pi_{+,+}(\theta_{k,m})$, non-local, then why is the additivity of Wigner's model a characteristic of locality, while dropping the hidden restriction of additivity leads to non-locality? We claim that both $|\Phi\rangle$ and the operators $\Pi_{+,+}(\theta_{k,m})$ are local in the sense of Einstein (1949), because there is no indication how a change in the state of the particle in the right-wing will change the behaviour of the particle in the left-wing and vice versa.

Generally speaking, the present paper questions the elimination of local causality in quantum mechanics by Bell inequalities. In addition, it is also claimed that the explicit construction of a local hidden variables model, such as that presented here, also contradicts the attempts to eliminate local hidden variables without making use of Bell inequalities (Greenberger and Zeilinger 1995).

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