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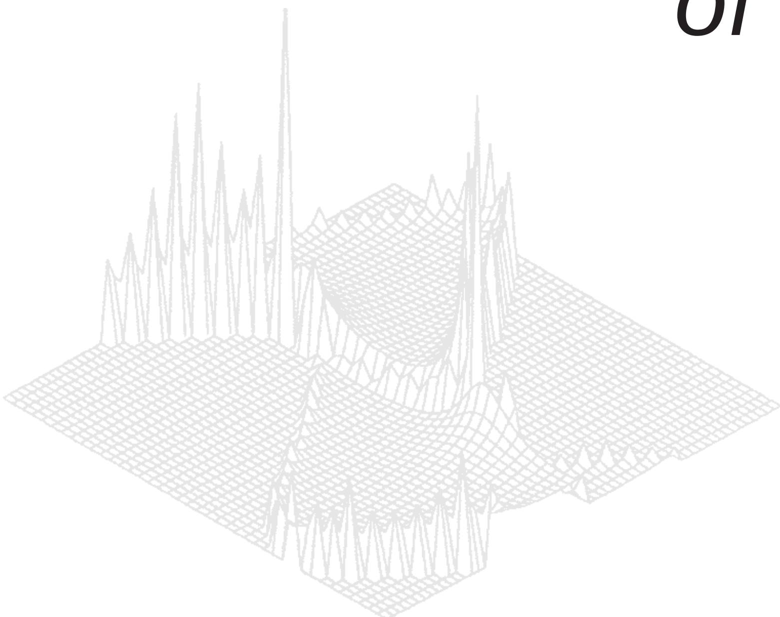

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Weak Plasma Turbulence Theory and Some Other Items of Plasma Kinetics

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Abstract

A new approach to a plasma kinetic description is discussed, the beginnings of which were published recently (Erofeev 1997*a*). It is shown that calculations of the three-wave collision integral following this approach confirm the intensity and structure of the three-wave collision integral obtained in the traditional theory. The reported kinetics extend the area of applicability for the weak plasma turbulence theory: apart from waves it properly accounts for the effect of various other plasma nonlinear structures of the type of solitons, drift vortices, collapsing cavities and so on. Some directions for further studies are also discussed.

1. Introduction

The existing machinery for the kinetic description of plasma collective motions is based on the Vlasov equation (Vlasov 1945). It is worth noting that the Vlasov equation coincides, in its form, with the Klimontovich–Dupree equation (see Klimontovich 1967 and Dupree 1963)— they differ only in the assumptions made about the solution to this equation. Namely, the Vlasov theory implies a smooth distribution function (the characteristic spatial scale of the function is large compared with the mean interparticle distance), whereas the Klimontovich–Dupree distribution consists of δ-functions corresponding to individual charged plasma particles. Conceptually, the Klimontovich–Dupree equation operates with a mixture of discrete charged particles, whereas the Vlasov equation operates eventually with an imaginary continuous liquids for the electron and ion components. Naturally, the substitution of the former by the latter may result in a diversity of opinion regarding the physical picture of plasma evolution.

It is worth noting also that the substitution of the Klimontovich–Dupree equation by the Vlasov equation is widely motivated by the assumption that some mean field exists in a plasma, and that it is via the mean field that the plasma particles influence each other. Meanwhile, the definition of a mean electromagnetic field seems to be problematic enough in the situation of an actual plasma with discrete charged particles, where the mean square of electromagnetic field far exceeds the square of the mean electromagnetic field.

Bearing in mind the above ideas, the author has developed an approach to a plasma kinetic description based on the Klimontovich–Dupree equation and applied it to the calculation of the three-wave collision integral. In an earlier

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paper (Erofeev 1997*a*) I concluded that the three-wave collision integral in a Klimontovich–Dupree collisionless plasma far exceeds its traditional analogue (calculated for the Vlasov plasma). Based on this, it was stated (Erofeev 1997*b*) that the difference in the intensities of the three-wave processes in Klimontovich–Dupree and Vlasov plasmas plainly indicates the inadequacy of the method of the Gibbsian probabilistic ensemble for studies on physical manifestations of thermodynamically nonequilibrium systems. Note that the method of the probabilistic ensemble is explicitly involved in some papers (via the substitution of an actual mixture of discrete charged particles by the ensemble of such mixtures) to justify the use of the Vlasov equation instead of the Klimontovich–Dupree equation (see Klimontovich 1967).

In reality, the conclusion regarding the increase in the intensity of the three-wave process turns out to be erroneous. A mistake was made at the last stage of the calculation in Erofeev (1997*a*), and the actual intensity of the three-wave process in a mixture of charged particles coincides with that of the traditional weak plasma turbulence theory. Therefore, the declaration in Erofeev (1997*b*) on the inadequacy of the method of Gibbsian probabilistic ensemble for evolving inhomogeneous physical systems has lost its basis. However, this declaration still corresponds to the usual physical commonsense, and it was an inadequate choice of the situation for illustration of its consistency that lead the author to this failure to substantiate this declaration.

Thus, our calculation (Erofeev 1997*a*) seems to have given nothing new: it turned out to be one more substantiation of the well known weak plasma turbulence theory. Nevertheless, one aspect of the kinetics in this calculation is worthy of attention. Within the framework of the above kinetics it becomes possible to generalise the notion of a weak plasma turbulent wave field. To show the way in which this approach extends this notion, let us first recall the history of the traditional weak plasma turbulence theory.

At the very beginning of plasma studies it was revealed that plasmas are usually unstable with respect to generation of various types of plasma waves. These waves in the leading order do not have an influence on distributions of plasma particles or on each other, and the first efforts of plasma theorists were spent on studies of different waves and their linear growth rates. Then an understanding developed that in actual situations, with plasmas being essentially far from thermodynamic equilibrium, the plasma waves lead to the quasilinear diffusion of charged particles, i.e. they determine the plasma evolution. It is not only that particle distributions define the growth (damping) rates of plasma waves, but also the evolution of particle distributions depend on the wave amplitudes. The plasma waves also interact, which complicates even more the problem of a self-consistent description of plasma evolution. The means for solving this problem were developed in the theory of weak plasma turbulence. The basis of the plasma description within the framework of this theory can be outlined as follows.

Suppose, for simplicity, that we have a homogeneous nonequilibrium plasma with slightly excited plasma waves, and we let it faithfully follow the Vlasov equation. Then the use of the Vlasov and Maxwell equations gives at least a formal opportunity to calculate particle distributions and the distribution of the electromagnetic field at any moment, provided the initial amplitudes of all plasma waves and initial distributions of plasma particles are known. But note, on the one hand, one never has full information on the wave amplitudes (at least one usually has no information on relative phase shifts) or on plasma distributions. On the other hand, the full integration of the Vlasov–Maxwell equations is technically problematic. In reality, a full knowledge of the plasma electromagnetic field is not necessary. From a practical standpoint, the spectrum of the turbulent wave field, i.e. the distribution of the energy density of plasma waves with respect to their wavelength, is a perfectly adequate characteristic for describing effects associated with the plasma waves. For instance, it is the wave spectrum that defines the plasma quasilinear diffusion. Also, it is the spectrum of drift waves that determines the plasma currents across the magnetic field due to the interaction of charged particles with drift waves. Factually, all the practically important aspects of nonlinear plasma evolution due to the development of a weakly turbulent wave field and the interaction of its waves with particles and other waves can be described in terms of the wave spectrum, i.e. in terms of squared wave amplitudes.

Conceptually, the traditional weak plasma turbulence theory deals only with slightly excited wave fields, when the actual electromagnetic field in the plasma can be decomposed into plane waves that are supposed to have a typical time of wave nonlinear interaction large compared with the time of wave separation in phase. In this case, it is usually said that the plasma waves possess random phases, and therefore to leading order the correlation in phases of the various waves can be ignored. (This is the essence of the so-called random phase approximation.)

It should be stressed that only a sufficiently restricted class of nonlinear phenomena in plasmas can be explored in this approximation. As a matter of fact, with an increase in the amplitudes of partial waves one quickly comes to a situation where the random phase approximation is violated (the phases of different waves cannot be regarded as independent). Consider, for instance, a soliton (solitary wave) in a plasma (Berezin and Karpman 1964, 1967; Krall 1969). It can be decomposed into plane waves, but the relative phases of its components are fixed (entirely correlated), due to the nonlinear interaction of the soliton spatial harmonics. To take another phenomenon, a drift plasma vortex (Hasegawa and Mima 1977, 1978; Petviashvili 1977) cannot even be decomposed into plane waves. Therefore, within the framework of the traditional weak plasma turbulence theory it was impossible to consider either the effect of a mixture of solitons, or that of a mixture of drift vortices. Meanwhile, from the same practical standpoint, the effect of both the homogeneous mixtures of the solitons and of the drift vortices on the macroscopic plasma evolution is similar to that of the traditional turbulent wave field. (It is worth noting that the problem of the influence of plasma solitons and drift vortices on the plasma macroscopic behaviour was regarded as a motivating force for the theoretical exploration of these objects.)

The basis of the kinetics of Erofeev (1997*a*) makes no distinction in the traditional weak turbulent wave field and homogeneous mixtures of solitons and drift vortices. Moreover, if other exotic objects of the existing plasma theory [for instance the phase space granulations of Dupree (1972, 1978) or the phase density holes of Dupree (1982)] had been of any importance for plasma physical manifestations, the effect of their homogeneous mixtures would have been described in these kinetics on common ground with that of the waves.

In the present paper we begin by briefly outlining the kinetics of Erofeev (1997a). Emphasis will be given to substantiating the declaration made above on extending of range of applicability of the weak plasma turbulence theory. Then we write the corrected results of the earlier calculation. In conclusion we discuss once more the problem of plasma kinetics.

2. Kinetics of the Klimontovich–Dupree Plasma

Klimontovich introduced the distribution function

$$N_\alpha(\mathbf{r},\mathbf{p},t) \;=\; \sum_{i=1} \delta^3(\mathbf{r}-\mathbf{r}_i(t))\,\delta^3(\mathbf{p}-\mathbf{p}_i(t)). \tag{1}$$

This function is composed of the terms corresponding to the individual charged plasma particles. The subscript α stands for the type of particle (electron, ion) and the subscript i represents different particles of a given kind; the functions $\mathbf{r}_i(t)$ and $\mathbf{p}_i(t)$ represent the particle trajectories. We call this distribution function a microdistribution.

The microdistribution evolves according to the Klimontovich–Dupree equation:

$$\frac{\partial N_\alpha}{\partial t} + \mathbf{v}\,.\,\nabla N_\alpha + e_\alpha\left(\mathbf{E}+\frac{1}{c}\,(\mathbf{v}\times\mathbf{B})\right).\,\frac{\partial N_\alpha}{\partial \mathbf{p}} \;=\; 0\,. \tag{2}$$

The trajectories of the plasma particles are the characteristics of this partial differential equation. The total electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields are solutions to the Maxwell equations, when the charge currents and densities are the corresponding integrals of the microdistributions N_α.

For definiteness, we shall focus on the problem of plasma diffusion across a magnetic field. The plasma is homogeneous along the yz plane. A nonlinear turbulent field of the drift waves is present in the plasma boundary layer. Due to the interactions of the plasma particles with the drift waves, the width of the plasma boundary increases with the course of time (which may lead to plasma losses in experiments with magnetic plasma confinement).

For studying the temporal extension of the plasma boundary it is most natural to consider the kinetic currents along the x axis, and therefore we average the microdistribution (1) along the yz plane. We regard the planes as 'thick': they have small but finite dimensions along x and the momentum components p_β. The averaged microdistribution is a well-defined and statistically reliable function. This function describes well the statistics of the particle distribution in the $\mathbf{r}$–$\mathbf{p}$ phase space, and it can be used for an objective description of plasma currents along the x axis. It is natural to regard this statistical function as a distribution function $f(\mathbf{r},\mathbf{p},t)$. It is clear that its time variation is strictly specified. Therefore, the problem of an objective plasma description is equivalent to the problem of the calculation of the rate of change of this function.

In the situation under consideration, the turbulent field of drift waves determines the evolution of the distribution functions and vice versa. The drift turbulence is characterised by the drift wave spectral density, and we derive the evolution equation for the wave spectral density. Similar to the traditional expression for the rate of change of the spectral density, our rate of change can be divided into a number of terms corresponding to different physical processes. In this way, we separate the part of the time derivative of the spectral density that corresponds to the three-wave interactions.

Calculating the rate of change of the distribution function, one obtains an evolution equation of traditional form:

$$\left[\frac{\partial}{\partial t} + v^{\beta}\frac{\partial}{\partial r^{\beta}} + \frac{e_\alpha}{c}\, v_i^0 F^{i\beta}\,\frac{\partial}{\partial p^{\beta}}\right] f_\alpha(\mathbf{r},\mathbf{p},t)$$
$$= -\frac{e_\alpha}{c}\, v_i\,\frac{\partial}{\partial p^{\beta}}\left\langle\, \delta F^{i\beta}(\mathbf{r},t)\, N_\alpha(\mathbf{r},\mathbf{p},t)\,\right\rangle . \qquad (3)$$

We see that the distribution function $f(\mathbf{r},\mathbf{p},t)$ is advanced in time by a 'two-point correlation function',

$$\langle\, \delta F^{i\beta}\,(\mathbf{r}',t')\; N_\alpha\,(\mathbf{r},\mathbf{p},t)\,\rangle\,. \qquad (4)$$

In this notation $\delta F^{i\beta}\,(\mathbf{r}',t')$ stands for the spatially fluctuating part of the electromagnetic field (EMF) tensor and the averaging is of the above type over the yz plane. The difference $\mathbf{r}-\mathbf{r}'$ should be fixed at the averaging, i.e. we should change the variables $\mathbf{r}$ and $\mathbf{r}'$ synchronously.

In turn, the two-point correlation function is advanced in time by a 'three-point correlation function',

$$\left\langle\, \delta F^{i\beta}\,(\mathbf{r}'',t'')\; \delta F^{j\gamma}\,(\mathbf{r}',t')\; N_\alpha\,(\mathbf{r},\mathbf{p},t)\,\right\rangle\,. \qquad (5)$$

(When averaging, all three variables $\mathbf{r}$, $\mathbf{r}'$ and $\mathbf{r}''$ change synchronously, for fixing the differences $\mathbf{r}'-\mathbf{r}$ and $\mathbf{r}''-\mathbf{r}$. The averaging with fixed shifts of variables $\mathbf{r}$ from one point to another is a general procedure in our consideration.)

Correspondingly, the three-point correlation function is advanced in time by the 'four-point correlation function', etc. In such a way, we obtain a definite hierarchy of equations. These equations account appropriately for many processes in a turbulent classical plasma: wave excitation and absorption by plasma particles, particle scattering on the waves, Coulomb collisions, and so on. But they are not suitable for a constructive plasma description. Nevertheless, the corresponding equations can be used for the derivation of more useful kinetic equations: the evolution equation for the function that we call a wave spectral density and the evolution equation for the actual distribution function. These equations are obtained as a result of a calculation carried out in two stages. For the first stage we derive a truncated closed evolution equation for the two-point correlation function. To obtain this equation, we use conventional expansion techniques, and the above hierarchy of equations (truncated up to necessary order) is used to start the iteration procedure.

The two-point correlation function is always integrated in all of the final equations. With an accuracy sufficient for integration, it can be expressed in terms of a more familiar 'two-time correlation function'. The two-time correlation function is determined by two electromagnetic field tensors, the product of which is averaged over the yz plane:

$$\Phi^{i\beta j\gamma}(\mathbf{r}', t', \mathbf{r}, t) = \left\langle \delta F^{i\beta}(\mathbf{r}', t')\, \delta F^{j\gamma}(\mathbf{r}, t) \right\rangle . \tag{6}$$

This function is typical for traditional (i.e. based on the Vlasov equation) weak plasma turbulence theory. It evolves in accordance with the Maxwell equations when the charge currents and densities are the corresponding integrals of the two-point correlation functions. The expression for the two-point correlation function in terms of the two-time correlation function is obtained by iterating the above-mentioned evolution equation for the two-point correlation function. (To leading order this equation represents a linear relationship between the two-point correlation function and the two-time correlation function.) Obtaining this expression (and consequently the evolution equation for the two-time correlation function) completes the first stage in the calculation of the final kinetic equations. For the second stage we express the two-time correlation function in terms of a function called the wave spectral density and obtain the time derivative of this function.

For an expanded description of this first stage the reader is referred to Erofeev (1997*a*). The corresponding calculation was performed by utilising graphical means for writing the analytical manipulations. Here we give only the final results of this stage: the evolution equations for the matrix elements of the two-time correlation function [see equations (24) and (25) in Erofeev (1997*a*)]

$$\begin{aligned} \frac{1}{c}\,\frac{\partial}{\partial t}\left\langle \widetilde{\delta F}_{\beta\gamma}(\mathbf{r},t)\,\widetilde{\delta F}^{kl}(\mathbf{r}',t') \right\rangle = &-\frac{\partial}{\partial r^{\beta}}\left\langle \widetilde{\delta F}_{\gamma 0}(\mathbf{r},t)\,\widetilde{\delta F}^{kl}(\mathbf{r}',t') \right\rangle \\ &+\frac{\partial}{\partial r^{\gamma}}\left\langle \widetilde{\delta F}_{\beta 0}(\mathbf{r},t)\,\widetilde{\delta F}^{kl}(\mathbf{r}',t') \right\rangle , \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{1}{c}\,\frac{\partial}{\partial t}\left\langle \widetilde{\delta F}^{\beta 0}(\mathbf{r},t)\,\widetilde{\delta F}^{kl}(\mathbf{r}',t') \right\rangle = &-\frac{\partial}{\partial r^{\gamma}}\left\langle \widetilde{\delta F}^{\beta\gamma}(\mathbf{r},t)\,\widetilde{\delta F}^{kl}(\mathbf{r}',t') \right\rangle \\ &-\frac{4\pi}{c}\int d^3\mathbf{r}_1\,\mathrm{d}t_1\,\sigma^{\beta m\cdot}_{\cdot\cdot\gamma}(\mathbf{r},t,\mathbf{r}_1,t_1)\left\langle \widetilde{\delta F}^{\cdot\,\gamma}_{m\cdot}(\mathbf{r}_1,t_1)\,\widetilde{\delta F}^{kl}(\mathbf{r}',t') \right\rangle \\ &-\frac{4\pi}{c}\sum_{\alpha} e_{\alpha}\int d^3\mathbf{p}\,v^{\beta}\,\mathcal{P}_{\alpha}{}^{kl}(\mathbf{r},\mathbf{p},t,\mathbf{r}',t') . \end{aligned} \tag{8}$$

In equation (8), $\sigma^{\beta m\cdot}_{\cdot\cdot\gamma}(\mathbf{r},t,\mathbf{r}_1,t_1)$ is a conductivity tensor. (For a detailed description of the various terms in these formulae see Erofeev 1997*a*.)

It should be emphasised that to this point we have not formulated any restrictions on the type of motion that the plasma particles participate in. There are only two essential restrictions that are implied. The first one is that the two-time correlation function has some characteristic decay length, the correlation length R_c, which is large compared to the Debye radius. In fact this is not even a restriction for ideal plasmas. (The correlation length R_c depends purely on the spectrum of plasma collective motions.) The second restriction is on the value of spatial variations of the electromagnetic field in the plasma: the absolute value of the energy density of the electromagnetic field (i.e. the absolute value of the two-point correlation function) should not exceed some threshold dictated by convergence of the iteration procedure. Speaking of its content, this restriction resembles the well-known applicability condition of the traditional weak plasma turbulence theory: the theory works only when the wave energy density does not exceed some threshold. For instance, for the case of Langmuir turbulence this threshold is the threshold of the modulational instability. The reader is reminded that above the latter threshold the time of interaction for separate waves becomes comparable with the time of their phase separation, i.e. the phases of different waves in the plasma become strongly correlated.

We reiterate that the plasma particles can exercise very complicated motions, and no restriction on the type motion exists. Some plasma particles can be involved in oscillatory motion, while others can be trapped inside drift vortices. One can speak also about phase space granulations (Dupree 1972, 1978) or density holes (Dupree 1982), and there even may be the possibility of the Bernstein–Green–Kruskal modes (see Bernstein *et al.* 1957). Even the influence of a random mixture of Zakharov's (1972) collapsing cavities (in the case of Langmuir turbulence) on the plasma behaviour may sometimes be studied. All the above-mentioned objects contribute to the two-time correlation function, and this function properly accounts for their integral effect.

Note also that equations (7) and (8) describe the effect of the Coulomb collisions. It seems that they can be used to check the well-known Lenard–Balescu equation (see Lenard 1960 and Balescu 1960).

In the case when the plasma collective motions define the main aspects of the plasma evolution (the Coulomb collisions can be neglected) a further calculation—the second stage of the derivation of the final kinetic equation—can be performed as follows. First, we perform a transition from the dependencies on the spatial variable $\mathbf{r} - \mathbf{r}'$ to the related dependencies on Fourier variables. In the case of a homogeneous plasma this transition is very useful for simplifying the treatment of the problem. In our problem the inhomogeneity effect cannot be neglected. But the typical scale of motion (of the order of the electron Larmor radius) is small compared with the inhomogeneity length. It helps to reduce (7) and (8) to comparatively simple equations for the Fourier transform of the two-time correlation function. We define the latter transform $\Phi_{\mathbf{k}}^{ijkl}(\mathbf{r}, t, t')$ by the identity

$$\Phi_{\mathbf{k}}^{ijkl}(\mathbf{r}, t, t') = \int \frac{d^3\mathbf{R}}{(2\pi)^3} \exp(-i\mathbf{k}\,.\,\mathbf{R}) \left\langle \delta F^{ij}(\mathbf{r} + \frac{\mathbf{R}}{2}, t)\, \delta F^{kl}(\mathbf{r} - \frac{\mathbf{R}}{2}, t') \right\rangle . \tag{9}$$

With this definition, the transform is self-adjoint:

$$\Phi_{\mathbf{k}}^{ijkl}(\mathbf{r}, t, t') = \left[\Phi_{\mathbf{k}}^{klij}(\mathbf{r}, t', t) \right]^* . \tag{10}$$

In the general case, the system of simultaneous equations that follows from (7) and (8) in Fourier variables consists of 36 independent equations in 36 independent components of the tensor $\Phi_{\mathbf{k}}$. For simplicity, we restrict ourselves to the situation where the following condition is satisfied:

$$\beta \equiv \frac{8\pi nT}{B^2} \ll \frac{m_e}{m_i}\,. \tag{11}$$

Under this condition, when the drift waves are potential, the only essential components of the tensor $\Phi_{\mathbf{k}}^{ijkl}(\mathbf{r},t,t')$ are the components $\Phi_{\mathbf{k}}^{\beta o\gamma o}(\mathbf{r},t,t')$. All the others can be expressed in terms of these by iterations. As a result, the system of equations reduces to one equation for one function:

$$\Phi_{\mathbf{k}}(\mathbf{r},t,t') = \frac{1}{k^2}\,k_\beta k_\gamma \Phi_{\mathbf{k}}^{\beta o\gamma o}(\mathbf{r},t,t')\,.$$

In other words, the field of the drift turbulence is a scalar one: it can be described by the scalar function $\Phi_{\mathbf{k}}(\mathbf{r},t,t')$. The evolution equation of this function is

$$\begin{aligned}\frac{\partial}{\partial t}\,\Phi_{\mathbf{k}}(\mathbf{r},t,t') = &-4\pi\int \mathrm{d}t_1\ \{\sigma_{\mathbf{k}}(\mathbf{r},t,t_1)\,\Phi_{\mathbf{k}}(\mathbf{r},t_1,t')\\ &-\frac{i}{2}\,\frac{\partial}{\partial k_\delta}\,\sigma_{\mathbf{k}}(\mathbf{r},t,t_1)\,\frac{\partial}{\partial r^\delta}\,\Phi_{\mathbf{k}}(\mathbf{r},t_1,t')\\ &+\frac{i}{2}\,\frac{\partial}{\partial r^\delta}\,\sigma_{\mathbf{k}}(\mathbf{r},t,t_1)\,\frac{\partial}{\partial k_\delta}\,\Phi_{\mathbf{k}}(\mathbf{r},t_1,t')\,\} - \mathcal{B}_{\mathbf{k}}(\mathbf{r},t,t')\,.\end{aligned} \tag{12}$$

In this formula $\sigma_{\mathbf{k}}(\mathbf{r},t,t')$ is a conductivity scalar, defined by

$$\sigma_{\mathbf{k}}(\mathbf{r},t,t') = \frac{k_\beta}{k^2}\left[k^\gamma + \frac{i}{2}\left(\delta^{\delta\gamma} - \frac{k^\gamma k^\delta}{k^2}\right)\frac{\partial}{\partial r^\delta}\right]\sigma_{\mathbf{k}\,\cdot\,\cdot\,\gamma}^{\beta o\,\cdot}\,, \tag{13}$$

where $\delta^{\delta\gamma}$ is the Kronecker delta of rank 2. The scalar $\mathcal{B}_{\mathbf{k}}(\mathbf{r},t,t')$ is

$$\mathcal{B}_{\mathbf{k}}(\mathbf{r},t,t') = 4\,\pi\sum_\alpha e_\alpha \int d^3\mathbf{p}\,\frac{k_\beta\,v^\beta\,k_\gamma}{k^2}\,\mathcal{P}_{\alpha\mathbf{k}}^{\ 0\gamma}(\mathbf{r},\mathbf{p},t,t')\,. \tag{14}$$

The formula (12) contains only the first non-vanishing terms of the expansion parameter ρ_{Li}/a (recall that a is a typical inhomogeneity length).

The instability of the drift waves may be revealed when one considers the first term in the braces on the right-hand side of this formula. The other terms within these braces contain derivatives of $\hat{\Phi}_{\mathbf{k}}(\mathbf{r},t,t')$ with respect to the coordinates and the wave vectors. They correspond to a wave drift in ($\mathbf{k}$,$\mathbf{r}$) space, which is induced by the plasma inhomogeneity.

We repeat once more that to this point we have not imposed any restrictions on the motions of plasma particles, or on the variety of physical structures present in the plasma. And if we use notions developed in the theory of plasma waves, it is only because of convenience, and because the essence of these notions is well known. In other words, an orientation on waves in a plasma in our treatment is the most transparent one for readers.

In calculations it is sufficient to regard (12) as an equation for the correlation function $\Phi_{\mathbf{k}}(\mathbf{r},t,t')$ in the region $t > t'$. In the region $t < t'$ the function can be reconstructed using equation (10).

Now let us consider the following aspect of our master equation (12). Suppose that we are dealing with a steady homogeneous plasma and that the last term on the right-hand side of the equation can be neglected. Then, the only solutions to the equation are natural oscillations: if we replace the function $\Phi_{\mathbf{k}}(t,t')$ by the oscillating function $\Phi_{\mathbf{k}}(t')\exp(-i\omega_{\mathbf{k}}(t-t'))$ then (12) reduces to a dispersion equation. Further, if we return to an inhomogeneous time-dependent collisionless plasma and restore the last term on the right-hand side, this will slightly modify the situation. Namely, the key part of the two-time correlation function comprises the natural oscillations, and all the remaining terms are the forced oscillations related to the natural oscillations in some way.

If one accepts the given image of the wave correlation function, one can conclude that the term $\mathcal{B}_{\mathbf{k}}(\mathbf{r},t,t')$ as a function of $t-t'$ damps for a time roughly equal to the inverse frequency width of the spectrum. Note that this width is small compared with the decay time of the oscillations. We suppose that the term $\mathcal{B}$ slightly modifies the natural oscillations. This is the case provided the energy density of the turbulent wave field is sufficiently low (or more correctly we are likely to have the well-known applicability condition of weak plasma turbulence theory—see above).

We stress that the introduction of natural oscillations here appears only as a method of calculation of the two-time correlation function. By no means do these natural oscillations separate the 'wavy' part of the two-time correlation function from the others. However, undoubtedly, the traditional waves are entirely included in our natural oscillations, and in the case of traditional weak plasma turbulence they are the only contributors to the spectral density.

A more expanded basis for the calculation of the two-time correlation function is given by Erofeev (1997*a*). Here we mention only that the leading order of the two-time correlation function has the form

$$\Phi_{\mathbf{k}}(\mathbf{r},t,t') = \sum_{s=\pm} n_{s\mathbf{k}}(\mathbf{r},t') \exp\left(-\, i \int_{t'}^{t} \omega^{s}_{\mathbf{k}}(\mathbf{r},\tau)\, d\tau \right), \tag{15}$$

with a positive real function $n_{\mathbf{k}}(\mathbf{r},t')$ corresponding to the wave spectral density (that is, had the two-time correlation function been composed of the contributions of waves only, this function is just the wave spectral density). A direct integration of (12) over time gives a correction to leading order of the two-time correlation function; within the accuracy of our treatment the corresponding correction is quite sufficient. The corrected two-time correlation function is

$$
\begin{aligned}
\Phi_{\mathbf{k}}(\mathbf{r},t,t') = \sum_s \Bigg\{ & [-13]\, n_{s\mathbf{k}} - 4\pi \left[\frac{\partial n_{s\mathbf{k}}}{\partial t'} - 2\gamma^s_{\mathbf{k}} n_{s\mathbf{k}} \right] \\
& \times \int_{t'}^{t} \mathrm{d}t_1 \int_{-\infty}^{t'} \mathrm{d}t_2\, \sigma_{\mathbf{k}}(\mathbf{r},t_1,t_2)\,(t_2 - t')\, \exp\left(i \int_{t_2}^{t_1} \omega^s_{\mathbf{k}}(\tau)\mathrm{d}\tau \right) \\
& + \frac{4\pi i(t-t')}{2} \Bigg\{ \left(\frac{i}{4\pi} - \frac{\partial \sigma_{\mathbf{k}\omega}}{\partial \omega} \right) \left[\frac{\partial \omega^s_{\mathbf{k}}}{\partial k_\delta} \frac{\partial n_{s\mathbf{k}}}{\partial r^\delta} - \frac{\partial \omega^s_{\mathbf{k}}}{\partial r^\delta} \frac{\partial n_{s\mathbf{k}}}{\partial k_\delta} \right] \\
& + n_{s\mathbf{k}} \left[\frac{\partial^2 \sigma_{\mathbf{k}\omega}}{\partial k_\delta \partial \omega} \frac{\partial \omega^s_{\mathbf{k}}}{\partial r^\delta} - \frac{\partial^2 \sigma_{\mathbf{k}\omega}}{\partial r^\delta \partial \omega} \frac{\partial \omega^s_{\mathbf{k}}}{\partial k_\delta} \right] \Bigg\} \Bigg\} \Bigg|_{\omega = \omega^s_{\mathbf{k}}(\mathbf{r},t')} \\
& \times \exp\left(-i \int_{t'}^{t} \omega^s_{\mathbf{k}}(\mathbf{r},\tau)\, d\tau \right) - \int_{t'}^{t} \mathcal{B}_{\mathbf{k}}(\mathbf{r},t_1,t')\, dt_1\,. \qquad (16)
\end{aligned}
$$

When omitted, the time variable t' and the variable $\mathbf{r}$ are implied in this expression.

The time derivative of the spectral density can be easily obtained from the following chain of equations:

$$
\begin{aligned}
\frac{\partial}{\partial t}\, \Phi_{\mathbf{k}}(\mathbf{r},t,t) &= \frac{\partial}{\partial t}\, \Phi_{\mathbf{k}}(\mathbf{r},t,t') \Bigg|_{t'=t-0} + \frac{\partial}{\partial t}\, \Phi_{\mathbf{k}}(\mathbf{r},t',t) \Bigg|_{t'=t-0} \\
&= 2\,\mathrm{Re} \left[\frac{\partial}{\partial t}\, \Phi_{\mathbf{k}}(\mathbf{r},t,t') \right] \Bigg|_{t'=t-0} . \qquad (17)
\end{aligned}
$$

Calculating here the time derivative of the autocorrelation function, i.e. the time derivative of $n_{\mathbf{k}} + n_{-\mathbf{k}}$, one can replace the two-time correlation function by the right-hand side of (16). The mistake in Erofeev (1997a) was made at this point. Instead of relation (39) in that paper, we have the equation

$$
\begin{aligned}
\sum_s \frac{\partial n_{s\mathbf{k}}}{\partial t} = 2 \sum_s \{ & \mathrm{Im}\, \omega^s_{\mathbf{k}}\, n_{s\mathbf{k}} + \mathrm{Re}\, A^s_{\mathbf{k}} \} \\
& + 4\pi \left[\frac{\partial n_{s\mathbf{k}}}{\partial t'} - 2\gamma^s_{\mathbf{k}} n_{s\mathbf{k}} \right] \frac{\partial \mathrm{Im}\, \sigma_{\mathbf{k}\omega}}{\partial \omega} \Bigg|_{\omega = \mathrm{Re}\, \omega^s_{\mathbf{k}}} - 2\,\mathrm{Re}\, \mathcal{B}_{\mathbf{k}}(\mathbf{r},t,t)\,, \\
A^s_{\mathbf{k}} = \frac{4\pi}{2} \Bigg[& \left(\frac{1}{4\pi} + i\, \frac{\partial \sigma_{\mathbf{k}\omega}}{\partial \omega} \right) \left(\frac{\partial \omega^s_{\mathbf{k}}}{\partial r^\delta} \frac{\partial n_{s\mathbf{k}}}{\partial k_\delta} - \frac{\partial \omega^s_{\mathbf{k}}}{\partial k_\delta} \frac{\partial n_{s\mathbf{k}}}{\partial r^\delta} \right) \\
& + i\, n_{s\mathbf{k}} \left(\frac{\partial^2 \sigma_{\mathbf{k}\omega}}{\partial k_\delta \partial \omega} \frac{\partial \omega^s_{\mathbf{k}}}{\partial r^\delta} - \frac{\partial^2 \sigma_{\mathbf{k}\omega}}{\partial \omega \partial r^\delta} \frac{\partial \omega^s_{\mathbf{k}}}{\partial k_\delta} \right) \Bigg] \Bigg|_{\omega = \mathrm{Re}\, \omega^s_{\mathbf{k}}} , \qquad (18)
\end{aligned}
$$

where $\sigma_{\mathbf{k}\omega}$ is the Laplace transform of the conductivity scalar.

Now let us present the corrected results of the calculation performed by Erofeev (1997a). The rate of change of the drift wave spectral density can be written in the form

$$\frac{\partial}{\partial t} n_{\mathbf{k}} \equiv \mathrm{St}\, n_{\mathbf{k}} = 2\gamma_{\,\mathrm{lin}}(\mathbf{k})\, n_{\mathbf{k}} + \mathrm{St}_{\mathrm{scat}}\, n_{\mathbf{k}} + \mathrm{St}_3\, n_{\mathbf{k}} + 2\,\frac{\mathrm{Re}\, A^{+}_{\mathbf{k}}}{\mathrm{Re}\mathcal{E}_{\mathbf{k}}\omega|_{\omega=\mathrm{Re}\,\omega^{s}_{\mathbf{k}}}}\,, \tag{19}$$

where

$$\mathcal{E}_{\mathbf{k}\omega} = 1 + 4\pi i\,\frac{\partial}{\partial\omega}\sigma_{\mathbf{k}\omega}. \tag{20}$$

In the expression (19) for the collision integral of drift waves, $\mathrm{St}\, n_{\mathbf{k}}$, the different terms correspond to different physical processes. The linear growth of the drift waves is given by the first term on the right-hand side: $\gamma_{\,\mathrm{lin}}$ in (19) is for the linear growth rate. The term $2\,\mathrm{Re}\, A^{+}_{\mathbf{k}}/\mathrm{Re}\mathcal{E}_{\mathbf{k}\omega}$ corresponds directly to wave drift in phase space:

$$\mathrm{Re}\, A^{+}_{\mathbf{k}} = \frac{4\pi}{2}\Bigg[\mathrm{Re}\mathcal{E}_{\mathbf{k}\omega}\left(\frac{\partial \mathrm{Re}\,\omega^{+}_{\mathbf{k}}}{\partial r^{\delta}}\,\frac{\partial n_{s\mathbf{k}}}{\partial k_{\delta}} - \frac{\partial \mathrm{Re}\,\omega^{+}_{\mathbf{k}}}{\partial k_{\delta}}\,\frac{\partial n_{s\mathbf{k}}}{\partial r^{\delta}}\right) - n_{\mathbf{k}}\left(\frac{\partial^2 \mathrm{Im}\,\sigma_{\mathbf{k}\omega}}{\partial k_{\delta}\partial\omega}\,\frac{\partial \mathrm{Re}\,\omega^{+}_{\mathbf{k}}}{\partial r^{\delta}} - \frac{\partial^2 \mathrm{Im}\,\sigma_{\mathbf{k}\omega}}{\partial\omega\partial r^{\delta}}\,\frac{\partial \mathrm{Re}\,\omega^{+}_{\mathbf{k}}}{\partial k_{\delta}}\right)\Bigg]\Bigg|_{\omega=\mathrm{Re}\,\omega^{s}_{\mathbf{k}}}. \tag{21}$$

The term $\mathrm{St}_{\mathrm{scat}}$ describes the nonlinear wave scattering induced by the plasma particles:

$$\mathrm{St}_{\mathrm{scat}}\, n_{\mathbf{k}} = \frac{2n_{\mathbf{k}}}{\mathrm{Re}\mathcal{E}_{\mathbf{k}\omega}} \sum_{\alpha,s} e_{\alpha}\,\mathrm{Re}\left\{\int d^3\mathbf{k}_1\, d^3\mathbf{k}_2\, d^3\mathbf{p}\, d^3\mathbf{p}_1 \times \frac{k_{\beta} v_1^{\beta}}{k}\,\delta^3(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\, n_{s\mathbf{k}_1} \times{}^0\, G_{\alpha\mathbf{k}\omega+i0}(\mathbf{p},\mathbf{p}_1)\,\frac{k_1^{\varepsilon}}{k_1}\,\frac{\partial}{\partial p_1^{\varepsilon}}\,\mathcal{H}_{\alpha\,\mathbf{k}\,\omega,\,-\mathbf{k}_1\,-\omega'}(\mathbf{p}_1)\right\}, \tag{22}$$

where

$$\omega = \mathrm{Re}\,\omega^{+}_{\mathbf{k}}, \quad \omega' = \mathrm{Re}\,\omega^{s'}_{\mathbf{k}_1}, \quad \omega' = \mathrm{Re}\,\omega^{s'}_{\mathbf{k}_2}\,. \tag{23}$$

Finally, the term $\mathrm{St}_3\, n_{\mathbf{k}}$ is the three-wave collision integral:

$$\mathrm{St}_3\, n_{\mathbf{k}} = \frac{2\pi}{\mathrm{Re}\mathcal{E}_{\mathbf{k}\omega}} \sum_{s',s'} \int d^3\mathbf{k}_1\, d^3\mathbf{k}_2\, \delta^3(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\, \delta(\omega-\omega'-\omega')$$
$$\times\ \mathrm{Re}\left\{V_{\mathbf{k}_1\omega',\,\mathbf{k}_2\omega''}\, V_{\mathbf{k}\omega,-\mathbf{k}_1-\omega''}\, n_{\mathbf{k}} n_{s'\mathbf{k}_1}\, (\mathcal{E}_{\mathbf{k}_2\omega''})^{-1}\right.$$
$$\left. + \tfrac{1}{2}\, |V_{\mathbf{k}_1\omega',\mathbf{k}_2\omega''}|^2\, n_{s'\mathbf{k}_1} n_{s''\mathbf{k}_2}\, (\mathcal{E}_{\mathbf{k}\omega})^{-1}\right\}. \tag{24}$$

In these formulae the matrix element of the three-wave interaction $V_{\mathbf{k}_1\omega',\mathbf{k}_2\omega'}$ is defined as follows:

$$V_{\mathbf{k}_1\omega',\mathbf{k}_2\omega'} = 4\pi \sum_\alpha e_\alpha \int d^3\mathbf{p}\, \frac{k_\beta v^\beta}{k}\, \mathcal{H}_{\alpha\,\mathbf{k}_1\,\omega',\,\mathbf{k}_2\,\omega''}(\mathbf{p})\,, \tag{25}$$

$$\mathcal{H}_{\alpha\,\mathbf{k}_1\,\omega',\,\mathbf{k}_2\,\omega'}(\mathbf{p}) = e_\alpha^2 \int d^3\mathbf{p}_1\, {}^0G_{\alpha\mathbf{k}_1+\mathbf{k}_2\,\omega'+\omega''+i0}(\mathbf{p},\mathbf{p}_1)$$
$$\times \left\{ \frac{k_2^\beta}{k_2} \frac{\partial}{\partial p_1^\beta}\, {}^0G_{\alpha\mathbf{k}_1\,\omega'+i0}(\mathbf{p}_1,\mathbf{p}_2)\, \frac{k_1^\gamma}{k_1} \frac{\partial f_\alpha}{\partial p_2^\gamma}\right.$$
$$\left. + \frac{k_1^\beta}{k_1} \frac{\partial}{\partial p_1^\beta}\, {}^0G_{\alpha\mathbf{k}_2\,\omega'+i0}(\mathbf{p}_1,\mathbf{p}_2)\, \frac{k_2^\gamma}{k_2} \frac{\partial f_\alpha}{\partial p_2^\gamma}\right\}. \tag{26}$$

The function $\mathcal{H}_{\alpha\,\mathbf{k}_1\,\omega',\,\mathbf{k}_2\,\omega''}(\mathbf{p})$ contains singularities as a function of ω' and ω''. This function was introduced to represent concisely the term $\mathcal{P}$ that is necessary for calculating the time derivative of the distribution function. The expression for $\mathcal{P}$ is

$$\mathcal{P}^{0\gamma}_{\alpha\mathbf{k}}(\mathbf{r},\mathbf{p},t,t') = -\frac{k^\gamma}{k} \int d\omega\, \exp(-i\omega(t-t')) \sum_{s,s'} \int d^3\mathbf{k}_1\, d^3\mathbf{k}_2\, \delta^3(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$$
$$\times\ \tfrac{1}{2}\, \delta(\omega - \mathrm{Re}\,\omega^s_{\mathbf{k}_1} - \mathrm{Re}\,\omega^{s'}_{\mathbf{k}_2})\, \mathcal{H}_{\alpha\,\mathbf{k}_1\,\omega',\,\mathbf{k}_2\,\omega''}(\mathbf{p})$$
$$\times\ n_{s\mathbf{k}_1}\, n_{s'\mathbf{k}_2}\, \left(V_{\mathbf{k}_1\,\omega'^*,\,\mathbf{k}_2\,\omega''^*}\right)^* \frac{1}{\left(-\,i\,\omega^* + 4\pi\,\sigma_{\mathbf{k}\omega^*}\right)^*}\,. \tag{27}$$

From a formal point of view, this expression diverges. However, to calculate the time derivative of the distribution function one needs to integrate $\mathcal{P}$ over $\mathbf{k}$. After this, the final contribution to the collision integral converges.

In the form presented here the three-wave collision integral coincides in its main features with that of the traditional calculations by Kadomtsev (1965), Davidson (1972) and Rogister and Oberman (1969).

3. Conclusion

In the paper here we have outlined the beginnings of a nontraditional approach to the derivation of kinetic equations for classical plasmas, first developed by Erofeev (1997*a*). We have shown that calculations following the framework of this earlier work confirm the structure and intensity of the traditional three-wave collision integral (contrary to the erroneous declaration made in Erofeev 1997*a* and then repeated in Erofeev 1997*b*). This means that the new calculations constitute one more substantiation of the existing weak plasma turbulence theory. We have shown that these calculations also extend the notion of the plasma weak turbulent wave field and the grounds for application of the weak plasma turbulence theory. Unlike the traditional weak plasma turbulence theory that was developed for treating wave fields only, in our new understanding the theory can be equally applied to studying macroscopic effects of mixtures of interacting solitons (Berezin and Karpman 1964, 1969; Krall 1969), vortices (Hasegawa and Mima 1977, 1978; Petviashvili 1977), the Bernstein–Green–Kruskal (1957) modes, Zakharov's (1972) collapsing caverns (in the case of Langmuir turbulence), Dupree's (1972, 1978) density holes and Dupree's (1982) phase space granulations (if the latter two notions are of any importance for plasma studies).

In reality, what was advanced in Erofeev (1997*a*) is but a most natural basis for studies of plasma macroscopic behaviour due to various nonlinear phenomena. According to this basis, there is no necessity to consider, for instance, the structures of various vortices, or questions of their stability and interaction. The reader knows that there exist plenty of publications devoted to the description of different vortex structures (surely, an infinite number of these structures in plasmas can be proposed), to studies of their stability and to discussions of various aspects of the interaction of plasma vortices. However, note that without accurate descriptions of vortex generation (which is thought to be due to the three-wave interaction, see Shukla 1991), no consistent quantitative description of the vortex role in a plasma can be developed. It is not only that a quantitative description of the vortex generation is absent at the moment, but it cannot even be developed due to the large variety of vortex structures. For this reason, when the question arises as to the role of vortices in plasma losses, only qualitative speculation about their contribution to the diffusion coefficient is usually proposed. In contrast, our approach from the very beginning is oriented towards extracting a quantitatively correct physical picture of plasma evolution, even when it is complicated by the generation of mixtures of vortices and the subsequent interaction of vortices.

It seems that at present the study of plasma vortices is popular because the vortices are regarded as more universal structural elements of the plasma physical picture compared to plasma waves. But in reality, the language of the drift vortices is a less rigorous one than that of the waves, and the methodology of this language is insufficiently well elaborated. After our treatment, the substitution of one set of structural elements, the plane waves, by another, the drift vortices, seems to be senseless.

Arguments similar to those above regarding the drift vortices can be advanced for all the other structures in plasmas that we have listed here. We suppose that there is no extensive use in studies of either separate solitons or Dupree's density holes and phase space granulations, and the same can be said about many other topics of plasma research.

In the case of four-wave interactions, the traditional calculations cannot give a full description of the collision integral, and only their modification following the reported ideas make it possible both to check the structure of different terms of the integral and to obtain a correct expression for the intensities in various channels of the process. Moreover, preliminary calculations revealed some new renormalisations of the four-wave matrix element when one starts from the Klimontovich–Dupree rather than the Vlasov equation. The reason is that while calculating the four-wave collision integral one should use not a linear relation between the two-point and two-time correlation functions in diagrams (which was easily sufficient for obtaining the three-wave collision integral), but a more corrected nonlinear relation of the former with the latter. It is anticipated that even some extra terms in the collision integral may appear that have structures different from the known ones.

Thus, the approach to the plasma kinetic description reported here seems to be a more rigorous one compared to its traditional analogue. This conclusion may be ascribed to the fact that we have constructed a perturbation theory that operates from the very outset with the correlation functions and not with wave amplitudes.

Now let us return to our earlier declaration concerning the inadequacy of the method of the Gibbsian probabilistic ensemble for studies of evolving inhomogeneous physical systems. According to usual physical commonsense, the traditional substitution of a certain thermodynamically nonequilibrium physical system by a system ensemble looks an unnatural one, and the results of this substitution should depend on the content of the ensemble. In particular, this can be said about substitution of a Klimontovich–Dupree plasma by a Vlasov plasma. Physically, there are no evolving ensembles in the Universe. An evolving ensemble is a theoretical abstraction only. For this reason it is only natural to anticipate some difference in the pictures of physical evolution of the mixture of discrete charged particles (such as the Klimontovich–Dupree plasma) and the imaginary continuous liquids of electron and ion components (such as the Vlasov plasma). The collective effects are not those where this difference manifests itself, and it is in the studies of collisions where one can hope to find a discrepancy in plasma evolution following the Klimontovich–Dupree equation and following the kinetic equations of Bogolubov (1946), Born and Green (1949), Kirkwood (1946) and Yvon (1935) (known collectively as the BBGKY kinetic equations). In particular, the first step to be done is to check the Lenard–Balescu equation (Lenard 1960; Balescu 1960). But the author has only a small hope for success in this direction. Even if there exists any difference in the physical pictures of Coulomb collisions within the framework of the reported kinetics and within the BBGKY kinetics, it should be only a small correction to the leading order of the process. At least for the plasma case, the substitution of a physical system by a system ensemble is performed in a situation where the discordance in the physical pictures of the process for a certain physical system and for a system ensemble seems to be negligibly small in all circumstances. That is, it may well be the case that one can always directly study physical manifestations of certain physical system (as was done for the case of the Klimontovich–Dupree plasma by Erofeev 1997*a*), and that one can also substitute it by a system ensemble (traditional BBGKY kinetics) with practically no change in the physical picture of the system evolution. We stress, nevertheless, that without a thorough study of this question the traditional substitution of a physical system by the system ensemble is not substantiated.

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