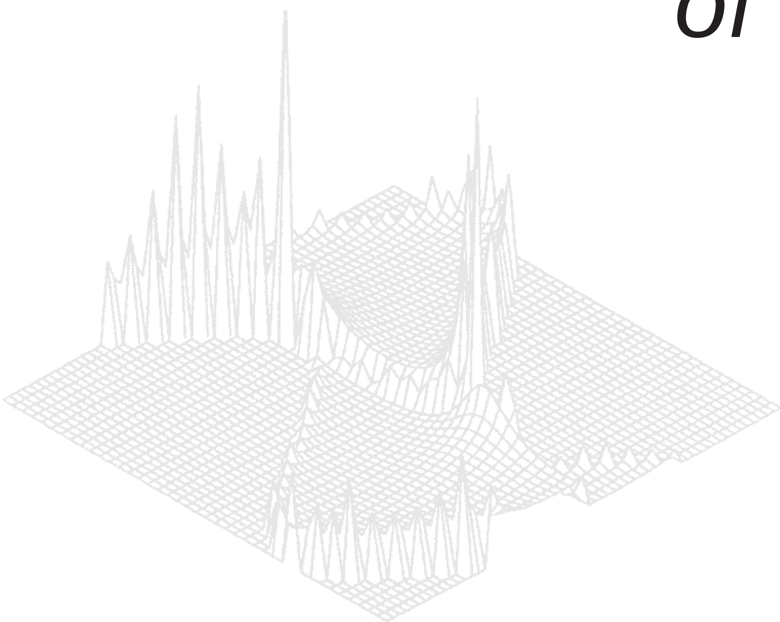
CSIRO PUBLISHING

Australian Journal of Physics

Volume 51, 1998



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia
Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: peter.robertson@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO and the
Australian Academy of Science

Aust. J. Phys., 1998, **51**, 477–507

The Physics of Tachyons
IV. Tachyon Electrodynamics*

Ross L. Dawe[A] *and Kenneth C. Hines*[B]

[A] Maritime Operations Division, Aeronautical and Maritime Research Laboratory,
Defence Science and Technology Organisation, PO Box 1500, Salisbury, SA 5108, Australia.
[B] School of Physics, University of Melbourne, Parkville, Vic. 3052, Australia.

Abstract

A new formulation of the theory of tachyons using the same two postulates as in special relativity is applied to the electrodynamics of material media. A discussion of Lagrange's equations and Hamilton's equations for 'classical' charged tachyons shows that such a formalism is a viable approach. An essay is included on why tachyons can be considered to be localised particles for the purpose of calculations. Tachyonic transformations of the electromagnetic fields **D**, **P**, **H** and **M** are shown to be the same as for bradyonic transformations. Examples discussed include the electric dipole moment of a tachyonic current loop, constitutive equations, polarisation in tachyonic dielectric materials and the velocity of light in tachyonic dielectric media. This is followed by discussions of the collision energy loss for charged tachyons interacting with a material medium and a mathematical proof that tachyons cannot emit Cherenkov radiation when passing through a bradyonic dielectric medium.

1. Introduction

The aim of this paper is to build upon the work of Dawe and Hines (1992*a*, 1992*b*, 1994) in the previous papers in this series on tachyon kinematics (hereafter referred to as Paper I), tachyon dynamics (Paper II) and tachyon electromagnetism (Paper III).

This paper investigates the subject of electrodynamics for charged tachyons. Lagrange's and Hamilton's equations for charged tachyons are considered. Transformations of the electric displacement and polarisation vectors, as well as the magnetic field intensity and magnetisation vectors, are derived. This is followed by discussions of the electric dipole moment of a tachyonic current loop, constitutive equations and the velocity of light in a tachyonic medium. The question of radiation emission by charged tachyons is considered through the case study of tachyons travelling through a dielectric medium. An essay is included on why tachyons are effectively localised particles for the purpose of calculations.

The development of tachyon mechanics in Papers I and II showed that the framework of special relativity (SR) can be extended to include particles having a relative speed greater than the speed of light in free space. The requirements necessary to allow this extension of special relativity into extended relativity (ER) are the switching principle (expressed mathematically as the 'γ-rule'), a standard

* Part I, *Aust. J. Phys.* **45**, 1992, 591. Part II, *Aust. J. Phys.* **45**, 1992, 725. Part III, *Aust. J. Phys.* **47**, 1994, 431.

 10.1071/PH97089 0004-9506/98/030477$10.00

convention for decomposing imaginary square roots and the minor modification of some familiar definitions such as 'source' and 'detector'. The results and definitions of ER automatically reduce to those of SR as soon as the objects appear to the observer to be bradyons.

Some terms will be in common usage throughout this work, so they will be defined here. 'Special relativity' (SR) refers to all currently accepted physics for particles which travel more slowly relative to the observer than the speed of light in free space, c. These particles will henceforth be called 'bradyons'. A 'tachyon' is defined to be a particle which is travelling relative to the observer at a speed greater than the speed of light in free space. 'Extended relativity' (ER) is the theoretical framework which describes the motion and interactions of tachyons. A 'bradyonic observer' travels at a speed less than c, while a 'tachyonic observer' travels at a speed greater than c relative to the inertial reference frame of the laboratory. Note that the version of ER being presented here has differences to the original version of ER as described in review articles by Recami and Mignani (1974) and Recami (1986): these differences will be discussed in detail at various points below.

2. Summary of ER developed thus far for Tachyons

The first paper in the present series dealing with tachyonic electrodynamics (Paper III) contained a summary of tachyon mechanics as presented in Paper I and Paper II. The reader is therefore referred to Section 2 of Paper III for a brief general introduction to tachyonic physics, including discussions of energy, momentum, mass, force and acceleration.

The theory being developed in this series of papers is founded upon the philosophy of Corben (1978) who has argued that tachyons, should they exist, 'are basically the same objects as ordinary particles (they just look different because they are moving so fast).' With this in mind, the same two postulates apply in ER as in SR:

Postulate 1: The laws of physics are the same in all inertial systems.

Postulate 2: The speed of light in free space has the same value c in all inertial systems.

Recami (1986) has also discussed this connection between ER and SR in detail. The term 'inertial system' now refers to any system travelling at a constant velocity with respect to an inertial observer, irrespective of whether the system is travelling slower than or faster than the speed of light. The postulates lead to the Lorentz transformations if the relative speed u between the two inertial reference frames is such that $u^2 < c^2$, and lead to the following superluminal Lorentz transformations (SLTs) if the relative speed between the two inertial reference frames is such that $u^2 > c^2$:

$$x' = i\gamma_u(x - ut),\ y' = iy,\ z' = iz,\ t' = i\gamma_u(t - ux/c^2)\,. \tag{1}$$

Here u is the relative speed along the common x, x' axes of an inertial reference frame Σ' with respect to an inertial reference frame Σ and

$$\gamma_u = \left(1 - u^2/c^2\right)^{-\frac{1}{2}} \tag{2}$$

for both $u^2 < c^2$ and $u^2 > c^2$.

Inverse tachyonic transformations can be obtained using the following rules: (i) interchange primed and unprimed quantities, (ii) reverse the sign of u and (iii) reverse the sign of i. A fourth rule which applies only to transformations involving the proper mass is discussed in Paper II.

When $u^2 > c^2$ the inertial frames Σ and Σ' are on opposite sides of the light barrier and a particle seen by Σ as a bradyon would be seen as a tachyon by Σ' and vice versa. Even though tachyonic transformations such as (1) indicate that the axes transverse to the boost are imaginary while the axis parallel to the boost remains real, an inertial observer in the rest frame of the tachyon considers all of the axes to be real.

In Paper I it was shown that tachyons can logically and consistently obey the conservation laws of energy, momentum and electric charge through the use of a 'switching principle' (expressed mathematically as the 'γ-rule'). A detailed numerical example was used to demonstrate that '*for switched tachyons the negative root of γ_u is used and for unswitched tachyons the positive root of γ_u is used.*' Written explicitly, this rule is

$$\gamma_u = \mathrm{sign}(\gamma_u)\left(1 - u^2/c^2\right)^{-\frac{1}{2}} , \tag{3}$$

where $\mathrm{sign}(\gamma_u) = +1$ if the particle appears to an observer to be an unswitched tachyon or a bradyon, and $\mathrm{sign}(\gamma_u) = -1$ if the particle appears to that observer to be a switched tachyon. Note that as there is no switching for a particle viewed by an observer to be a bradyon, then $\mathrm{sign}(\gamma_u)$ is always $+1$ and the standard result of SR is automatically recovered. Here the speed u is the relative speed between two inertial reference frames and should not be confused with the speed of the particle in the observer's inertial reference frame. For other discussions of the switching principle for tachyons, see for example Bilaniuk and Sudarshan (1969) and also Recami and Mignani (1974).

The tachyonic velocity transformations, which are exactly the same in ER and SR, automatically show whether the particle is switched or unswitched relative to a particular observer. Let v_x be the speed of the particle in the initial frame Σ, while u is the speed of the final frame Σ' relative to Σ along the common x, x' axes. The particle will appear to Σ' to be switched if:

$$c > u > c^2/v_x \text{ for } v_x > c \text{ and } |u| < c, \text{ or} \tag{4}$$

$$c < u < c^2/v_x \text{ for } v_x < c \text{ and } |u| > c\,. \tag{5}$$

To be consistent in the calculations of ER, a convention is used to deal with imaginary square roots such that when $u^2 > c^2$ then

$$\left(1 - u^2/c^2\right)^{\frac{1}{2}} = i\left(u^2/c^2 - 1\right)^{\frac{1}{2}} , \tag{6}$$

so that γ_u can be written as

$$\gamma_u = -\mathrm{sign}(\gamma_u) i |\gamma_u| \,. \tag{7}$$

It was shown in Paper III that one consequence of the second postulate of ER is that charged tachyons obey the same set of Maxwell's equations in free space that are obeyed by bradyons:

$$\nabla \,.\, \mathbf{B} = 0 \,, \tag{8}$$

$$\nabla \times \mathbf{E} = -\,\partial \mathbf{B}/\partial t \,, \tag{9}$$

$$\nabla . \mathbf{E} = \rho/\epsilon_0 \,, \tag{10}$$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{j} + \epsilon_0 \partial \mathbf{E}/\partial t) \,. \tag{11}$$

For a boost $-\infty < u < \infty$ along the common x, x' axes the electromagnetic field components transform between inertial reference frames as follows:

$$E_{x'} = E_x ,\ E_{y'} = \gamma_u (E_y - u B_z),\ E_{z'} = \gamma_u (E_z + u B_y) \,, \tag{12}$$

$$B_{x'} = B_x ,\ B_{y'} = \gamma_u (B_y + u E_z/c^2),\ B_{z'} = \gamma_u (B_z - u E_y/c^2) \,. \tag{13}$$

Here the inertial reference frames can be bradyonic or tachyonic as (12) and (13) are valid for both SR and ER. This in turn means that the scalar and vector potentials ϕ and $\mathbf{A}$ are related to the fields $\mathbf{E}$ and $\mathbf{B}$ via the same expressions in both SR and ER:

$$\mathbf{E} = -\,\nabla \phi - \partial \mathbf{A}/\partial t \,, \tag{14}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \,. \tag{15}$$

The form of the Lorentz force law and the transformations of the electromagnetic field tensor or stress-energy tensor are also unchanged when dealing with tachyons. Fundamental constants such as the permittivity and permeability of free space are the same regardless of whether the observer's inertial reference frame is bradyonic or tachyonic, so that $\epsilon_0 \mu_0 = c^{-2}$ in both SR and ER.

The total electric charge in any given inertial reference frame is always conserved, but the apparent charge is no longer the same when measured by different observers due to some of the tachyons appearing to be switched in some frames. As long as one notes which tachyons are switched and unswitched, this effect causes no insurmountable difficulties.

The metric used throughout this paper is $(+1, +1, +1, +1)$ as detailed on p. 593 of Paper I and a discussion of the reason for this choice of metric is also to be found there. Note that the results of this work can also be constructed with the more usual Minkowski metric. SI units are used throughout this work.

Four-vectors in ER transform according to:

$$B'_\lambda = \pm i \sum_{\nu=1}^{4} L_{\lambda\nu} B_\nu \,, \tag{16}$$

with the inverse transformation being

$$B_\nu = \mp i \sum_{\lambda=1}^{4} L'_{\nu\lambda} B'_\lambda \,, \tag{17}$$

where frame Σ' is a tachyonic inertial reference frame and Σ is a bradyonic inertial reference frame. The SR equivalent transformations omit the factor of $\pm i$ and $\mp i$ as appropriate. The 4×4 matrices $L_{\lambda\nu}$ and $L'_{\nu\lambda}$ are defined by:

$$L_{\lambda\nu} = \begin{bmatrix} \gamma_u & 0 & 0 & iu\gamma_u/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -iu\gamma_u/c & 0 & 0 & \gamma_u \end{bmatrix} , \tag{18}$$

$$L'_{\nu\lambda} = \begin{bmatrix} \gamma_u & 0 & 0 & -iu\gamma_u/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ iu\gamma_u/c & 0 & 0 & \gamma_u \end{bmatrix} . \tag{19}$$

Examples of four-vectors which obey (16) and (17) include:

- spacetime position $X_\lambda = (\mathbf{x}, ict)$,
- energy–momentum $P_\lambda = (\mathbf{p}, iE/c)$,
- charge and current density $J_\lambda = (\mathbf{j}, ic\rho)$,
- electromagnetic potential $A_\lambda = (\mathbf{A}, i\phi/c)$,
- partial derivatives $D_\lambda = (\partial/\partial\mathbf{x}, (ic)^{-1}\partial/\partial t)$.

The upper sign in (16) and (17) applies to the four-vectors X_λ, P_λ and A_λ, while the lower sign applies to the four-vectors J_λ and D_λ. The square of all the four-vectors listed above is

$$\sum_{\lambda=1}^{4} B'^2_\lambda = \pm \sum_{\lambda=1}^{4} B^2_\lambda \,, \tag{20}$$

where the upper $(+)$ sign applies for $u^2 < c^2$ and the lower $(-)$ sign applies for $u^2 > c^2$.

A third class of transformations in SR and ER applies to quantities which normally only make up three-vectors, such as velocity and force. These quantities have the common properties that their transformations are exactly the same for $-\infty < u < \infty$ and that transformations of the components perpendicular to the boost contain factors of γ.

The electric and magnetic fields, or alternatively the scalar and vector potentials, produced by a charged tachyon travelling through a vacuum at constant velocity are real and in principle detectable inside a Mach cone having semivertex angle $|c/u| = |\sin\theta|$. As the field is real and moves with constant speed c regardless of the source speed, any point lying outside the cone corresponds to a position where the field is purely imaginary and is therefore undetectable. At the instant of contact with the cone describing the propagation of the field, any detection instruments would register a sudden jump called an 'optic boom', in analogy with the 'sonic boom' generated by supersonic aircraft. This effect has been studied in detail by Recami *et al.* (1986). After the instant of initial contact, the image of the tachyon splits into two images travelling in opposite directions, an effect called the 'two source effect'.

3. Lagrange's Equations

The purpose of this section is to demonstrate that ER has the equivalent level of internal consistency as SR. The Lagrangian for bradyons L_B and tachyons L_T in the absence of external fields or potentials was discussed in Paper II:

$$L_B = -m_0c^2(1-v^2/c^2)^{\frac{1}{2}},\ v^2 < c^2\,, \tag{21}$$

$$L_T = -m_*c^2(1-v^2/c^2)^{\frac{1}{2}},\ v^2 > c^2\,. \tag{22}$$

Here $\mathbf{v}$ is the velocity of the particle, m_0 is the proper mass if the particle is a bradyon and m_* is the proper mass if the particle is a tachyon, with the two proper masses being related by $m_* = im_0$.

Now suppose the particle carries charge Q and is moving in a region containing an electromagnetic field described by the scalar and vector potentials ϕ and $\mathbf{A}$ respectively. In this case the Lagrangians are

$$L_B = -m_0c^2(1-v^2/c^2)^{\frac{1}{2}} - Q\phi + Q(\mathbf{v}.\mathbf{A})\,, \tag{23}$$

$$L_T = -m_*c^2(1-v^2/c^2)^{\frac{1}{2}} - Q\phi + Q(\mathbf{v}.\mathbf{A})\,. \tag{24}$$

Here both L_B and L_T are real, which corresponds to the charged particle having real energy.

Lagrange's equations are defined in the same way for both SR and ER as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0\,, \tag{25}$$

where L represents either the bradyonic Lagrangian L_B or the tachyonic Lagrangian L_T and the q_i are generalised coordinates $q_1, q_2, \ldots, q_n$ $(i = 1, 2, \ldots, n)$. Taking the partial derivative of L_T with respect to x and remembering that ϕ and $\mathbf{A}$ depend upon the position of the charge, but not its velocity, leads to

$$\frac{\partial L_T}{\partial x} = -Q\frac{\partial \phi}{\partial x} + Q\left(v_x\frac{\partial A_x}{\partial x} + v_y\frac{\partial A_y}{\partial x} + v_z\frac{\partial A_z}{\partial x}\right)\,. \tag{26}$$

Partially differentiating the Lagrangian with respect to v_x leads to

$$\frac{d}{dt}\left(\frac{\partial L_T}{\partial \dot{x}}\right) = \frac{d}{dt}\left(\frac{m_* v_x}{(1 - v^2/c^2)^{\frac{1}{2}}}\right) + Q\frac{dA_x}{dt}\,. \tag{27}$$

Since A_x is a function of the spacetime coordinates x, y, z, t then using the chain rule and dividing through by dt gives

$$\frac{dA_x}{dt} = \frac{\partial A_x}{\partial x}v_x + \frac{\partial A_x}{\partial y}v_y + \frac{\partial A_x}{\partial z}v_z + \frac{\partial A_x}{\partial t}\,. \tag{28}$$

Combining (27) and (28) shows that the left-hand side of Lagrange's equation in ER leads to an expression which can be simplified using the relations between the electromagnetic fields and potentials (14) and (15) to give

$$\frac{d}{dt}\left(\frac{\partial L_T}{\partial \dot{x}}\right) - \frac{\partial L_T}{\partial x} = \frac{dp_x}{dt} - QE_x - Q(\mathbf{v}\times\mathbf{B})_x\,, \tag{29}$$

where $p_x = m_* v_x(1 - v^2/c^2)^{-1/2}$ is the tachyon's momentum in the x direction.

The x component of an applied mechanical force is defined to be $F_x = dp_x/dt$, while the x component of the Lorentz force is $F_{x(L)} = QE_x + Q(\mathbf{v}\times\mathbf{B})_x$. Hence the x component of Lagrange's equation in ER as expressed in (29) gives

$$\frac{d}{dt}\left(\frac{\partial L_T}{\partial \dot{x}}\right) - \frac{\partial L_T}{\partial x} = F_x - F_{x(L)} = 0\,, \tag{30}$$

and so the tachyonic Lagrangian leads directly to the Lorentz force equation. Equation (30) is identical in form to the corresponding bradyonic result, the only difference arising from the appearance of m_* in L_T. Similar derivations for the y and z components of the tachyonic Lagrangian will confirm the result.

4. Hamilton's Equations

Having demonstrated that the tachyonic Lagrangian of (24) is correct for the present formulation, the tachyonic Hamiltonian can be defined as

$$H_T = \sum_i p_i \dot{q}_i - L_T\,, \tag{31}$$

where again q_i are generalised coordinates $q_1, q_2, \ldots, q_n$ $(i = 1, 2, \ldots, n)$. The generalised momentum p_i is defined by $p_i = \partial L_T/\partial \dot{q}_i$. These definitions have the same form in SR, with L_T replaced by L_B. If a single tachyon has charge Q and moves through a region in which scalar and vector potentials are present, then the Hamiltonian is

$$H_T = Q\phi + [c^2(\mathbf{p} - Q\mathbf{A})^2 - m_0^2 c^4]^{\frac{1}{2}}\,. \tag{32}$$

If the scalar and vector potentials are set to zero, then the Hamiltonian becomes equivalent to the tachyon's energy as discussed in Paper II:

$$H_T = (p^2c^2 - m_0^2c^4)^{\frac{1}{2}} = E_T\,. \tag{33}$$

Hamilton's equations for tachyons can be written as

$$\dot{p}_i = -\,\partial H_T/\partial q_i\,, \tag{34}$$

$$\dot{q}_i = \partial H_T/\partial p_i\,. \tag{35}$$

For a charged tachyon (35) gives

$$v_x = c(p_x - QA_x)[(\mathbf{p} - Q\mathbf{A})^2 - m_0^2c^2]^{-1/2} \tag{36}$$

with similar expressions for the y and z components of the velocity $\mathbf{v}$. Using (36) allows the first of Hamilton's equations to be written as

$$\dot{p}_x \equiv \frac{dp_x}{dt} = -Q\frac{\partial\phi}{\partial x} + Q\left(v_x\frac{\partial A_x}{\partial x} + v_y\frac{\partial A_y}{\partial x} + v_z\frac{\partial A_z}{\partial x}\right)\,. \tag{37}$$

Combining this with (28) and using (14) and (15) leads to

$$\frac{d}{dt}(p_x - QA_x) = QE_x + Q(\mathbf{v}\times\mathbf{B})_x, \tag{38}$$

which is the equation of motion in the x direction for a charged tachyon. Similar calculations apply for the y and z components.

Thus it can be seen that the Hamiltonian formalism gives the same results as those that would be obtained using the ordinary mechanical formalism developed in Paper II. This means that an extension of relativistic quantum mechanics into the tachyonic realm should have excellent prospects of success if the Hamiltonian formalism is used.

5. Localisation of Tachyons

In Paper III it was pointed out that the 'two source effect' means that after the time of initial contact, tachyons appear to a bradyonic observer to be in two places at once. This effect is due purely to the finite transmission time of the electromagnetic field which carries the information to the observer: the tachyonic particle is not intrinsically dissociated or dismembered in any way. Relative to its own rest frame, the tachyon is a localised bradyon and generates electromagnetic potentials and associated fields just like a bradyon in our rest frame. For the purpose of calculations when the two source effect is occurring, each tachyonic source is effectively a localised tachyon with an appropriate time delay due to finite information transmission time.

In Recami's (1986) formulation of the theory of tachyons, the two source effect is taken as evidence that tachyons are nonlocalised particles: see also Barut (1978), Duffey (1975, 1980), Vysin (1977), Soucek (1981) and in particular Barut *et al.* (1982). Further evidence cited by Recami for nonlocalisation is that tachyons can appear to have infinite speed relative to certain bradyonic reference frames: this is given by the dual speed condition and the velocity transformations discussed in Paper I.

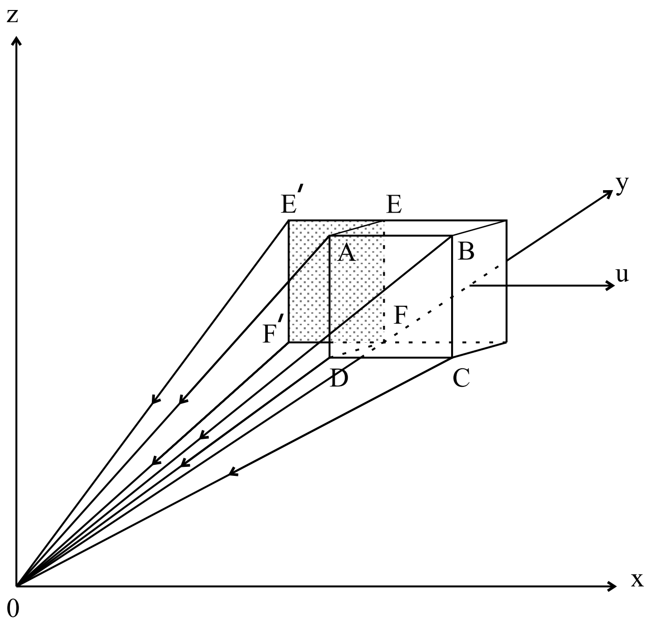


Fig. 1. A demonstration of the apparent rotation of a tachyonic cube as seen by an observer at O.

It was shown in Paper I that a tachyonic cube moving perpendicularly to the observer at a large distance will appear to be rotated so that its side face can be seen: this is illustrated in Fig. 1. The face ABCD which normally was facing towards the observer appeared to be contracted for $u^2 < 2c^2$ and appeared to be dilated for $u^2 > 2c^2$. The side face ADFE appeared to be rotated into the view of the observer and appeared to be dilated as the speed increased. At no stage does the image of the cube lose its integrity and appear to be fragmented, as would happen under Recami's (1986) interpretation. In this example the two source effect would produce, after an initial optical contact, two mutually receding cubes each undergoing contraction and dilation effects depending on the speed relative to the observer, with the side face appearing to be rotated into view for both images.

Now consider an object which is a sphere in its own rest frame Σ'. The equation describing the sphere in Σ' is

$$0 \le {x'}^2 + {y'}^2 + {z'}^2 \le {r'}^2 \,. \tag{39}$$

In bradyonic frame Σ the apparent shape of this object when moving with speed u such that $u^2 < c^2$ is

$$0 \le \frac{(x-ut)^2}{1-u^2/c^2} + y^2 + z^2 \le r^2 \,. \tag{40}$$

In his formulation of tachyon theory, Recami (1986) claims that such a particle, if instead it were a tachyon, would occupy the space bounded by a double, unlimited cone and a two-sheeted hyperboloid connected at a point. This conclusion was reached because Recami argues that if the object is seen as a tachyon, its coordinates should transform according to the (Recami) SLTs so that, for $u^2 > c^2$,

$$0 \geq -\frac{(x-ut)^2}{u^2/c^2-1} + y^2 + z^2 \geq -r^2 \,, \tag{41}$$

which yields the hyperboloid to which he refers.

It is here that one of the differences between the Recami formulation and the one developed in this series of papers becomes apparent. Even though tachyons use a complex spacetime rather than the real spacetime used by bradyons, the length, area and volume of a tachyon must be real and positive due to the properties of numbers in the complex plane. In Paper I it was shown that for $u^2 > c^2$ the apparent length l of a rod moving parallel to the boost is given by $l = l_0|(1-u^2/c^2)^{\frac{1}{2}}|$, where l_0 is the rod's proper (i.e. rest) length. The apparent length of the tachyonic rod measured perpendicularly to the boost is $l = |il_0|$ and not $l = -il_0$. Since the equation for the sphere is essentially measuring lengths in each spatial dimension, then the components of (39) are transformed in this case according to $x'^2 \to |i\gamma_u(x-ut)|^2$, $y'^2 \to |iy|^2 = y^2$ and $z'^2 \to |iz|^2 = z^2$. Hence the equation describing the apparent shape of the object in frame Σ for $u^2 > c^2$ in the present formulation is not (41), but is instead:

$$0 \leq \frac{(x-ut)^2}{u^2/c^2-1} + y^2 + z^2 \leq r^2 \,. \tag{42}$$

This equation describes an ellipsoid and shows that the image is still connected. It does not take into account the apparent elongation of the side as the relative speed increases (this corresponds to the side face ADFE in the cube example): this must be left to a more detailed analysis than the one presented here. A similar equation describing an ellipsoid applies to the second image seen as a consequence of the two source effect.

The example of a moving sphere highlights one of the major problems inherent in developing a logical and consistent formulation of the theory of tachyons: that of misinterpretation. Tachyonic observers consider their transverse axes to be real, but bradyonic observers consider these same transverse axes to be imaginary. Similarly, transverse axes which are real for bradyonic observers are imaginary for tachyonic observers. However, lengths, areas and volumes are always positive and real for both bradyons and tachyons as these quantities are based on the magnitude of the distance between points. Thus to avoid misinterpretation it is always necessary to consider carefully what is being transformed or described in each reference frame.

The discussion of the tachyonic charge's electromagnetic field and its scalar and vector potentials in Paper III used the SLTs (1) without requiring any modulus signs for transforming coordinates as it was not necessary to calculate any quantity which is intrinsically positively real, such as length, area or volume. The proof of the conservation of electric charge in all inertial reference frames given in

Paper III used modulus signs for superluminal relative speeds when calculating the volume transformations, but when combined with the other quantities in the derivation the correct (i.e. observed) sign of the electric charge was still obtained in every case.

Therefore, so long as one considers carefully what is being derived or calculated, problems of misinterpretation can be avoided. Tachyons can be treated as being effectively localised for the purpose of calculations, and indeed have been treated that way throughout Papers I to III. Apart from the possibility of switching in some reference frames due to the relative speed and the two source effect arising from finite information transmission time, tachyons behave at the classical (i.e. nonquantum) level just like normal point particles for the purpose of generating fields and potentials. As well as providing consistency and convenience, this interpretation of tachyon behaviour has the advantage of removing the inherent difficulties associated with dealing with nonlocalised particles, for example the problem of finite time extension for tachyons (Recami 1986).

6. Transformations of D and H

The first postulate of ER has been taken to mean that the form of Maxwell's equations in free space is the same for both bradyonic and tachyonic inertial observers. It will now be shown to cover the form of Maxwell's equations which uses the electric displacement vector **D** and the magnetic field intensity vector **H**. Two of Maxwell's equations in bradyonic frame Σ can be written as (Rosser 1964)

$$\nabla \times \mathbf{H} = \partial \mathbf{D}/\partial t + \mathbf{j}\,, \tag{43}$$

$$\nabla . \mathbf{D} = \rho\,. \tag{44}$$

In tachyonic frame Σ' the corresponding equations are

$$\nabla' \times \mathbf{H}' = \partial \mathbf{D}'/\partial t' + \mathbf{j}'\,, \tag{45}$$

$$\nabla' . \mathbf{D}' = \rho'\,. \tag{46}$$

These expressions are just restatements of (10) and (11) with the replacements $\mathbf{E} \to \mathbf{D}/\epsilon_0$, $\mathbf{B}/\mu_0 \to \mathbf{H}$, $\mathbf{E}' \to \mathbf{D}'/\epsilon_0$ and $\mathbf{B}'/\mu_0 \to \mathbf{H}'$. It was shown in Paper III that the permittivity and permeability constants of free space, ϵ_0 and μ_0 respectively, are unchanged when transforming between tachyonic and bradyonic inertial reference frames.

The tachyonic transformations of $\partial/\partial x$, $\partial/\partial z$, $\partial/\partial t$ and j_y can be found from (17) and substituted into the y component of (43). Rearranging terms, cancelling a common factor of i and comparing the result with the y' component of (45) shows that

$$H_{x'} = H_x,\ H_{z'} = \gamma_u(H_z - uD_y),\ D_{y'} = \gamma_u(D_y - uH_z/c^2)\,. \tag{47}$$

These steps can be repeated by substituting the tachyonic transformations of $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial t$ and j_z into the z component of (43), then comparing with the z' component of (45) to give

$$H_{x'} = H_x,\ H_{y'} = \gamma_u(H_y + uD_z),\ D_{z'} = \gamma_u(D_z + uH_y/c^2)\,. \tag{48}$$

By substituting appropriate transformations into (44) and comparing the result with (46) it can be shown that $D_{x'} = D_x$. Hence for a boost along the common x, x' axes the tachyonic transformations of $\mathbf{D}'$ and $\mathbf{H}'$ are

$$D_{x'} = D_x,\ D_{y'} = \gamma_u(D_y - uH_z/c^2),\ D_{z'} = \gamma_u(D_z + uH_y/c^2)\,, \tag{49}$$

$$H_{x'} = H_x,\ H_{y'} = \gamma_u(H_y + uD_z),\ H_{z'} = \gamma_u(H_z - uD_y)\,. \tag{50}$$

The inverse transformations are

$$D_x = D_{x'},\ D_y = \gamma_u(D_{y'} + uH_{z'}/c^2),\ D_z = \gamma_u(D_{z'} - uH_{y'}/c^2)\,, \tag{51}$$

$$H_x = H_{x'},\ H_y = \gamma_u(H_{y'} - uD_{z'}),\ H_z = \gamma_u(H_{z'} + uD_{y'})\,. \tag{52}$$

These are exactly the same transformations as those used in SR for transforming between two bradyonic frames, so that (49) to (52) are valid for $-\infty < u < \infty$. In vector form the transformations (49) and (50) are

$$\mathbf{D}'_{\|} = (\mathbf{D} + \mathbf{u} \times \mathbf{H}/c^2)_{\|},\ \mathbf{D}'_{\perp} = \gamma_u(\mathbf{D} + \mathbf{u} \times \mathbf{H}/c^2)_{\perp}\,, \tag{53}$$

$$\mathbf{H}'_{\|} = (\mathbf{H} - \mathbf{u} \times \mathbf{D})_{\|},\ \mathbf{H}'_{\perp} = \gamma_u(\mathbf{H} - \mathbf{u} \times \mathbf{D})_{\perp}\,, \tag{54}$$

where the subscripts $\|$ and $\perp$ refer to components parallel and perpendicular to the boost $\mathbf{u}$ respectively. Of course $(\mathbf{u} \times \mathbf{H})_{\|} = (\mathbf{u} \times \mathbf{D})_{\|} = 0$, but these terms have been explicitly included for the appearance of symmetry. Note that the axes perpendicular to the boost are imaginary, while the axis parallel to the boost is real. This conforms with comments made in previous papers in this series on the nature of various axes in ER.

7. Electromagnetic Invariances

The electromagnetic field tensor $F_{\mu\nu}$ in frame Σ was defined in Paper III as

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_z & -B_y & -iE_x/c \\ -B_z & 0 & B_x & -iE_y/c \\ B_y & -B_x & 0 & -iE_z/c \\ iE_x/c & iE_y/c & iE_z/c & 0 \end{bmatrix}. \tag{55}$$

It was shown in Paper III that the corresponding field tensor $F'_{\alpha\beta}$ in tachyonic frame Σ' could be found by using either the general transformation matrices such that

$$F'_{\alpha\beta} = \sum_{\mu=1}^{4} \sum_{\nu=1}^{4} L_{\alpha\mu} F_{\mu\nu} L'_{\nu\beta}\,, \tag{56}$$

or by simply transforming each component of $F_{\mu\nu}$ into the corresponding component of $F'_{\alpha\beta}$ using the transformations of the fields **E** and **B**. This was a direct consequence of the fact that the transformations of **E** and **B** are the same in SR and ER.

Other invariances involving electrodynamic quantities can also be shown to hold for both $u^2 < c^2$ and $u^2 > c^2$. The four-tensors $F^*_{\mu\nu}$, $G_{\mu\nu}$ and $G^*_{\mu\nu}$ are defined to be (Rosser 1964):

$$F^*_{\mu\nu} = \begin{bmatrix} 0 & -iE_z/c & iE_y/c & B_x \\ iE_z/c & 0 & -iE_x/c & B_y \\ -iE_y/c & iE_x/c & 0 & B_z \\ -B_x & -B_y & -B_z & 0 \end{bmatrix}, \tag{57}$$

$$G_{\mu\nu} = \begin{bmatrix} 0 & H_z & -H_y & -icD_x \\ -H_z & 0 & H_x & -icD_y \\ H_y & -H_x & 0 & -icD_z \\ icD_x & icD_y & icD_z & 0 \end{bmatrix}, \tag{58}$$

$$G^*_{\mu\nu} = \begin{bmatrix} 0 & -icD_z & icD_y & H_x \\ icD_z & 0 & -icD_x & H_y \\ -icD_y & icD_x & 0 & H_z \\ -H_x & -H_y & -H_z & 0 \end{bmatrix}. \tag{59}$$

The corresponding tensors in frame Σ', $F'_{\mu\nu}$, ${F'}^*_{\mu\nu}$, $G'_{\mu\nu}$ and ${G'}^*_{\mu\nu}$, are defined similarly to (55) to (59), except that primed variables replace unprimed variables. The Euclidean metric $(+1, +1, +1, +1)$ is being used with coordinates given by (x, y, z, ict), and so there is no distinction made here between covariant and contravariant quantities. It can be shown by substitution that the following six quantities are invariant for $-\infty < u < \infty$:

$$\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} F^2_{\mu\nu} = 2(B^2 - E^2/c^2) = 2({B'}^2 - {E'}^2/c^2) = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} {F'}^2_{\mu\nu}\,, \tag{60}$$

$$\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} F_{\mu\nu}F^*_{\mu\nu} = -4i\mathbf{B}.\mathbf{E}/c = -4i\mathbf{B}'.\mathbf{E}'/c = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} F'_{\mu\nu}{F'}^*_{\mu\nu}\,, \tag{61}$$

$$\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} G^2_{\mu\nu} = 2(H^2 - c^2D^2) = 2({H'}^2 - c^2{D'}^2) = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} {G'}^2_{\mu\nu}\,, \tag{62}$$

$$\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} G_{\mu\nu}G^{*}_{\mu\nu} = -4ic\mathbf{H}.\mathbf{D} = -4ic\mathbf{H}'.\mathbf{D}' = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} G'_{\mu\nu}{G'}^{*}_{\mu\nu}\,, \tag{63}$$

$$\begin{aligned}\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} F^{*}_{\mu\nu}G_{\mu\nu} &= -2i(c\mathbf{D}.\mathbf{B} + \mathbf{H}.\mathbf{E}/c) \\ &= -2i(c\mathbf{D}'.\mathbf{B}' + \mathbf{H}'.\mathbf{E}'/c) = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} {F'}^{*}_{\mu\nu}G'_{\mu\nu}\,,\end{aligned} \tag{64}$$

$$\sum_{\mu=1}^{4}\sum_{\nu=1}^{4} F_{\mu\nu}G_{\mu\nu} = 2(\mathbf{B}.\mathbf{H} - \mathbf{D}.\mathbf{E}) = 2(\mathbf{B}'.\mathbf{H}' - \mathbf{D}'.\mathbf{E}') = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} F'_{\mu\nu}G'_{\mu\nu}\,. \tag{65}$$

In the Recami (1986) formulation of tachyon theory, (60) to (65) do not apply as these quantities are not invariant for $u^2 > c^2$: this is one of several points of difference between his work and the present formulation. The cause of the noninvariance in the Recami formulation is the factor of $\pm i$ in his electromagnetic field transformations, which result in minus signs appearing on the right-hand sides of (60) to (65) when transforming from bradyonic to tachyonic frames and vice versa.

Using the above definitions of $F_{\mu\nu}$ and $G_{\mu\nu}$ allows Maxwell's equations to be written in a more compact, though less transparent, form (Rosser 1964). The first pair of Maxwell's equations can be written as a single tensor equation:

$$\frac{\partial F_{\nu\sigma}}{\partial X_\alpha} + \frac{\partial F_{\sigma\alpha}}{\partial X_\nu} + \frac{\partial F_{\alpha\nu}}{\partial X_\sigma} = 0\,, \tag{66}$$

where ν, σ and α run from 1 to 4 and $X_\alpha = (x, y, z, ict)$. The second pair of Maxwell's equations, (43) and (44), can be combined into a single tensor equation:

$$\partial G_{\nu\sigma}/\partial X_\sigma = J_\nu\,. \tag{67}$$

The two compact equations (66) and (67) can be written in frame Σ' simply by replacing unprimed quantities with primed quantities. By substituting the SLTs from (1) and the appropriate transformations of **E**, **B**, **D** and **H** into each component, it can be verified that these compact forms apply when $u^2 > c^2$, as well as for the standard SR case of $u^2 < c^2$.

8. Transformations of P and M

For stationary matter the polarisation vector **P** is defined as the dipole moment induced per unit volume of a dielectric due to the influence of an applied electric field (Rosser 1964). The relationship between **P**, **D** and **E** is expressed as

$$\mathbf{P} = \mathbf{D} - \epsilon_0\mathbf{E}\,, \tag{68}$$

and is applicable to both stationary and moving frames. For stationary matter the magnetisation vector **M** is defined as the magnetic dipole moment induced per unit volume by an applied magnetic field. The relationship between **M**, **B** and **H** is expressed as:

$$\mathbf{M} = \mathbf{B}/\mu_0 - \mathbf{H}\,, \tag{69}$$

which also applies in both stationary and moving frames.

Now suppose that the medium is at rest in tachyonic frame Σ', so that its polarisation and magnetisation are given by

$$\mathbf{P}' = \mathbf{D}' - \epsilon_0 \mathbf{E}'\,, \tag{70}$$

$$\mathbf{M}' = \mathbf{B}'/\mu_0 - \mathbf{H}'\,. \tag{71}$$

In bradyonic frame Σ (the laboratory frame) the apparent polarisation is given by (68). By substituting the transformations of **D** and **E** into each component of **P** and then comparing with the appropriate components of (70) and (71), it can be shown that

$$P_x = P_{x'},\ P_y = \gamma_u(P_{y'} - uM_{z'}/c^2),\ P_z = \gamma_u(P_{z'} + uM_{y'}/c^2)\,. \tag{72}$$

The inverse transformations of these components are

$$P_{x'} = P_x,\ P_{y'} = \gamma_u(P_y + uM_z/c^2),\ P_{z'} = \gamma_u(P_z - uM_y/c^2)\,. \tag{73}$$

Just as for the transformations of **E**, **B**, **D** and **H**, the transformation of the electric polarisation is the same in both SR and ER. This was to be expected considering how **P** is directly related to **D** and **E**.

Now consider the magnetisation of the material. In tachyonic frame Σ' the medium is at rest so that its magnetisation $\mathbf{M}'$ is given by (71). The magnetisation in bradyonic frame Σ is given by (69). Substituting the transformations of **B** and **H** into each component of (69) and comparing each of the resulting relations with the corresponding components of (70) and (71) shows that

$$M_x = M_{x'},\ M_y = \gamma_u(M_{y'} + uP_{z'}),\ M_z = \gamma_u(M_{z'} - uP_{y'})\,. \tag{74}$$

The inverse transformations are

$$M_{x'} = M_x,\ M_{y'} = \gamma_u(M_y - uP_z),\ M_{z'} = \gamma_u(M_z + uP_y)\,, \tag{75}$$

and so the magnetisation also has the same form of transformation for SR and ER.

The transformations of $\mathbf{P}'$ and $\mathbf{M}'$ can be written in vector form as

$$\mathbf{P}'_{\parallel} = (\mathbf{P} - \mathbf{u} \times \mathbf{M}/c^2)_{\parallel},\ \mathbf{P}'_{\perp} = \gamma_u(\mathbf{P} - \mathbf{u} \times \mathbf{M}/c^2)_{\perp}\,, \tag{76}$$

$$\mathbf{M}'_{\parallel} = (\mathbf{M} + \mathbf{u} \times \mathbf{P})_{\parallel},\ \mathbf{M}'_{\perp} = \gamma_u(\mathbf{M} + \mathbf{u} \times \mathbf{P})_{\perp}\,. \tag{77}$$

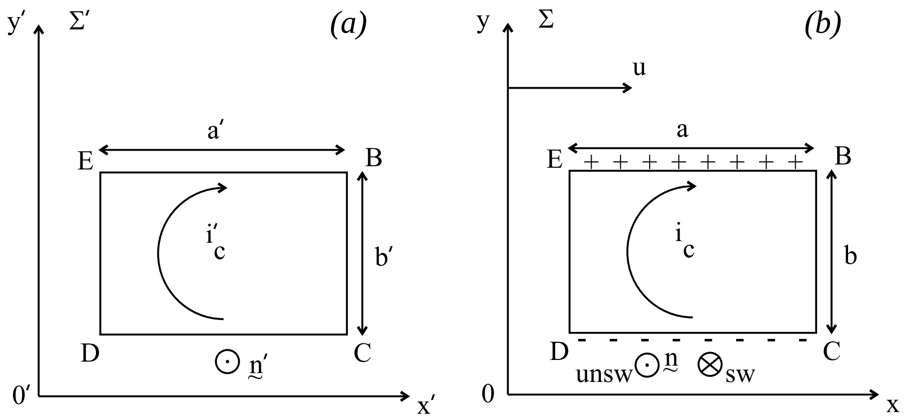


Fig. 2. Electric dipole moment of a moving coil. In Fig. 2a the coil is at rest in frame Σ' and carries a conduction current i'_c. The wire is a rectangle of sides a' and b' and has a uniform cross-sectional area A'. In Fig. 2b the coil has speed u along the common x, x' axes relative to frame Σ, and so has an electric dipole moment due to an apparent separation of charge along the arms EB and CD. The possibility of switching in frame Σ if $u > c$ means that the unit vector normal to the plane of the coil may appear to reverse direction.

9. Electric Dipole Moment of a Tachyonic Current Loop

In this section the electric dipole moment of a simple current loop moving with speed $u > c$ will be investigated. As with much of the material in this paper, the following discussion is adapted from the corresponding relativistic case given by Rosser (1964).

Suppose there is a current-carrying coil at rest in frame Σ', as shown in Fig. 2a. For simplicity the coil is treated as a rectangle with area $EB \times BC = a' \times b'$. The coil is in the x', y' plane and carries a conduction current i'_c. The wire itself has a uniform rectangular cross-sectional area A', so that the current density inside the wire is $j_{x'} = j_{y'} = i'_c/A'$. The magnetic moment of the coil in frame Σ' is

$$\mathbf{m}' = \mathbf{n}' i'_c a' b' \,, \tag{78}$$

where $\mathbf{n}'$ is a unit vector normal to the plane of the coil and $\mathbf{m}'$ is real. For the coil illustrated in Fig. 2a the components of $\mathbf{m}'$ are

$$m_{x'} = m_{y'} = 0, \; m_{z'} = -i'_c a' b' \hat{n}_{z'} \,, \tag{79}$$

where $\hat{n}_{z'}$ is a unit vector in the z' direction.

Fig. 2b shows the same coil as seen by bradyonic observer Σ. In this frame the coil has speed u along the common x, x' axes, so that due to length contraction (or dilation) effects the apparent length a of sides EB and DC as measured in frame Σ is

$$\begin{aligned} a &= a'(1 - u^2/c^2)^{\frac{1}{2}} \text{ for } u^2 < c^2 \,, \\ a &= a'|(1 - u^2/c^2)^{\frac{1}{2}}| \text{ for } u^2 > c^2 \,. \end{aligned} \tag{80}$$

Even for tachyonic objects all lengths are positive and real: see Paper I for further discussion of this point. For the transverse sides DE and BC the apparent length is

$$b = b' \text{ for } u^2 < c^2 ,$$
$$b = |-ib'| = b' \text{ for } u^2 > c^2 . \tag{81}$$

In frame Σ the apparent cross-sectional area of the wire, A, is also affected by length contraction (or dilation). For sides EB and CD the apparent cross-sectional area in frame Σ is $A = A'$ for $-\infty < u < \infty$. For sides BC and DE the apparent cross-sectional area is

$$A = A'(1-u^2/c^2)^{\frac{1}{2}} \text{ for } u^2 < c^2 ,$$
$$A = A'|(1-u^2/c^2)^{\frac{1}{2}}| \text{ for } u^2 > c^2 . \tag{82}$$

Now assume that there is an equal number of positive and negative charges in each of the four sides of the coil, so that the effective charge density in frame Σ' is $\rho' = 0$. In frame Σ the apparent electric charge density ρ is found using the SR and ER transformations of charge density (see Paper III) to be

$$\rho = u\gamma_u j_{x'}/c^2 \text{ for } u^2 < c^2 ,$$
$$\rho = iu\gamma_u j_{x'}/c^2 \text{ for } u^2 > c^2 . \tag{83}$$

The total electric charge Q along the arm EB is $Q = a\rho A$, so that

$$Q = a'ui'_c/c^2 \text{ for } u^2 < c^2 ,$$
$$Q = \mathrm{sign}(\gamma_u)a'ui'_c/c^2 \text{ for } u^2 > c^2 . \tag{84}$$

The γ-rule and the convention for decomposing imaginary square roots (6) have been used in obtaining (84). As the current is in the opposite direction along the arm CD, the total electric charge along that arm as measured in frame Σ is

$$-Q = -a'ui'_c/c^2 \text{ for } u^2 < c^2 ,$$
$$-Q = -\mathrm{sign}(\gamma_u)a'ui'_c/c^2 \text{ for } u^2 > c^2 . \tag{85}$$

Therefore in frame Σ there is an effective charge separation between the arms EB and CD, and so there must be an electric dipole moment in the y direction given by $p_y = Qb = a\rho Ab$ for $-\infty < u < \infty$, with Q being given by (84).

The apparent magnetic moment as observed in frame Σ can be affected by switching, due to the possible reversal of the unit vector. In Fig. 2a the unit vector $\mathbf{n}'$ points out of the page. When the coil moves, as shown in Fig. 2b, the arm BC leads DE in the increasing x direction for unswitched frames. However, in frames in which the coil appears to have undergone switching, the arm BC still leads DE but in the decreasing x direction. (See Paper I for a full discussion of this effect for rods.) Hence the unit vector normal to the plane of the coil in frame Σ points out of the page for an unswitched coil and into the page for a switched coil: this is expressed as $\mathbf{n} = \mathrm{sign}(\gamma_u)\mathbf{n}'$. Using this expression in (79) shows that the magnetic moment is

$$m_{z'} = -\mathrm{sign}(\gamma_u) i'_c a' b' \hat{n}_{z'} \,. \tag{86}$$

Combining the above relations for $m_{z'}$, b and $a\rho A$ gives the apparent electric dipole moment of the coil measured in frame Σ as

$$p_y = -u m_{z'}/c^2 \tag{87}$$

for $-\infty < u < \infty$. This result can be generalised, so that a current loop which is at rest in frame Σ' has an electric dipole moment in bradyonic frame Σ equal to

$$\mathbf{p} = \mathbf{u} \times \mathbf{m}'/c^2 \tag{88}$$

for $-\infty < u < \infty$. Here $\mathbf{m}'$ is the magnetic moment measured in frame Σ' and $\mathbf{u}$ is the uniform velocity of frame Σ' relative to frame Σ.

10. Constitutive Equations

Consider a material medium for which the relative permeability and permittivity are not necessarily equal to one. In such a medium the values of the field quantities become dependent upon the medium's properties, while Maxwell's equations have to be supplemented by the constitutive equations in order to solve problems. In the discussion that follows, which has been adapted from Rosser (1964), it will be assumed that κ_m, ϵ and σ can be considered as constants. Here κ_m is the relative permeability, ϵ is the dielectric coefficient and σ is the electrical conductivity. While this assumption is not appropriate for ferromagnetic materials, in which κ_m is not a constant but instead depends on $\mathbf{H}$, there is a wide range of materials for which κ_m, ϵ and σ are virtually constant.

When the material is at rest in frame Σ' the constitutive equations take the form:

$$\mathbf{B}' = \kappa_m \mu_0 \mathbf{H}' \,, \tag{89}$$

$$\mathbf{D}' = \epsilon\epsilon_0 \mathbf{E}' \,, \tag{90}$$

$$\mathbf{j}' = \sigma \mathbf{E}' \,. \tag{91}$$

Combining (53) with $\mathbf{E}'_\parallel = (\mathbf{E} + \mathbf{u} \times \mathbf{B})_\parallel$ and $\mathbf{E}'_\perp = \gamma_u(\mathbf{E} + \mathbf{u} \times \mathbf{B})_\perp$ means (90) can be written in frame Σ as

$$\mathbf{D} + \mathbf{u} \times \mathbf{H}/c^2 = \epsilon\epsilon_0(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \,, \tag{92}$$

which applies for $-\infty < u < \infty$. For $\mathbf{u} = \mathbf{0}$ this reduces to the standard relation $\mathbf{D} = \epsilon\epsilon_0 \mathbf{E}$.

In a similar manner, the parallel and perpendicular components of $\mathbf{B}'$ and $\mathbf{H}'$ can be used to rewrite the constitutive equation (89) as

$$\mathbf{B} - \mathbf{u} \times \mathbf{E}/c^2 = \kappa_m \mu_0 (\mathbf{H} - \mathbf{u} \times \mathbf{D}) \,, \tag{93}$$

which also applies for $-\infty < u < \infty$.

The tachyonic transformations of $\mathbf{j}'$ and $\mathbf{E}'$ can be used to transform the third constitutive equation from the tachyonic frame Σ' to the bradyonic frame Σ. In component form (91) becomes

$$-i\gamma_u (j_x - u\rho) = \sigma E_x, \; -ij_y = \sigma \gamma_u (E_y - uB_z), \; -ij_z = \sigma \gamma_u (E_z + uB_y) \,. \tag{94}$$

These expressions can be written in vector form as

$$-i\gamma_u (\mathbf{j} - \mathbf{u}\rho)_\parallel = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B})_\parallel, \; -i(\mathbf{j} - \mathbf{u}\rho)_\perp = \sigma \gamma_u (\mathbf{E} + \mathbf{u} \times \mathbf{B})_\perp \,, \tag{95}$$

where again the subscripts $\parallel$ and $\perp$ refer to components parallel and perpendicular to the boost respectively. Here the tachyonic transformations have caused the appearance of a factor of $-i$, so that (95) only applies for $u^2 > c^2$. The corresponding SR version is similar to (95), except that the factor of $-i$ does not appear. As γ_u is imaginary for $u^2 > c^2$, then the imaginary factors cancel out to give real quantities as measured in bradyonic frame Σ. Note that as $\mathbf{j}$ is the total current density, it includes the convection current density $\mathbf{u}\rho$. Thus the term $\mathbf{j} - \mathbf{u}\rho$ is equal to the conduction current density.

The scalar and vector potentials are also modified in media for which κ_m and ϵ are not unity. Using the constitutive equations in one of Maxwell's equations (11) in frame Σ' gives

$$\nabla' \times \mathbf{B}' = \kappa_m \mu_0 \mathbf{j}' + (\kappa_m \epsilon / c^2) \partial \mathbf{E}' / \partial t' \,. \tag{96}$$

Using the primed equivalents of (14) and (15) and the vector identity $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla.\mathbf{A}) - \nabla^2 \mathbf{A}$ in (96) leads to

$$\nabla'^2 \mathbf{A}' - \frac{\kappa_m \epsilon}{c^2} \frac{\partial^2 \mathbf{A}'}{\partial t'^2} = -\kappa_m \mu_0 \mathbf{j}' \,, \tag{97}$$

where use has been made of the Lorentz condition

$$\nabla'.\mathbf{A}' + \frac{\kappa_m \epsilon}{c^2} \frac{\partial \phi'}{\partial t'} = 0 \tag{98}$$

to simplify the expression.

From Maxwell's equations, if ρ' is the proper charge density at a point then $\nabla'.\mathbf{E}' = \rho'/\epsilon\epsilon_0$. Substituting for $\mathbf{E}'$ from the primed equivalent of (14) and using (98) leads to

$$\nabla'^2 \phi' - \frac{\kappa_m \epsilon}{c^2} \frac{\partial^2 \phi'}{\partial t'^2} = -\rho'/\epsilon\epsilon_0 \,. \tag{99}$$

Equations (97), (98) and (99) apply for both bradyons and tachyons. The corresponding equations in bradyonic frame Σ can be written simply by unpriming all of the variables.

11. Polarisation of a Tachyonic Dielectric Medium

In free space the velocity of light has been postulated to be equal to c for all inertial reference frames, regardless of whether the observer's frame is bradyonic or tachyonic. However, the speed of light in a material is modified by the properties of that material and is no longer equal to c. It is known that for a bradyonic medium with refractive index $n > 1$ the apparent speed of light is less than c, and as n increases the speed of light in that bradyonic medium decreases. In the next section the speed of light in a tachyonic medium will be investigated for a simple material in which $\kappa_m = 1$ but $\epsilon \neq 1$. The magnetisation and polarisation of such a medium will first be investigated, using a discussion adapted from the relativistic case given by Rosser (1964).

Suppose that light is travelling through a material medium whose speed is such that $u^2 > c^2$ relative to the observer. In the tachyonic inertial reference frame Σ' the medium is at rest and has a dielectric coefficient $\epsilon \neq 1$. It is assumed that the material is nonmagnetic, so that the relative permeability is $\kappa_m = 1$. Hence $\mathbf{M}' = 0$ and so the magnetic field is given by $\mathbf{B}' = \mu_0 \mathbf{H}'$.

In bradyonic frame Σ the material has velocity $\mathbf{u}$ such that $u^2 > c^2$. The polarisation in frame Σ is given by

$$\mathbf{P}_{\parallel} = \mathbf{P}'_{\parallel},\ \mathbf{P}_{\perp} = \gamma_u \mathbf{P}'_{\perp}\,, \tag{100}$$

while the magnetisation is

$$\mathbf{M}_{\parallel} = \mathbf{M}'_{\parallel} = 0,\ \mathbf{M}_{\perp} = -(\mathbf{u} \times \mathbf{P})_{\perp}\,. \tag{101}$$

Note that (100) and (101) would also apply if $u^2 < c^2$. As $(\mathbf{u} \times \mathbf{P}')_{\parallel} = 0$ and $\mathbf{P}_{\parallel} = \mathbf{P}'_{\parallel}$ in this example, then $\mathbf{M}_{\parallel}$ can be written as $\mathbf{M}_{\parallel} = -(\mathbf{u} \times \mathbf{P})_{\parallel} = 0$, which allows the magnetisation vector to be written in a more general form as $\mathbf{M} = -\mathbf{u} \times \mathbf{P}$. In frame Σ this gives $\mathbf{H} = \mathbf{B}/\mu_0 + \mathbf{u} \times \mathbf{P}$, so that one of Maxwell's equations can be rearranged to give

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \nabla \times (\mathbf{P} \times \mathbf{u}) + \epsilon_0 \partial \mathbf{E}/\partial t + \partial \mathbf{P}/\partial t\right)\,, \tag{102}$$

where $\mathbf{P} \times \mathbf{u} = -\mathbf{u} \times \mathbf{P}$ and $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$. As a check it can be seen that setting $\mathbf{P} = \mathbf{0}$ recovers (11).

In tachyonic frame Σ' the constitutive equation $\mathbf{D}' = \epsilon\epsilon_0 \mathbf{E}'$ means that $\mathbf{P}' = \epsilon_0(\epsilon - 1)\mathbf{E}'$ and combining this expression with the components of $\mathbf{E}'$ leads to

$$\mathbf{P}'_{\parallel} = \epsilon_0(\epsilon - 1)(\mathbf{E} + \mathbf{u} \times \mathbf{B})_{\parallel},\ \mathbf{P}'_{\perp} = \epsilon_0 \gamma_u (\epsilon - 1)(\mathbf{E} + \mathbf{u} \times \mathbf{B})_{\perp}\,. \tag{103}$$

Combining (100) and (103) shows that

$$\mathbf{P}_{\parallel} = \epsilon_0(\epsilon - 1)(\mathbf{E} + \mathbf{u} \times \mathbf{B})_{\parallel},\ \mathbf{P}_{\perp} = \epsilon_0 \gamma_u^2 (\epsilon - 1)(\mathbf{E} + \mathbf{u} \times \mathbf{B})_{\perp}\,, \tag{104}$$

which applies for $-\infty < u < \infty$ as the equations and transformations used thus far in the discussion are the same in SR and ER.

The perpendicular component of the polarisation has an extra factor of γ_u^2 compared to the parallel component and so the polarisation is anisotropic. For $u^2 > c^2$ the factor $\gamma_u^2(\epsilon - 1)$ becomes negative, whereas it is always positive for $u^2 < c^2$. Thus there is a clear difference in the possible values measured for $\mathbf{P}_\perp$ in ER and SR. For large relative speeds such that $u^2 \gg c^2$ the polarisation becomes heavily anisotropic and greatly favours the direction parallel to the boost. In the limit as $u \to c^+$ (i.e. u is just above c) the transverse polarisation $\mathbf{P}_\perp$ differs by a sign from the SR case in which $u \to c^-$ (i.e. u is just below c). For $u \to c^+$ the parallel polarisation $\mathbf{P}_\parallel$ becomes virtually the same as for $u \to c^-$. Note that the magnitude of $\mathbf{P}_\perp$ is extremely large in the limits $u \to c^\pm$.

12. Velocity of Light in a Tachyonic Dielectric Medium

Many textbooks on special relativity which treat the problem of the speed of light in a material medium only present a first order theory, for which $u^2 \ll c^2$. In that case $\gamma_u^2 \approx 1$ and the polarisation becomes virtually isotropic. However, the approximation $\gamma_u^2 \approx 1$ is generally invalid in ER unless $u^2 \approx 2c^2$, and so in ER the solution for terms including u^2/c^2 needs to be calculated.

In order to simplify the following derivation, it is now assumed that the medium is uncharged so that $\rho' = 0$. The medium is nonmagnetic so that $\kappa_m = 1$ and the current in Σ' is $\mathbf{j}' = \mathbf{0}$. The medium, which is at rest in tachyonic frame Σ', is moving in the positive x direction with uniform velocity $\mathbf{u}$ relative to a bradyonic frame Σ. It is now assumed that there is a plane-polarised plane wave moving in the positive x direction parallel to $\mathbf{u}$. The electric and magnetic vectors of the plane wave are in the y and z direction respectively. For this case $v_x = u$, $v_y = v_z = 0$, $\mathbf{B} = B_z$ and $\mathbf{E} = E_y$ and so $\mathbf{P} \times \mathbf{u} = -\hat{\mathbf{k}} P_y u$, where $\hat{\mathbf{k}}$ is a unit vector in the z direction. The only component of $\mathbf{P}$ which is nonzero is

$$P_y = \epsilon_0 \gamma_u^2 (\epsilon - 1)(E_y - uB_z)\,. \tag{105}$$

The y component of (102) is

$$-\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} + \mu_0 \frac{\partial P_y}{\partial t} + \mu_0 u \frac{\partial P_y}{\partial x}\,. \tag{106}$$

The only nonzero component of (9) is $\partial E_y/\partial x = -\partial B_z/\partial t$, so that taking the partial derivative with respect to x gives $\partial^2 E_y/\partial x^2 = -\partial^2 B_z/\partial t \partial x$. The expression for P_y given by (105) can be substituted into (106) and, after collecting terms and then taking the partial derivative with respect to t, we have

$$-\gamma_u^2 \left(1 - \frac{\epsilon u^2}{c^2}\right) \frac{\partial^2 B_z}{\partial x \partial t} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} + \frac{\gamma_u^2(\epsilon - 1)}{c^2} \left(\frac{\partial^2}{\partial t^2} + 2u \frac{\partial^2}{\partial x \partial t}\right) E_y\,. \tag{107}$$

Combining this with the above expression for $\partial^2 E_y/\partial x^2$ leads to the wave equation for the propagation of light in a stationary dielectric medium:

$$\frac{\partial^2 E_y}{\partial x^2} = \left(\frac{\epsilon - u^2/c^2}{c^2 - \epsilon u^2}\right) \frac{\partial^2 E_y}{\partial t^2} + \left(\frac{2u(\epsilon - 1)}{c^2 - \epsilon u^2}\right) \frac{\partial^2 E_y}{\partial t \partial x}\,. \tag{108}$$

This expression is valid under the above assumptions for $-\infty < u < \infty$. In the limiting case of $u \to 0$ then (108) reduces to

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\epsilon}{c^2}\frac{\partial^2 E_y}{\partial t^2}\,, \tag{109}$$

which is the ordinary wave equation for the propagation of light in a stationary medium having dielectric coefficient ϵ. In the opposite limit of $u \to \infty$ then (108) instead reduces to

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{\epsilon c^2}\frac{\partial^2 E_y}{\partial t^2}\,. \tag{110}$$

In the ultrarelativistic limit as $u \to c^{\pm}$ then (108) reduces to

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} - \frac{2}{c}\frac{\partial^2 E_y}{\partial t \partial x} \tag{111}$$

for both bradyons and tachyons.

As the electric and magnetic fields form a plane wave in this example, it is assumed that the solution of (108) is of the form

$$E_y = E_0 e^{2\pi i(x/\lambda - \nu t)}\,, \tag{112}$$

where $v = \lambda\nu$ is the velocity of the plane wave, λ is its wavelength and ν is its frequency. The idea of a tachyonic plane wave with regard to tachyonic optics was discussed in Paper III, where it was pointed out that a plane wave in a tachyonic frame is also a plane wave when viewed from a bradyonic frame. Hence the assumed form of the solution to (108) is expected to be valid for the full range of relative speeds $-\infty < u < \infty$. From (112) we have

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{4\pi^2\nu^2 E_y}{v^2},\ \frac{\partial^2 E_y}{\partial t^2} = -4\pi^2\nu^2 E_y,\ \frac{\partial^2 E_y}{\partial t \partial x} = \frac{4\pi^2\nu^2 E_y}{v}\,. \tag{113}$$

Substituting these expressions into (108), cancelling common factors and solving the resulting quadratic in v leads to

$$v_{\pm} = \frac{u(\epsilon - 1) \pm c\epsilon^{\frac{1}{2}}(1 - u^2/c^2)}{\epsilon - u^2/c^2}\,. \tag{114}$$

Here the upper signs apply for light travelling in the positive x direction and the lower signs apply for light travelling in the negative x direction. The speeds v_+ and v_- are the speed of light inside a dielectric material as measured by a bradyonic observer Σ. The material itself travels with speed u relative to Σ such that $0 \le u < \infty$ for the upper signs and $-\infty < u \le 0$ for the lower signs, so that (114) is valid in both SR and ER.

In the limit as $u \to c^{\pm}$ then the speed of light in the dielectric measured by Σ is $v_{\pm} \to +c$, and in the limit as $u \to -c^{\pm}$ then $v_{\pm} \to -c$ as expected. In the limit as $u \to \pm\infty$ then $v_\pm \to \pm c\epsilon^{\frac{1}{2}}$. When u is small such that $u^2 \ll c^2$ then (114) reduces to the first order theory result for SR:

$$v_\pm \approx \pm c\epsilon^{-1/2} + u(1-\epsilon^{-1})\,. \tag{115}$$

As $u \to 0$ this gives the standard result $v_\pm \approx \pm c\epsilon^{-1/2}$.

The dielectric coefficient ϵ is related to the refractive index n via the relation $n = \epsilon^{\frac{1}{2}}$. Here n is defined to be the refractive index of the medium when it is at rest, that is, n is its 'proper refractive index'. Rewriting (114) and factorising the denominator gives

$$v_\pm = \frac{c(nu \pm c)}{nc \pm u}\,, \tag{116}$$

which is valid for $-\infty < u < \infty$. Equation (116) agrees with the SR expression given by Bohm (1989) for the observed phase velocity when light is travelling through a moving fluid of refractive index n and speed u. Reversing the sign of u gives $v_+ = -v_-$, so that reversing the direction of motion of the medium has no effect on the speed of light in that material, regardless of whether the motion is bradyonic or tachyonic.

13. Tachyon Collision Energy Loss in a Dielectric Medium

This section studies the energy loss incurred by a charged particle in distant collisions with atoms of a medium, assuming that a continuum approximation of a macroscopic dielectric constant $\epsilon(\omega)$ can be used. The discussion closely follows the SR case given by Jackson (1975).

Consider a charged particle travelling through a medium with speed u along the x axis. The electromagnetic four-potential $A_\lambda = (\mathbf{A}, i\phi/c)$ and the source density $J_\lambda = (\mathbf{j}, ic\rho)$ can be Fourier transformed in space and time to give modified wave equations:

$$(k^2 - \omega^2\epsilon(\omega)/c^2)\phi(\mathbf{k},\omega) = \rho(\mathbf{k},\omega)/\epsilon_0\epsilon(\omega)\,, \tag{117}$$

$$(k^2 - \omega^2\epsilon(\omega)/c^2)\mathbf{A}(\mathbf{k},\omega) = \mu_0\mathbf{j}(\mathbf{k},\omega)\,. \tag{118}$$

Here ω is the frequency and $\mathbf{k}$ is the wavenumber. The Fourier transforms of ρ and $\mathbf{j}$ are found to be

$$\rho(\mathbf{k},\omega) = Q\delta(\omega - \mathbf{k}.\mathbf{u})/2\pi\,, \tag{119}$$

$$\mathbf{j}(\mathbf{k},\omega) = \mathbf{u}\rho(\mathbf{k},\omega)\,, \tag{120}$$

so that the scalar and vector potentials become

$$\phi(\mathbf{k},\omega) = \frac{Q\delta(\omega - \mathbf{k}.\mathbf{u})}{2\pi\epsilon_0\epsilon(\omega)(k^2 - \omega^2\epsilon(\omega)/c^2)}\,, \tag{121}$$

$$\mathbf{A}(\mathbf{k},\omega) = \epsilon(\omega)\mu_0\mathbf{u}\phi(\mathbf{k},\omega)\,. \tag{122}$$

From the definitions of the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ in terms of their potentials (14) and (15) we obtain their Fourier transforms:

$$\mathbf{E}(\mathbf{k},\omega) = i(\omega\epsilon(\omega)\mathbf{u}/c^2 - \mathbf{k})\phi(\mathbf{k},\omega)\,, \tag{123}$$

$$\mathbf{B}(\mathbf{k},\omega) = i\epsilon(\omega)\mu_0\mathbf{k}\times\mathbf{u}\phi(\mathbf{k},\omega)\,. \tag{124}$$

To calculate the energy loss we require the Fourier transform in time of the electromagnetic fields at a perpendicular distance b from the path of the particle:

$$\mathbf{E}(\omega) = (2\pi)^{-3/2}\int d^3k\mathbf{E}(\mathbf{k},\omega)e^{ibk_y}, \tag{125}$$

where the observation point has coordinates $(0, b, 0)$. Using (121) to (124) in (125) and solving the integrals using the same steps as in Jackson (1975) yields separate solutions based on whether $u^2 < c^2$ or $u^2 > c^2$. For $u^2 < c^2$ we have

$$E_x(\omega) = -\frac{iQ\omega}{(2\pi)^{\frac{3}{2}}\epsilon_0 u^2}\left(\frac{1}{\epsilon(\omega)} - \frac{u^2}{c^2}\right)K_0(\lambda b)\,, \tag{126}$$

$$E_y(\omega) = \frac{Q\lambda}{(2\pi)^{\frac{3}{2}}\epsilon_0\epsilon(\omega)u}K_1(\lambda b)\,, \tag{127}$$

$$B_z(\omega) = \epsilon(\omega)uE_y(\omega)/c^2\,, \tag{128}$$

where

$$\lambda^2 = \omega^2(1 - u^2\epsilon(\omega)/c^2)/u^2\,. \tag{129}$$

Here $K_\nu(\lambda b)$ are modified Bessel functions. For $u^2 > c^2$ we obtain instead

$$E_x(\omega) = \frac{i\pi Q\omega}{2(2\pi)^{\frac{3}{2}}\epsilon_0 u^2}\left(\frac{1}{\epsilon(\omega)} - \frac{u^2}{c^2}\right)N_0(\lambda_T b)\,, \tag{130}$$

$$E_y(\omega) = -\frac{Q\lambda_T}{4(2\pi)^{\frac{1}{2}}\epsilon_0\epsilon(\omega)u}N_1(\lambda_T b)\,, \tag{131}$$

$$B_z(\omega) = \epsilon(\omega)uE_y(\omega)/c^2\,, \tag{132}$$

where

$$\lambda_T^2 = -\omega^2(1 - u^2\epsilon(\omega)/c^2)/u^2\,. \tag{133}$$

Here $N_\nu(\lambda_T b)$ are Neumann functions. In Paper III it was noted that the test particle experiences two apparent fields generated by the passing tachyon: one field is generated by the tachyon as it recedes and the second field is simply the time delayed field generated by the tachyon during its approach. This 'two-source effect' is accounted for by the selection of appropriate sets of limits during the integrations. In solving the integrals for $u^2 > c^2$ particular attention must be paid to the conditions under which the integral solutions in references such as

Gradshteyn and Ryzhik (1994) are valid: in some cases it is necessary to go to prior sources to determine if a particular solution is valid for the conditions. In the ultrarelativistic limit $u \to c^{\pm}$ the corresponding bradyon and tachyon expressions for the electric fields give the same asymptotic behaviour, again demonstrating the indistinguishability of effects produced by these particles in such a limit.

To find the energy transferred to the atom at an impact parameter b we use

$$\Delta E(b) = 2q \sum_j f_j \mathrm{Re} \int_0^\infty i\omega \mathbf{x}_j(\omega).\mathbf{E}^*(\omega) d\omega \,, \tag{134}$$

where q is the charge of the atomic electron interacting with the incoming charged particle, $\mathbf{x}_j(\omega)$ is the amplitude of the jth type of electron in one atom and f_j is the number of electrons having the same harmonic binding frequency: $\sum f_j = Z$. There are N atoms per unit volume with Z electrons per atom. The sum of dipole moments is expressed in terms of the molecular polarisability, or rather the dielectric constant:

$$-q \sum_j f_j \, \mathbf{x}_j(\omega) = \frac{\epsilon_0}{N}(\epsilon(\omega) - 1)\mathbf{E}(\omega) \,. \tag{135}$$

The energy transfer can be written as

$$\Delta E(b) = \frac{\epsilon_0}{N} \mathrm{Re} \int_0^\infty -i\omega\epsilon(\omega)|\mathbf{E}(\omega)|^2 d\omega \,. \tag{136}$$

This expression applies regardless of whether the incoming particle is a bradyon or a tachyon.

The energy loss per unit distance is proportional to the energy lost per unit time as tachyons obey the law of conservation of energy (Papers I and II). The energy lost per unit time by the incident particle can be obtained by calculating the electromagnetic energy flow through a cylinder of radius a around the path of the particle. Thus we have

$$\left(\frac{dE}{dx}\right)_{b>a} = \frac{1}{u}\frac{dE}{dt} = -\frac{1}{\mu_0 u}\int_{-\infty}^{\infty} 2\pi a B_z E_x dx \,. \tag{137}$$

Converting the integral over dx at one instant of time to an integral at one point on the cylinder over all time, then in turn converting this to a frequency integral gives

$$\left(\frac{dE}{dx}\right)_{b>a} = -\frac{4\pi a}{\mu_0}\mathrm{Re}\int_0^\infty B_z^*(\omega)E_x(\omega)d\omega \,, \tag{138}$$

where the asterisk denotes complex conjugation. Substituting the fields (130) to (132) for $u^2 > c^2$ gives:

$$\left(\frac{dE}{dx}\right)_{b>a} = \frac{Q^2}{8\epsilon_0 u^2}\mathrm{Re}\int_0^\infty i\omega\lambda_T^* a N_1(\lambda_T^* a) N_0(\lambda_T a)\left(\frac{1}{\epsilon(\omega)} - \frac{u^2}{c^2}\right) d\omega \,. \tag{139}$$

This expression describes the observed energy loss as the charged tachyon travels through the dielectric medium. The corresponding bradyonic expression is

$$\left(\frac{dE}{dx}\right)_{b>a} = \frac{Q^2}{2\pi^2\epsilon_0 u^2}\mathrm{Re}\int_0^\infty i\omega\lambda^* aK_1(\lambda^* a)K_0(\lambda a)\left(\frac{1}{\epsilon(\omega)} - \frac{u^2}{c^2}\right)d\omega\,. \quad (140)$$

Now assume that the radius a around the path of the tachyon is of the order of atomic dimensions, and that $|\lambda_T a| \sim (\omega a/c) \ll 1$. In the ultrarelativistic limit $u/c \approx 1$ and the Neumann functions can be approximated by their small argument limits, so that (132) becomes

$$\left(\frac{dE}{dx}\right)_{b>a} \approx \frac{Q^2}{2\pi^2\epsilon_0 c^2}\mathrm{Re}\int_0^\infty i\omega\left(\frac{1}{\epsilon(\omega)} - 1\right)\left[\ln\left(\frac{1\cdot 123c}{a\omega}\right) - \tfrac{1}{2}\ln(1-\epsilon(\omega))\right]d\omega\,. \quad (141)$$

Performing the integral as per Jackson (1975) gives

$$\left(\frac{dE}{dx}\right)_{b>a} = \frac{Q^2\omega_p^2}{4\pi\epsilon_0 c^2}\ln\left(\frac{1\cdot 123c}{a\omega_p}\right), \quad (142)$$

where ω_p is the plasma frequency, defined by

$$\omega_p^2 = NZq^2/\epsilon_0 m_0\,. \quad (143)$$

This result is the same as for the ultrarelativistic bradyonic case with $|\lambda a| \sim (\omega a/c) \ll 1$ and $u/c \approx 1$. Thus the charged tachyon exhibits similar behaviour to a charged bradyon under these conditions.

14. Tachyons and Cherenkov Radiation

In Paper II it was argued that tachyons travelling through a vacuum would not spontaneously emit Cherenkov radiation. However, it is necessary to consider the behaviour of tachyons as they travel through a material medium in some detail. It has already been seen how charged tachyons can interact electromagnetically with test particles and produce electromagnetic fields which are real inside a Mach cone and so are intrinsically detectable. The derivation below closely follows that given by Jackson (1975) for the case of relativistic charged particles moving with constant velocity through a dielectric medium.

Consider the opposite limit to that discussed in the previous section: here $|\lambda_T a| \gg 1$. In this case the Neumann functions again take asymptotic forms so that the electromagnetic fields generated by the charged tachyon (130) to (132) become

$$E_x(\omega, b) \to \frac{iQ\omega}{4\pi\epsilon_0 c^2}\left(\frac{c^2}{u^2\epsilon(\omega)} - 1\right)\frac{\sin(\lambda_T b - \pi/4)}{(\lambda_T b)^{\frac{1}{2}}}\,, \quad (144)$$

$$E_y(\omega, b) \to -\frac{Q(\lambda_T/b)^{\frac{1}{2}}}{4\pi\epsilon_0\epsilon(\omega)u}\sin(\lambda_T b - 3\pi/4)\,, \tag{145}$$

$$B_z(\omega, b) \to \epsilon(\omega)uE_y(\omega, b)/c^2\,. \tag{146}$$

The integrand in (138) in this limit is

$$-\frac{4\pi a}{\mu_0}B_z^*(\omega, b)E_x(\omega, b) \to$$

$$\frac{Q^2\omega}{8\pi\epsilon_0 c^2}\left(-i\sqrt{\frac{\lambda_T^*}{\lambda_T}}\right)\left(\frac{c^2}{u^2\epsilon(\omega)}-1\right)\left[\sin(\lambda_T^* b - \lambda_T b) + \cos(\lambda_T^* b + \lambda_T b)\right]\,. \tag{147}$$

The real part of this expression, integrated over frequencies, gives the energy deposited far from the path of the tachyon. The trigonometric factor is complex since λ_T is a complex number and b is real: this replaces an exponential term in the bradyon case (Jackson 1975) of $e^{-(\lambda^*+\lambda)a}$.

Now assume that $\epsilon(\omega)$ is purely real: this is also done in the analogous bradyonic case. When λ_T is purely real the expression (147) becomes purely imaginary and so the charged tachyon does not lose energy as radiation as it travels through the medium. At the other extreme, when λ_T is purely imaginary then (147) becomes real and so the charged tachyon can lose energy via radiation. However, (133) shows that for λ_T to be purely imaginary requires that $u^2\epsilon(\omega) < c^2$. Thus the condition for the tachyon to emit Cherenkov radiation when travelling through the bradyonic dielectric medium is

$$u < c/\sqrt{\epsilon(\omega)}\,. \tag{148}$$

Hence the charged tachyon can only emit Cherenkov radiation while travelling through a bradyonic dielectric medium if the tachyon's relative speed is less than the phase velocity of the electromagnetic fields at frequency ω. Since tachyons by definition have $u^2 > c^2$ and $\epsilon(\omega) > 1$ for all bradyonic materials, then the condition specified by (148) cannot be met and so tachyons will not spontaneously emit Cherenkov radiation when travelling through bradyonic dielectric media. Any energy lost by the tachyon is therefore deposited near the tachyon's path in collisions with the atoms of the medium.

The analysis just presented ignores the effect of the angle between the velocity of the charged tachyon and the wave vector of the emitted radiation (if any). We treat this more subtle effect using a method given by Melrose and McPhedran (1991) which also applies to tachyons. The Cherenkov condition is $\omega - \mathbf{k}.\mathbf{u} = 0$, which corresponds to a resonance at zero frequency in the rest frame of the particle. The Cherenkov condition may be written, for both bradyons and tachyons, in the form:

$$1 - n\kappa.\mathbf{u}/c = 1 - (nu/c)\cos\chi = 0\,, \tag{149}$$

where $\kappa = \mathbf{k}/k$ and χ is the angle between κ and $\mathbf{u}$. For tachyons $u > c$ and $\cos\chi \leq 1$. If we first consider $\chi = 0$, then $nu/c = 1$ and $n = c/u$, which is clearly the situation discussed above. In the limit $u \to c^+$ (the lowest tachyon velocity as viewed by a bradyonic observer) then $n \cos \chi_{\min} = 1$ and $\chi_{\min} = \arccos(1/n)$. This is the same limiting value as for bradyons on the other side of the light barrier. For tachyons though this is a minimum, not a maximum as it is for bradyons. When u increases from c, $\cos\chi$ must become smaller to satisfy the Cherenkov condition (149). If χ becomes large enough, then for u not much greater than c the condition may be satisfied with $n > 1$. In this connection it is interesting to note that Folman and Recami (1995) have pointed out that plasmas can constitute an electrodynamic medium with $n < 1$ under certain circumstances. We note also that one of the pioneering studies of Cherenkov radiation from tachyons is that of Mignani and Recami (1973).

The next question to be considered is whether a bradyonic observer will see Cherenkov radiation emitted by a tachyon as it travels through a tachyonic dielectric medium. Consider a reference frame Σ' which is tachyonic relative to the laboratory frame Σ. A dielectric medium with constant $\epsilon'(\omega')$ is at rest in Σ'. Frame Σ' and the dielectric medium move with constant speed u relative to Σ, with $u^2 > c^2$. Now suppose a charged particle with speed v' relative to Σ' travels through the dielectric medium.

In frame Σ' the particle will emit Cherenkov radiation if it has a relative speed such that $v' > c/\sqrt{\epsilon'(\omega')}$: this is the standard condition for emission of such radiation by bradyons, and relative to frame Σ' the particle is a bradyon if ${v'}^2 < c^2$. From the case studied above we know that if ${v'}^2 > c^2$ then the particle will not emit Cherenkov radiation. Any electromagnetic radiation emitted will travel with phase velocity $v'_{\rm ph} = c/\sqrt{\epsilon'(\omega')}$ relative to Σ', where $v'_{\rm ph} < c$.

Now consider this system as viewed relative to frame Σ. Whereas v' and $v'_{\rm ph}$ are both less than c relative to Σ', the velocity transformations (see Paper I) show that in frame Σ these speeds will appear to be greater than c. The apparent speed of the particle in Σ is

$$v = \frac{v' + u}{1 + uv'/c^2} > c\,, \tag{150}$$

while the apparent phase velocity of the radiation in frame Σ is

$$v_{\rm ph} = \frac{v'_{\rm ph} + u}{1 + uv'_{\rm ph}/c^2} = \frac{u + c[\epsilon'(\omega')]^{-\frac{1}{2}}}{1 + u/c[\epsilon'(\omega')]^{\frac{1}{2}}} > c\,. \tag{151}$$

Thus the apparent phase velocity of the electromagnetic radiation as it travels through the tachyonic dielectric medium appears to be superluminal relative to a bradyonic observer. As the phase velocity in Σ is given by $v_{\rm ph} = c/\sqrt{\epsilon(\omega)}$ and $v_{\rm ph} > c$ in this case, then the tachyonic dielectric has an apparent dielectric constant $\epsilon(\omega) < 1$ and so it appears to be 'antirefractive' to a bradyonic observer.

From the definitions of phase velocity in frames Σ' and Σ we see that the dielectric constant transforms according to

$$[\epsilon(\omega)]^{-1/2} = \frac{u/c + [\epsilon'(\omega')]^{-\frac{1}{2}}}{1 + u/c[\epsilon'(\omega')]^{\frac{1}{2}}}\,, \tag{152}$$

while its inverse transformation is

$$[\epsilon'(\omega')]^{-1/2} = \frac{-u/c + [\epsilon(\omega)]^{-\frac{1}{2}}}{1 - u/c[\epsilon(\omega)]^{\frac{1}{2}}}\,. \tag{153}$$

Both these transformations apply for $-\infty < u < \infty$. Defining the refractive indices in frames Σ' and Σ as $n' = \sqrt{\epsilon'(\omega')}$ and $n = \sqrt{\epsilon(\omega)}$ respectively gives the apparent phase velocity in Σ as

$$v_{\mathrm{ph}} = \frac{c(c + un'(\omega'))}{cn'(\omega') + u}\,. \tag{154}$$

As Σ' is the rest frame of the dielectric in this case, then (154) agrees with the apparent phase velocity calculated by considering the wave equation in a dielectric medium (116).

15. Conclusion

It has been shown in this paper that the overall structure of ER is consistent with the standard electrodynamics of SR. As with dynamics, some care must be exercised when considering tachyonic objects and normal vectors (e.g. the tachyonic current loop) due to the effect of switching on what a bradyonic observer sees in that particular reference frame.

The discussion of Lagrange's equations and Hamilton's equations for 'classical' charged tachyons showed that such a formalism is a viable approach. This indicates that using a Hamiltonian formalism in the development of quantum mechanics for charged tachyons may lead to concepts and results which are conducive to logical and consistent interpretation, as was the case with tachyon dynamics and electrodynamics. A detailed study of tachyonic quantum mechanics should answer the question as to why, if tachyons can exist at the classical (i.e. nonquantum) level in nature, do we not observe their effects?

It has been the intention throughout this and the previous three papers in this series to develop a formulation of the theory of tachyons which is logical and consistent, yet allows tachyons to interact with ordinary matter. To this end, several derivations have been included to demonstrate the rigorous nature of the results and to emphasise important points. The overall result of this work, based on ideas by Bilaniuk and Sudarshan (1969) and Corben (1975, 1976, 1978), has been to develop a formulation which is different from that proposed by other authors, most notably Recami and Mignani (1974) and Recami (1986), in subtle but important ways. If allowance is made for the different metrics used, the form of the tachyonic transformations of spacetime coordinates, velocities, momenta, energies and forces are all similar between the formulations proposed by the present authors and by Recami, only differing in signs on some components. The differences have arisen due to the way these two formulations have been set up: in the present work these transformations are derived from the fundamental postulates, whereas Recami (1986) considers symmetry arguments. However, when dealing with electrodynamics the differences between the two formulations become more pronounced. For example, the transformations of the electromagnetic field vectors **E**, **B**, **D** and **H** are different in the two formulations, which of course leads to considerably different results upon application to specific systems.

What has been described here as an electrically charged tachyon is instead a tachyonic magnetic monopole in the work of Recami and Mignani (1974) and Mignani and Recami (1975). This difference in ideas as to the nature of charged tachyons also carries over into how electromagnetic scalar and vector potentials are handled in the two formulations. The present formulation maintains that Maxwell's equations are the same for bradyons and tachyons, even to the extent of including them in the laws of physics covered by the second postulate of ER which is the same as for SR: this has resulted in a rigorously developed theory of tachyon electromagnetism and electrodynamics. In Recami's formulation, terms involving tachyonic charge and current densities are added to Maxwell's equations in order to symmetrise them, which obviously leads to a completely different set of results for tachyon electrodynamics.

There are other differences between the two formulations: these are in some cases due to the difference in interpretation of results involving a switched tachyon. For example, there are different interpretations of what would be the apparent shape of a tachyon which is a sphere when viewed in its own rest frame. When viewed by a bradyonic observer, Recami (1986) considers such a tachyon to appear to occupy the whole space bounded by a double, unlimited cone and a two-sheeted hyperboloid connected at a point: this in turn has major consequences as to whether a tachyon can be considered to be a localised particle for the purpose of calculations. In the formulation proposed here the tachyon would instead appear to the bradyonic observer to be an ellipsoid which undergoes various degrees of elongation according to the relative speed of the observer and the tachyon. This allows a tachyon to be considered to be localised for the purpose of generating fields and effects, except of course in the dual frame in which the tachyon appears to the bradyonic observer to have infinite relative speed.

The study of a charged tachyon colliding with the atoms of a dielectric medium has shown that tachyons do not emit Cherenkov radiation when passing through normal, i.e. bradyonic, matter. The only possible exception involves a particular range of values of the angle between the particle trajectory and the direction of radiation emission and may not correspond to any realistic dielectric medium. The tachyons lose energy to the medium via collisions and not via radiation. The principal way Cherenkov radiation can happen is if the tachyon passes through a dielectric medium which is itself tachyonic: in the tachyon's own reference frame this then reduces to the bradyonic case of a charged particle passing through a dielectric medium. Radiation emitted in that case is consistent with the dielectric appearing to the observer to be antirefractive. The consequences of this behaviour, coupled with the two source effect for tachyons, suggest that objects such as some quasars which exhibit apparent superluminal expansion in two separate directions may indeed contain tachyonic objects. However, a detailed discussion of this phenomenon is beyond the scope of the present work and will be left to a later date.

Acknowledgments

This work was completed with the assistance of an ARC grant. The authors are grateful to Dr R. A. Treumann of the Max-Planck-Institut for Physics and

Astrophysics, Garching bei München, Germany, for invaluable discussions during the course of this work.

References

Barut, A. O. (1978). *In* 'Erice-1976: Tachyons, Monopoles and Related Topics' (Ed. E. Recami), p. 143 (North Holland: Amsterdam).
Barut, A. O., Maccarrone, G. D., and Recami, E. (1982). *Nuovo Cimento* A **71**, 509.
Bilaniuk, O. M. P., and Sudarshan, E. C. G. (1969). *Phys. Today* **22** (5), 43.
Bohm, D. (1989). 'The Special Theory of Relativity' (Addison–Wesley: Reading, Mass.).
Corben, H. C. (1975). *Nuovo Cimento* A **29**, 415.
Corben, H. C. (1976). *Int. J. Theor. Phys.* **15**, 703.
Corben, H. C. (1978). *In* 'Erice-1976: Tachyons, Monopoles and Related Topics' (Ed. E. Recami), p. 31 (North Holland: Amsterdam).
Dawe, R. L., and Hines, K. C. (1992*a*). *Aust. J. Phys.* **45**, 591.
Dawe, R. L., and Hines, K. C. (1992*b*). *Aust. J. Phys.* **45**, 725.
Dawe, R. L., and Hines, K. C. (1994). *Aust. J. Phys.* **47**, 431.
Duffey, G. H. (1975). *Found. Phys.* **5**, 349.
Duffey, G. H. (1980). *Found. Phys.* **10**, 959.
Folman, R., and Recami, E. (1995). *Found. Phys. Lett.* **8**, 127.
Gradshteyn, I. S., and Ryzhik, I. M. (1994). 'Table of Integrals, Series, and Products', 5th edn (Ed. A. J. Jeffrey) (Academic: San Diego).
Jackson, J. D. (1975). 'Classical Electrodynamics', 2nd edn (Wiley: New York).
Melrose, D. B., and McPhedran, R. C. (1991). 'Electromagnetic Processes in Dispersive Media' (Cambridge University Press).
Mignani, R., and Recami, E. (1973). *Lett. Nuovo Cimento* **7**, 388.
Mignani, R., and Recami, E. (1975). *Nuovo Cimento* A **30**, 533.
Recami, E. (1986). *Riv. Nuovo Cimento* **9** (4).
Recami, E., and Mignani, R. (1974). *Riv. Nuovo Cimento* **4**, 209.
Recami, E., Castellino, A., Maccarrone, G. D., and Rodonò, M. (1986). *Nuovo Cimento* B **93**, 119.
Rosser, W. G. V. (1964). 'An Introduction to the Theory of Relativity' (Butterworths: London).
Soucek, J. (1981). *J. Phys.* A **14**, 1629.
Vysin, V. (1977). *Nuovo Cimento* A **40**, 113.

Manuscript received 9 December 1997, accepted 3 June 1998