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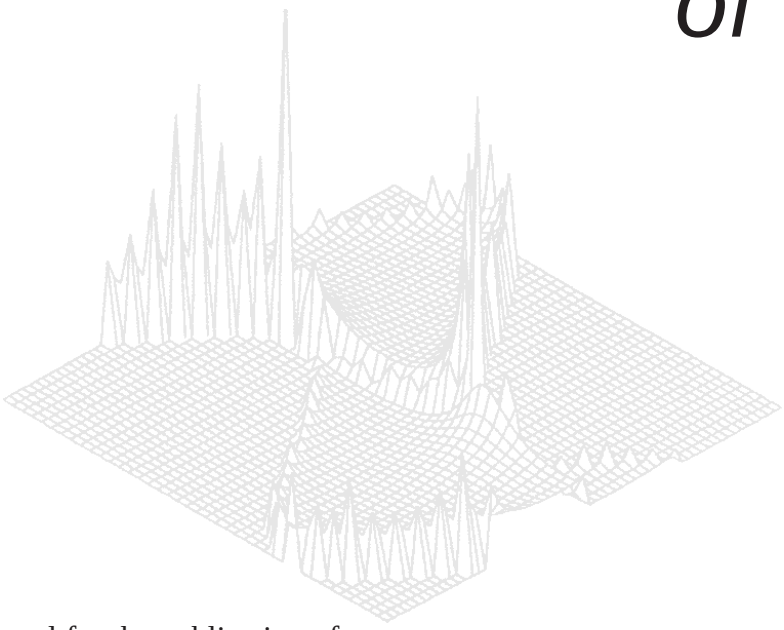
Australian Journal of Physics

Volume 51, 1998



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Aust. J. Phys., 1998, **51**, 763–834

One-dimensional Self-organised Structures in Dusty Plasmas

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Contents

 10.1071/PH98007 0004-9506/98/050763$30.00

Abstract

A dusty plasma is an open system in which the formation of structures is a natural phenomenon. The first part of the present review is devoted to an introduction to dusty plasma physics and a formulation of the new theoretical kinetic and new hydrodynamic approaches used in subsequent parts. In the new hydrodynamic approach only dust–plasma particle collisions are taken into account and the usual binary plasma collisions are neglected. This description takes into account the possibility of plasma absorption on dust particles and is appropriate for open systems. The second part of the review contains a description of one-dimensional plasma–dust self-organised structures where the energy source corresponds to the plasma fluxes towards the dust regions from the plasma regions where the dust is absent. The fluxes are created by the structures in a self-consistent manner to increase the ion density and decrease the electron density, drift velocity and electrostatic potential inside the structure. The third part contains the theory of shock waves in dusty plasmas, modification of the Gugoniot relation and analyses of the possibility of an increase of dust density behind the shock front. All parts are based on the modern point of view of the physics of dusty plasmas and mainly new material is presented. The review is written in the form of lectures and therefore the references to the existing literature are given in the text, with more details in the last section devoted to some historical comment. The references to general reviews are given separately as [A]–[D].

PART I. INTRODUCTION TO ELEMENTARY DUSTY PLASMA PHYSICS, KINETIC AND HYDRODYNAMIC DESCRIPTION OF DUSTY PLASMAS

1. Dusty Plasma as a New State of Matter

Dusty plasmas have become a fashionable subject for research and this is driven mainly by several important applications. The plasma physics aspect is driven by the problems of controlled thermonuclear research (CTR) and the development of active space research, while the dusty plasma physics aspect is driven by the industrial applications of etching and plasma deposition problems [ref. D], by the possibility of production of new materials, by new space research related to recent space missions, by the discovery of many new aspects of dust in space including the structures of planetary rings, star formation, the creation of dust in supernovae explosions and finally by pollution problems (dust in the lower ionosphere and lower atmosphere) [refs A–C].

The problem of whether other states of matter can exist is an important general physical problem. In conventional matter the only possible particle binding has a quantum nature since the exchange interactions are related to wave function overlapping. This interaction can produce particle attraction, all chemical bindings and form such a state as the solid state of matter. Dust in plasmas provides a new type of dust–dust attraction (ref. [A]; Tsytovich 1994; Khodataev *et al.* 1996*a*; Tsytovich *et al.* 1996*a*; Ignatov 1996; Vladimirov and Tsytovich 1998) which is classical in nature (see below) and therefore can be considered as an element of new matter which includes crystals and dust liquids. In this sense dust in plasmas can form very different structures with properties quite different from usual matter and one can speak about a new state of matter. The presence of new forces related to the openness of a dusty plasma system (Tsytovich 1994; Khodataev *et al.* 1996*a*, 1996*b*; Tsytovich *et al.* 1996*a*; Ignatov 1996; Vladimirov and Tsytovich 1998) opens new possibilities for the creation of classically bound molecules (Tsytovich *et al.* 1996*c*) and crystals, which can

be considered as the starting point of super-chemistry (Tsytovich *et al.* 1996*c*) in which the elementary interacting elements will be not atoms but dust particles in plasmas.

Thus one should understand why a dust particle in a plasma can really attract another regardless of the Coulomb repulsion of their equal sign large charges (Tsytovich *et al.* 1998). The key point for understanding the dusty plasma as a new state of matter is its openness (ref. [A]). A plasma with dust cannot survive in the absence of an external source of electrons and ions or in the absence of a flux of them from the region where the dust is absent. The dust cloud is usually automatically charged in the way to create such fluxes. The reason for the presence of fluxes of plasma particles on the dust particles is the presence of dust charging processes which, even after reaching the equilibrium charge, do not vanish when the currents on dust particles vanish. The openness of the dusty plasma systems and the presence of plasma particle fluxes both on each dust particle and on the dust cloud as a whole creates additional forces between the dust particles which are usually of attractive type. This effect is explained in a hand-waving way as a pressure produced by shadowing of the plasma flux on one dust particle by another dust particle.

Another fundamental property of the dusty plasma is that it is a highly dissipative system (Benkadda *et al.* 1996), the dissipation being caused by absorption of plasma fluxes on dust particles. The rate of formation of the self-organised structures is usually measured by the degree of dissipation which in equilibrium is compensated by external sources. A big flux of energy in the system creates new possibilities for the development of self-organised structures in dusty plasmas. One can say that the dusty plasma is a system extremely well adapted to the formation of structures. Thus the structures which can be created in the absence of dust will be modified by the presence of dust, but this is a minor effect. In dusty plasmas very new structures can be created with no analogy to the usual structures in the absence of dust. The variety of structures which can be created in dusty plasmas is much larger than in usual matter in the absence of dust.

Making an analogy with biological creatures we can say that plasma particles absorbed by dust in the structure play the role of 'food' for these structures, without which the structure is disorganised. The competition for this 'food' can be of importance for evolution in the system of many such structures. There appears also a possibility of modelling biological structures using dusty plasmas—a topic already discussed in the current literature. As well, the question of the existence of these structures in space plasmas was raised and the problem of storage of information in these structures was mentioned. It is possible that the search for extra-terrestrial life has been made inappropriately since there are conditions in space where the presence of dust in plasmas is very often found and thus in space conditions the most natural type of life could be related to these self-organised dust–plasma structures.

This means that the problems of dusty plasmas are of general importance in present physics research and the dusty plasma is not only an example of a new physical object. *Therefore, structures in dusty plasmas play an especially important role being a necessary component of the dusty plasma system.* The turbulence in dusty plasmas is the state of interaction of these structures.

Dust in space is an old subject for research (Spitzer 1975; Kaplan and Pikel'ner 1974, 1979). The new recent developments in understanding dusty plasma physics entail reconsideration of many old problems. Among them is the problem of star formation. According to the standard point of view, stars are formed by gravitational contraction when the shock in the galaxy arms propagates through the dust–molecular clouds (Kaplan and Pikel'ner 1974).

One of the well known dust structures in space are *interstellar dust–molecular clouds* (Kaplan and Pikel'ner 1974). The dust in these clouds is cold, with cooling occurring via radiation losses, and the molecules serve as material for dust growing and are also formed on dust surfaces. The ionisation is mainly produced by cosmic and subcosmic rays, and the relative degree of ionisation is not extremely small, of the order of $10^{-6} - 10^{-5}$, which is lower but close to the degree of ionisation in most low temperature laboratory experiments. Thus many of the phenomena of laboratory dusty physics can be applied to dust–molecular clouds.

Stars are examples of space self-organised structures formed in dusty plasmas in space (Kaplan and Pikel'ner 1974). The main question arises of whether the new physics of dusty plasmas can improve the physical picture of star formation and whether only gravitational contraction is responsible for star creation. The question is what kind of micro-contractions can occur due to dust attraction? The recent observations of star formation indicate that the interaction of radiation with dust plays an important role. What kind of collective dust behaviour is responsible for this interaction? The question is also about the properties of the shock wave in dusty plasmas, since it is believed in the present picture of star formation that the shock wave creates the initial disturbance from which the stars are formed.

Planetary rings are another example of complicated dust structures in space (Havnes *et al.* 1984; Havnes and Morfill 1984; Melanso and Havnes 1991; Aslaksen and Havnes 1992; Gor'kavyi and Fridman 1994; ref. [A]). The main problem is that the dust ring is embedded in plasma and therefore should be charged as a whole and create the plasma fluxes toward them. This problem has not yet been treated properly. We will discuss how at least to formulate this problem. The presence of plasma fluxes obviously leads to plasma energy dissipation in the ring and the important question is whether this dissipation is comparable to that due to dust collisions in the ring occurring during their Keplerian movement in the plane of the ring? The plasma fluxes perpendicular to the ring plane create the plasma mechanisms of self-organisation in the ring and plasma induced structure formation. The question is whether this structure formation can compete with the gravitational structure formation due to the appearance of spiral vortex structures in the plane of the ring? The question is also about the nonlinearities which relate the motions perpendicular to the ring with that in the plane of the ring (ref. [A]).

Cometary tails are another structure observed in dusty plasmas. The interaction of the neutral plasma component plays an important role similar to that in many laboratory experiments where the degree of ionisation is low and the interactions of dust particles with neutrals is an important process in dust–dust interactions. Damping of collective modes by dust particles should be a common phenomenon due to the intense collisions of dust particles with the neutrals of plasma. Such

structures as cometary tails are the ones in moving plasmas when the plasma fluxes are asymmetric and therefore are of general interest in dusty plasma physics.

Recent observations of *supernovae shocks* suggest that the dust is usually formed behind them. The Gugonoit relations for shocks should be modified in the presence of dust, not only because the dusty plasma is an open system but also because of radiation losses by the dust particle behind the shock. The shock structure should include the dust and plasma distribution in the shock, agglomeration of dust particles behind the supernovae shock and dust cooling. These shock waves are examples of *another dust–plasma structure in space*.

Dust plays an important role in most low temperature laboratory plasmas, and in etching and plasma material deposition. A review of these problems can be found in ref. [D] and the author's paper ref. [A], where also reviewed are the experiments on *plasma dust crystals*—another type of dust structure observed experimentally. We will not consider these problems here (except in Part IV which is devoted to historical comments), giving consideration only to the case where dust is in its gaseous state. This means that the dust particles will not have regular positions in space as in dust–plasma crystals.

Dust plays an important role close to the wall in tokamak plasmas. Recently, the agglomeration of dust particles in tokamak plasmas was observed. A review of the role of dust in problems of controlled thermonuclear research is given in ref. [B]. We will not touch on these problems here (except in Part IV).

The author's review on 2D vortex dust structures in dusty plasmas was given in the Proceedings of the Workshop on 2D Structures and 2D Turbulence (ref. [C]). Similarly we will not discuss these problems here. Several important reviews on dust in etching devices can be found in ref. [D].

2. Main Physical Mechanisms of Dust–Dust Interactions, Attraction and Repulsion in Plasmas

We should start by giving an introduction to elementary dust–dust interactions in plasmas since they will be used in discussions of 1D self-organised structures. The difference between dust particles and other charged particles is that their charge, being very large, is not fixed and depends on the surroundings since the ion and electron fluxes creating the dust charges depend on local plasma parameters and plasma particle distribution functions. Therefore, the presence of a neighbour dust particle changes the plasma flux and also the dust charges and their interactions. One should expect the presence of new forces between dust particles even for fixed charges. The interactions could be different for the case where the distance between dust particles is less that the Debye screening length and in the case where it is larger than this length. The first case is usually found in etching experiments (ref. [D]; Selwyn *et al.* 1990*a*, 1990*b*; Boufendi and Bouchoule 1994) and the second case is usually found in plasma–dust crystal experiments (Morfill 1992, 1994; Chu and Lin 1994; Melzer *et al.* 1994; Hayashi and Tachibana 1994; Fortov *et al.* 1996; Allen *et al.* 1996). For the latter case the Coulomb interaction is screened very strongly and the interactions due to mutual shadowing of plasma fluxes are most important (ref. [A]). Only the shadowing of the neutral plasma particle bombardment can compete with electrostatic forces

for distances less that the Debye screening distance. This bombardment force is usually an attractive force as we will demonstrate later and can lead to dust agglomeration (Tsytovich *et al.* 1998)—the effect usually observed in etching experiments (Boufendi and Bouchoule 1994). In the conditions of plasma-dust crystal experiments, the bombardment by neutrals is almost equal or less than the plasma charged particle fluxes bombardment. But these forces are always dominant for distances larger than the Debye screening distance.

The non-Coulomb forces have a nature similar to the usual electrostatic forces which, according to the general concept of forces, are due to an exchange of photons between the interacting particles (or for electrostatic forces due to the exchange of virtual photons). In this picture the electrostatic forces arise due to the process where one particle emits the photon and another particle absorbs this photon. This leads to an exchange of momenta and therefore leads to the existence of forces. If a single spherical dust particle is present in the plasma, the plasma particle bombardment does not transfer the momenta in an isotropic plasma. In the presence of another dust particle at a certain finite distance from the first one the fluxes on the first particle are shadowed by the second dust particle and vice versa. Since the total spherical flux does not produce any force one can subtract it and what is left is the absence of flux in a certain angular interval due to shadow effects, which is equivalent to emission of this flux by one of the dust particle necessary to compensate the part of the spherical flux in the shadow region. This flux acts on another dust particle in the same way as the flux of emitted photons and transfers the momenta. It is easy to see that these forces are determined by the solid angle of the shadow and therefore for distances larger than the size of dust particles these forces are inversely proportional to the square of the distance between the dust particles, as it is for Coulomb forces. It is clear also that these bombardment forces should be an attractive one and should both increase proportionally to the pressure times the surface area of the dust particle and to the solid angle related to the shadow effect, which is proportional to the square of the dust radius, i.e. the total force should be proportional to the fourth power of the dust radius. This conclusion is independent of whether the bombardment is produced by neutral plasma particles or by charged plasma particles (electrons and ions). This also means that for large dust sizes the attraction forces can dominate, which seems to be a physical reason for the possibility of dust agglomeration in the presence of large Coulomb repulsion. The presence of plasma fluxes on a single dust particle also changes the shielding of its electrostatic field since the shadow effect creates additional charge distribution around the dust particles with a potential proportional to the solid angle of the shadow. This leads to the presence of electrostatic non-screened repulsion between dust particles inversely proportional to the cube of the distance between dust particles (the derivative of additional potential which is inversely proportional to the square of the distance). This repulsion is small compared with the attraction for large distances since the attractive force is inversely proportional to the square of the inter-dust distance, while the repulsive force is proportional to the cube of the inter-dust distance. This gives a molecular-type of potential for the interaction of two dust particles in plasmas. Schematically the possible potentials for pair dust–dust interactions in dusty plasmas are shown in Fig. 1.

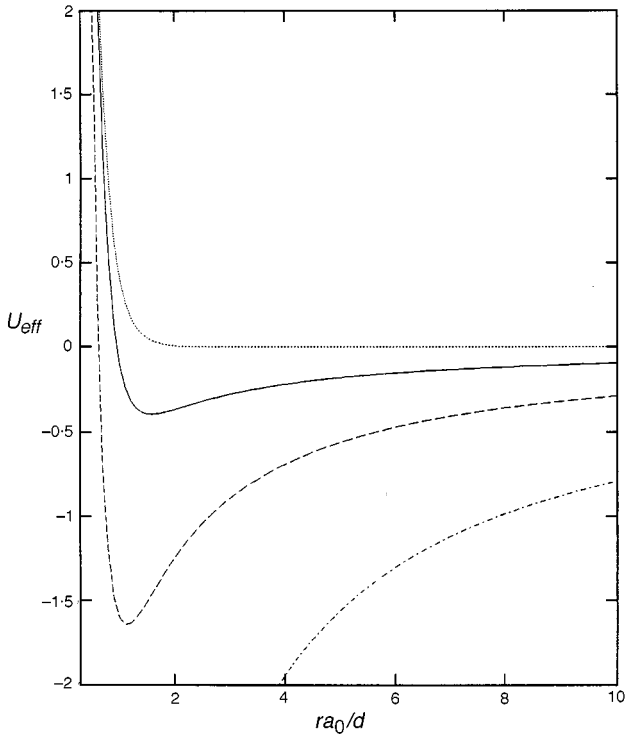

Fig. 1. Schematic dependence of the normalised effective potential $U_{eff} = U/U_0; U_0 = (Z_d^2 e^2/a_0)(a_0/d)^4$ for interaction of two dust particles on the normalised inter-dust distance ra_0/d^2 where a_0 is the size of dust particles, Z_d is the dust charge in units of electron charge and d is the Debye screening radius. The U_{eff} includes both the electrostatic potential, the Debye screened potential $\propto \exp(-r/d)(1/r)$ and the repulsion potential $\propto a_0/r^2$, and the potential of attractive bombardment forces. The dependence of the effective potential on the normalised inter-dust distance ($r \to ra/d^2$ is described in Fig. 1 by the simplified expression $U_{eff} = (1/r)[(1/a^2)\exp(-r/a) + 1/2r^2 - 1/r]$, where $a = a_0/d$ is the dust size in units of the Debye length (the simplification is that the repulsion and attraction coefficients are taken as unity; the dependence of these coefficients on plasma parameters is shown in Fig. 2). The solid line shows the behaviour of the total potential for $a = 0{\cdot}3$ and the neglect of attraction by neutral bombardment. The dotted line shows the potential for a pure Debye screening potential. The dashed and dash–dot lines are the same as the solid line but for increasing neutral bombardment (a decreasing degree of ionisation). The solid and dashed lines correspond to the degree of ionisation where the bounded state of two dust particles can be formed. The dash–dot line represents the lowest degree of ionisation where the attraction dominates up to an interdust distance equal to the dust size leading to dust agglomeration.

In the absence of dust, the plasma component has several characteristic lengths such as the Debye screening length, the Larmour radius or the length of density inhomogeneities. Dust–plasma physics needs to introduce an additional characteristic length related to the dust sizes. We call it the *dust fundamental length* (DFL) since in the case where all other characteristic lengths are unimportant (absence of external electric and magnetic fields and absence of inhomogeneity of plasma) the characteristics of dust–plasma structures can be expressed through this DFL.

This new fundamental length is

$$L_f \equiv d\,\frac{d}{a_0}\,, \tag{1}$$

where d is the Debye screening length and a_0 is the size of dust particles. The DFL plays a role in dusty plasmas similar to the atomic size in atomic physics. A dust molecule—the molecule formed from two equally charged dust particles—has a size equal to or of the order of L_f, while the dust–crystal lattice size is of the order of L_f, and even the plasma particle mean free path in dusty plasma is of the same order, the structures themselves having a size of order L_f etc. (for examples see below). One of the most important points is that the equations describing self-consistently the dusty plasma self-organised structures in the dimensionless length measured in units of L_f often do not have any small or large parameters. These equations include all the attractive and repulsive forces mentioned above. This is true only in the case where lengths such as the Larmour radius or Rossby radius do not play any important role in the structure (in the case where they play a role the situation is more complicated but the L_f and its ratio to other lengths plays a crucial role in the structures). This is another reason to call L_f the fundamental length.

The new attractive forces are similar to gravity and have the same dependence with respect to the inter-dust distance as the gravity forces. For the attraction produced by charged plasma particle fluxes one can express the forces in the form (Tsytovich 1996*c*; ref. [A]):

$$\mathbf{F}_{attr} = -\frac{\mathbf{r}}{r^3} Z_d^2 e^2 \frac{a_0}{L_f} \eta_a\,, \tag{2}$$

where η_a is the numerical coefficient which depends on the plasma parameters and is of the order of unity for the simplest conditions (see below). For the repulsive forces $\mathbf{F}_{rep}$ the expression a_0/r should be substituted in (2) for a_0/L_f and η_r should be substituted for η_a, where η_r is the numerical coefficient characterising the repulsive forces. It is easy to see that for numerical coefficients η_a and η_r of the order 1 the repulsive and attractive forces balance each other for $r = L_f$. The forces induced by neutrals contain a factor equal to the inverse degree of ionisation. The total potential of these forces is either of molecular-type or pure attraction in the case where the neutral bombardment force dominates. In the latter case the dust agglomeration occurs (see Fig. 1).

In the 19th century Lesage proposed a model of Newtonian gravity by assuming that each particle absorbs the ether and the shadowing of this absorption creates the gravitational force. The special theory of relativity by excluding the ether showed that such a model is unrealistic and it was forgotten. Dust in plasma absorbs electrons and ions and this creates a real flux which leads to dust attraction. Thus the dust attraction is the realisation of the Lesage model of gravity.

This analogy shows that there should exist a deep similarity between dust attraction and gravity, although gravity is universal while the attraction we are discussing acts only on dust particles but not on electrons and ions.

Nevertheless, an instability similar to the gravitational instability and caused by dust attraction should exist in dusty plasmas. We write down the dispersion relation for the gravitational instability together with the dust attraction electrostatic instability to point out this similarity. The equation for the gravitational instability has the well known form

$$\omega^2 = k^2 v_s^2 - Gnm\,, \tag{3}$$

where G is the gravitational constant, n is the density of gravitating matter and m is the mass of gravitating particles. In the case where the dust particles give the main contribution to the mass density in dusty plasmas $n \approx n_d$, where n_d is the dust density and $m = m_d$, and where m_d is the mass of dust particles. In the latter case

$$\omega^2 = k^2 v_s^2 - Gn_d m_d\,, \tag{4}$$

The instability occurs where ω^2 is negative and determines the Jeans critical length. The dispersion relation for the dust attraction instability has the form (ref. [A]; Tsytovich *et al.* 1996*a*, 1997)

$$\omega^2 = k^2 v_{ds}^2 - \omega_{pd}^2 \eta \frac{a_0}{L_f}\,, \tag{5}$$

where v_{ds} is the dust sound velocity ($v_{sd}^2 = \omega_{pd}^2 d_i^2$, d_i is the ion Debye radius) and ω_{pd} is the dust plasma frequency ($\omega_{pd}^2 = 4\pi n_d Z_d^2 e^2/m_d$). From (5) it follows that the critical length for development of the electrostatic dust attraction instability is of the order of the length L_f, another reason to call this length fundamental. The reason why this length is relatively small is that the dust sound velocity entering (5) is much less than the usual sound velocity entering (4). By comparing the last terms of (4) and (5) we find that the effective gravity introduced by dust attraction can be described by

$$G_{eff} = \frac{4\pi Z_d^2 e^2}{m_d^2} \eta \frac{a_0}{L_f}\,. \tag{6}$$

The development of dust attraction is the starting point for formation of self-organised structures in dusty plasma.

3. General Theoretical Concepts, Kinetic and Hydrodynamic Descriptions of Dusty Plasmas

The force of dust attraction acts to move the dust particles. This creates some disturbances in plasmas. But electrostatic disturbances can be caused by dust without any dust movement since the charge on dust particles is usually changed in any disturbance. To describe a dusty plasma one needs to use a new kinetic approach and new hydrodynamics in which the charges on the particles are not fixed but are determined self-consistently from the currents approaching the dust particles. The microscopic kinetic equation for the dust particle distribution function $f_d(\mathbf{p}, \mathbf{r}, t, q)$ will contain a new variable q, the charge

of the dust particle. Obviously it should have the form (for simplicity we write the case for electrostatic fields)

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\frac{\partial f_d}{\partial \mathbf{r}} + \frac{\partial q\mathbf{E} f_d}{\partial \mathbf{p}} + \frac{\partial J f_d}{\partial q} = 0\,, \tag{7}$$

where J is the current on the dust particle. The averaging of fluctuations will lead on the right-hand side to a new collision integral which describes both the influence of variability of dust charges on dust–dust and dust–plasma particle interactions and the new attractive and repulsive forces due to the presence of plasma fluxes on dust particles (Tsytovich and de Angelis 1998). The result can be used for example to calculate explicitly the coefficients η in the attractive and repulsive forces. This way is cumbersome but straight forward. We will use a more simple physical approach which gives the same results (see below). The general approach using equation (7) is a direct generalisation of the approach usually used in plasma physics.

An important consequence of (7) is the possibility of obtaining a *new hydrodynamics* (Benkadda *et al.* 1996) by taking the moments of (7) with respect to velocity and with respect to charge. The most important is the estimate of the domain of validity of the new hydrodynamics, which obviously should coincide with low frequency and the low wave number domain, or with sufficiently slow processes compared with the effective collision frequency, and sufficiently large scale processes compared with the particle mean free path with respect to these collisions. In the presence of dust the most important are the collisions of plasma particles with dust particles. It is easy to show that these collision frequencies are approximately Z_d times larger than the binary plasma particle collision frequencies. Since Z_d is usually a very large number the domain of validity of the new hydrodynamics is much broader than the domain of validity of conventional hydrodynamics. This means that all plasma modes with frequencies much less than the effective plasma particle–dust collision frequency can be strongly changed. This is the physical reason why the dusty plasma introduces qualitatively new effects unknown in usual plasma physics.

The new hydrodynamics is obtained from the kinetic equations by considering the moments of the particle distribution function and taking into account only the plasma particle collisions with dust particles and neglecting the binary plasma particle collisions. On the right-hand side of the continuity equations for electrons and ions the terms $-\bar{\nu}_{e,i} n_{e,i}$ will enter and in the momentum equations for electrons and ions the friction forces due to ion/electron friction on the dust with the collision frequencies $\tilde{\nu}_{e,i}$ will enter, and the viscosity terms will be negligible compared with these friction terms. We will not write down these equations since they have a trivial and standard form (except of course that instead of the usual frequencies they contain the frequencies of plasma collisions with dust introduced above). Concerning the dust component, the corresponding momentum hydrodynamic equation can be obtained from the general kinetic equations written above, having in mind that the frequencies for which the hydrodynamic description is valid, the dust motions correspond to a quasi-adiabatic charge ajustment to the local surrounding plasma moment quantities, and thus the dust distribution in its charges should be Gaussian.

Thus, in averaging quantities for the dust, such as the average dust density and average dust momentum, one uses the Gaussian dust charge distribution. Also the standard Klimontovich-type of procedure for the kinetic equation of dust leads to dust–dust collision integrals which contain the attractive and repulsive forces described in elementary fashion above. This interaction should be taken into account in describing the self-organised structures. These structures are nonlinear and dissipative and their description can be obtained by numerical solution of the nonlinear equations.

4. Some Useful Relations in Dusty Plasma Physics

We give here the simplest relations in dusty plasma physics (refs [A,C]). The following 16 relations useful in most applications:

(1) Dust size a_0 and Debye length d

$$a_0 \ll d; \qquad \frac{1}{d^2} = \frac{1}{d_i^2} + \frac{1}{d_e^2}\,, \tag{8}$$

where $d \approx d_i$ for $T_e \gg T_i$. We can introduce the dimensionless dust size a:

$$a \equiv \frac{a_0}{d_i}\,. \tag{9}$$

(2) Charging of dust

Floating potential arguments give an estimate of the dust charge in units of the electron charge $Z_d e^2/a_0 \sim T_e$. We will use the dimensionless charge z defined by

$$z \equiv \frac{Z_d e^2}{a_0 T_e}\,. \tag{10}$$

(3) Temperature ratio

$$\tau \equiv \frac{T_i}{T_e}\,, \tag{11}$$

where $\tau \ll 1$ in laboratory experiments and $\tau \approx 1$ in astrophysical applications.

(4) Dimensionless dust density

$$P \equiv \frac{n_d Z_d}{n_{0,i}}\,, \tag{12}$$

where $n_{0,i}$ is the unperturbed ion plasma density. In plasma etching experiments P is of the order of 10, in plasma crystal experiments it is of the order of 1 and in astrophysical conditions it varies from 10 to 10^{-3}.

The condition of quasi-neutrality of the background dusty plasma reads

$$n_{0,i} = n_{0,e}(1 + P)\,. \tag{13}$$

(5) Charging cross sections

The subscript ch is used for charging cross sections. For electrons we have

$$\sigma_{ch}^{e} = \pi a_0^2 \left(1 - \frac{2Z_d e^2}{m_e v^2}\right), \tag{14}$$

and for ions

$$\sigma_{ch}^{i} = \pi a_0^2 \left(1 + \frac{2Z_d e^2}{m_i v^2}\right). \tag{15}$$

The impact parameter for electrons is $p_e < a_0$ and for ions $p_i > a_0$, where $p_i \gg a_0$ for $\tau \ll 1$. The plasma particle fluxes on dust particles are

$$\frac{\langle \sigma_{ch}^{e} v\rangle}{\pi a_0^2 v_{Te}} = 2\sqrt{\frac{2}{\pi}}\exp(-z), \tag{16}$$

$$\frac{\langle \sigma_{ch}^{i} v\rangle}{\pi a_0^2 v_{Ti}} = 2\sqrt{\frac{2}{\pi}}\left(1 + \frac{\tau}{z}\right). \tag{17}$$

(6) Coulomb elastic collision cross sections

The usual estimates $e^2 Z_d/p_{i,e} \approx T_{i,e}$ and $\sigma_{id,ed} \approx \pi p_{i,e}^2$ give two important results (subscript C is used for Coulomb cross sections)

$$\frac{\sigma_C^{ed,id}}{\sigma_C^{ee,ii}} \approx Z_d^2 \gg 1;$$

$$\sigma_C^{ed} \approx \pi a_0^2 z^2 \ln(d/a_0), \tag{18}$$

$$\sigma_C^{id} \approx \pi a_0^2 \frac{z^2}{\tau^2} \ln(d/a_0). \tag{19}$$

For $\tau \ll 1$ we find $\sigma_C^{id} \gg \{\sigma_C^{ed}, \sigma_{ch}^{i}\}$ and $\sigma_C^{id} \approx (z/\tau)\sigma_{ch}^{i}$.

(7) Charging frequency

The current balance equation is

$$\frac{d\delta Z_d}{dt} = n^e \langle \sigma_{ch}^{e}\rangle - n^i \langle \sigma_{ch}^{i}\rangle \approx -\pi a_0^2 v_{Ti} n^i \delta \left(\frac{z}{\tau}\right)$$

$$= -\pi n^i a_0^2 v_{Ti} \frac{\delta Z_d e^2}{a_0 T_i} = -\nu_{ch} \delta Z_d, \tag{20}$$

$$\exp(-z) = \frac{1+P}{\sqrt{\tau\mu}}(\tau + z); \quad \mu = \frac{m_i}{m_e}. \tag{21}$$

The charging frequency ν_{ch} is

$$\nu_{ch} \approx \frac{a_0}{d_i^2} v_{Ti} = \frac{v_{Ti}}{L_f} \,. \tag{22}$$

An important estimate is

$$\frac{\nu_{ch}}{\nu_{ii}} \approx Z_d \frac{\tau}{z} \,. \tag{23}$$

The exact expression for the charging frequency for the equilibrium dust charge is

$$\nu_{ch} = \frac{\omega_{pi}}{\sqrt{2\pi}} \frac{a_0}{d_i} (1 + \tau + z) = \frac{v_{Ti}}{\sqrt{2\pi} L_f} (1 + \tau + z) \,. \tag{24}$$

(8) Frequencies of plasma particle collisions with dust

Only charging collisions enter in the continuity equation. The collision frequency in the continuity equation is denoted as $\bar{\nu}_{e,i}$:

$$\bar{\nu}_{e,i} \approx n^d \langle \sigma_{ch}^{e,i} \rangle v_{Te,Ti} \,, \tag{25}$$

$$\bar{\nu}_i \approx \pi a^2 n^d \frac{z}{\tau} v_{Ti} \approx \frac{P \nu_{ch}}{1 + P} \,. \tag{26}$$

This collision describes the rate of particle recombination on the dust. An exact expression for these frequencies for the equilibrium dust charge is

$$\bar{\nu}_e = \nu_{ch} \frac{P}{z} \frac{(\tau + z)}{(1 + \tau + z)} \,;$$

$$\bar{\nu}_i = \frac{\bar{\nu}_e}{1 + P}. \tag{27}$$

The frequency describing the momentum transfer from plasma particles to dust particles is usuallly larger since both the charging and the Coulomb collisions contribute and the Coulomb frequency is larger. We denote these frequencies as $\tilde{\nu}_{e,i}$. The exact expressions are

$$\tilde{\nu}_e = \nu_{ch} \frac{P(\tau + z)}{z(1 + \tau + z)} \left(4 + z + \frac{2z^2}{3} e^z \ln \frac{d}{a} \right) \,, \tag{28}$$

$$\tilde{\nu}_i = \nu_{ch} \frac{P}{(1 + P) z (1 + \tau + z)} \left(z + \tfrac{4}{3}\tau + \frac{2z^2}{3\tau} \ln \frac{d}{a} \right) \,. \tag{29}$$

For $\tau \ll 1$ we have

$$\tilde{\nu}_i \approx (z/\tau) \bar{\nu}_i \approx \nu_{ch} P z/\tau \approx (\nu_{ii} Z_d \tau / z) P z/\tau = \nu_{ii} P Z_d \,. \tag{30}$$

(9) Plasma particles mean free path

The mean free path for the charging process is

$$L_{ch} \approx \frac{\bar{\nu}_i}{v_{Ti}} \approx L_f \,, \tag{31}$$

and for Coulomb collisions it is

$$L_{C,i} = L_f \frac{\tau}{z} \,. \tag{32}$$

(10) Dust sound waves

The ion pressure against dust inertia describes the simplest short-wavelength ($k \gg 1/L_f$) dust sound waves (Resendes *at al.* 1996; Resendes and Tsytovich 1997):

$$\frac{1}{k^2 d_i^2} = \frac{\omega_{pd}^2}{\omega^2} \,, \tag{33}$$

$$\omega = k v_{sd}; \qquad v_{sd} = \sqrt{\frac{P Z_d T_i}{m_d (1+P)}} \,. \tag{34}$$

This expression can be compared with that for ion-sound waves:

$$\omega = k v_s; \qquad v_s = \sqrt{\frac{Z_i T_e}{m_i}} \,. \tag{35}$$

The expression (33) is valid only for $\tau \ll 1$. In the more general case

$$v_{sd} = \frac{P Z_d T_e}{m_d} \frac{\tau}{\tau + 1 + P} \,.$$

The long-wavelength ($k \ll 1/L_f$) dust sound waves are determined for $\tau \ll 1$ by the electron temperature (not the ion temperature as in equation 34). For arbitrary τ the long-wavelength dust sound velocity is

$$\omega = k v_{sd}; \qquad v_{sd} = \frac{Z_d T_e}{m_d} \frac{P(\tau + z) + (1+P)(1+\tau+z)}{\tau + z + 1 + P} \,. \tag{36}$$

(11) Dust temperature

The thermal dust velocity is $v_{Td} = \sqrt{T_d/m_d}$. The dimensionless temperature can be defined as

$$\tau_d = \frac{v_{Td}}{v_{sd}} = \sqrt{\frac{T_d(1+P)}{P Z_d T_i}} \,. \tag{37}$$

This is written for a short-wavelength dust sound speed.

(12) Electrostatic energy accumulated on dust particles

$$\frac{Z_d^2 e^2 N_d}{a N_e T_e} = Pz\,. \tag{38}$$

For typical values $z \approx 2$ and $P \approx 1$ the energy accumulated on dust particles is rather large and can drive instabilities.

(13) Some analytical expressions for attraction and repulsion coefficients

Analytic results can be obtained using the cross sections of charging and Coulomb scattering (Tsytovich *et al.* 1996*c*), assuming $r \gg a_0$ (which allows the plasma fluxes to be locally almost parallel), using arguments of flux conservation (which allows us to find the flux at distances less than Debye radius and use them for larger distances) and using the expressions for the shadow angular interval found from energy and momentum conservation. The results obtained in this manner are valid for any plasma particle distribution, but to derive explicit results we average on the thermal particle distribution. The repulsion of dust particles due to non-Debye screening at distances much larger than the Debye radius is described by (Tsytovich *et al.* 1996*c*):

$$\eta_r = \frac{T_e}{T_e + T_i}\left(Z_i + \frac{\tau}{2z}\right) \approx Z_i\,. \tag{39}$$

This expression is written for $\tau \ll 1$.

The attractive forces can be found by calculating the change in ion momentum for ions directly bombarding the dust particle (subscript b) or Coulomb scattered by the dust particle (subscript c). The total attraction coefficient η_a is (Tsytovich *et al.* 1996*c*):

$$\eta_a = \eta_b + \eta_c\,, \tag{40}$$

$$\eta_b = \frac{1}{2\sqrt{\pi}}\left(\frac{aT_i}{Z_d e^2}\right)^2 \int_{y_{min}}^{\infty}\left(1 + \frac{z}{\tau y^2}\right)^{\frac{5}{2}} y^4 e^{-y^2} dy\,, \tag{41}$$

$$\eta_c = \frac{1}{4\sqrt{\pi}} \int_{y_{min}}^{\infty}\left(1 + \frac{z}{y^2\tau}\right) \ln\Lambda e^{-y^2} dy\,, \tag{42}$$

$$y_{min} = \frac{a}{d}\sqrt{\frac{z}{\tau}}, \qquad \Lambda = \frac{a^{-2} + \dfrac{z^2}{4\tau^2 y^4}}{\left(1 + \dfrac{z}{\tau}\right) + \dfrac{z^2}{4\tau^2 y^4}}\,.$$

The dependence of the coefficients η_r, η_b and η_c on τ is shown in Fig. 2.

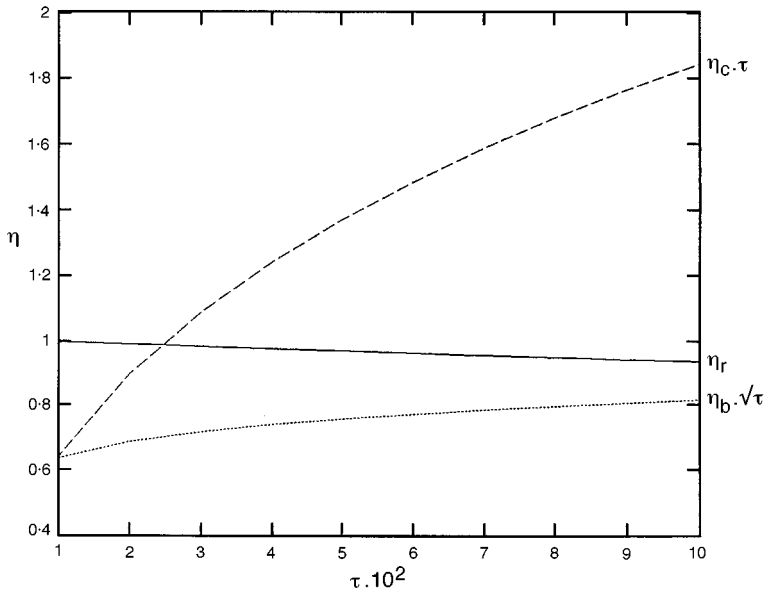

Fig. 2. Dependence of the coefficients of attraction (direct bombardment η_b and Coulomb scattering η_c) and repulsion η_r on the temperature ratio τ (Tsytovich *et al.* 1996*c*).

(14) Linear dusty plasma responses

In a linear description of dust plasma responses one should start at equilibrium and the deviations from it. In deriving these responses we suppose that in equilibrium the rate of electron and ion recombination on the dust particles is compensated by external ionisation creation of them in a way that the equilibrium values of the dust and plasma densities is established. We can find the linear dust plasma responses using the new hydrodynamic equations (Resendes *et al.* 1996).

In a total dusty plasma linear response to the electrostatic field, the dust response enters. For cold dust particles the dust response is

$$\epsilon^d_{\omega,k} = 1 - \frac{\omega^2_{pd}}{\omega^2}\,, \tag{43}$$

where the dust plasma frequency ω_{pd} is determined by the relation

$$\omega^2_{pd} = \frac{4\pi n_d e^2 Z_d^2}{m_d}\,.$$

Although the mass of the dust particle is large, Z_d^2 is also large, and the dust plasma frequency is usually small compared with plasma frequencies, but is not extremely small.

By taking into account the dust attraction and repulsion we have

$$\epsilon^d_{\omega,k} = 1 - \frac{\omega^2_{pd}}{\omega^2 - \dfrac{\pi}{4}\eta_r\omega^2_{pd}(ka) + (\eta_b+\eta_c)\omega^2_{pd}\dfrac{a_0}{L_f}}\,. \tag{44}$$

The *new hydrodynamics* (Benkadda *et al.* 1996), with the expressions for the collision frequencies of plasma particles with dust particles introduced above, can be used to find the dusty plasma dielectric function taking into account the change of dust charges in the wave and the dust movement in the wave (with the response to the wave described by equation 44) (Resendes *et al.* 1996):

$$\epsilon_{\omega,k} = A(k^2) + (\epsilon^d_{\omega,k} - 1)B(k^2)\,, \tag{45}$$

where

$$A(k^2) = 1 + \Omega_e^2 \frac{\left[k_i^2 + \dfrac{\tau+z}{1+\tau+z} + \dfrac{1+P}{1+\tau+z}\right]}{\left[\dfrac{k_e^2(\tau+z)}{1+\tau+z} + \dfrac{k_i^2}{1+\tau+z} + k_i^2 k_e^2\right]} + \Omega_i^2 \frac{\left[k_e^2 + \dfrac{1}{1+\tau+z} + \dfrac{\tau+z}{(1+\tau+z)(1+P)}\right]}{\left[\dfrac{k_e^2(\tau+z)}{1+\tau+z} + \dfrac{k_i^2}{1+\tau+z} + k_i^2 k_e^2\right]}\,, \tag{46}$$

and

$$B(k^2) = 1 + \frac{k_e^2(1+P) - k_i^2}{P\left[\dfrac{k_e^2(\tau+z)}{1+\tau+z} + \dfrac{k_i^2}{1+\tau+z} + k_i^2 k_e^2\right]}\,, \tag{47}$$

where

$$k_e^2 = \frac{k^2 v_{Te}^2}{\bar{\nu_e}\tilde{\nu_e}}; \qquad k_i^2 = \frac{k^2 v_{Ti}^2}{\bar{\nu_i}\tilde{\nu_i}}\,, \tag{48}$$

and $\bar{\nu}_e, \bar{\nu}_i$ and $\tilde{\nu}_e, \tilde{\nu}_i$ are the effective electron and ion collision frequencies in the continuity equations and in the momentum equations respectively.

Equation (45) is valid also for the case where dust is treated kinetically, neglecting the close dust–dust collisions except the long range attraction and repulsion. In this case in (45) we have

$$\epsilon^d_{\mathbf{k},\omega} = 1 + \frac{\epsilon^{d,0}_{\mathbf{k},\omega} - 1}{1 + \left(\dfrac{\pi}{4}\eta_r(ka) - \dfrac{a^2}{d_i^2}(\eta_b + \eta_c)\right)(\epsilon^{d,0}_{\mathbf{k},\omega} - 1)}\,, \tag{49}$$

and $\epsilon^{d,0}_{\mathbf{k},\omega}$ is the kinetic dust plasma response which does not take into account the long-range repulsion and attraction of dust particles:

$$\epsilon^{d,0}_{\mathbf{k},\omega} = 1 + \frac{4\pi(Z^d)^2 e^2}{k^2}\int \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\Phi^d_{\mathbf{p}}\frac{d^3p}{(2\pi)^3}\,. \tag{50}$$

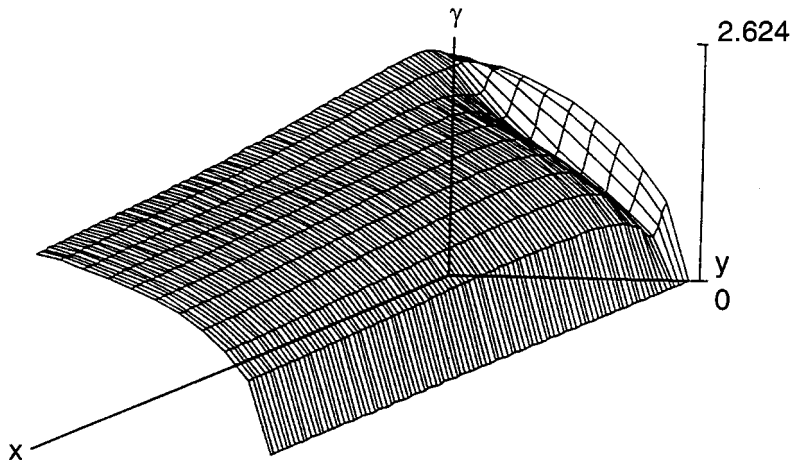

Fig. 3. Dependence of the growth rate of the attraction instability (the z gives values of the dimensionless growth rate γ normalised on $\omega_{pd}a/d_i$) on the temperature ratio τ (the x axis gives values of τ from $0{\cdot}01$ at left up to 1 at the axis crossing) and the dimensionless wave number (the y axis gives values of the wave number in units of the critical value of the wave number—the points to the right correspond to a dimensionless wave number equal to 1). The curves represent the results of numerical solution of the general dispersion relation for hydrogen and dust density $P = 1$ (Resendes and Tsytovich 1997).

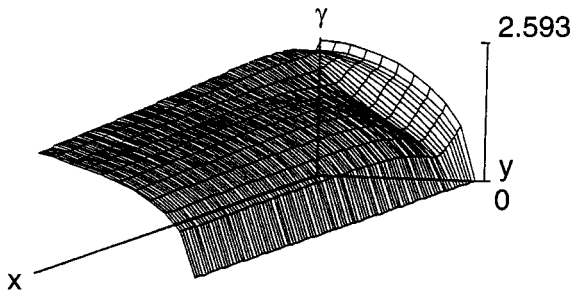

Fig. 4. The same as Fig. 3 but for silicon plasma ions (Resendes and Tsytovich 1997).

These responses can be used for numerical investigation of dispersion properties and the electrostatic dust attraction instabilities. The result is illustrated in Figs 3–6. These responses can also be used for investigation of the dust–dust binary correlation function $h(\rho)$ as a function of the inter-dust distance in units of the fundamental length $\rho = r/L_f$. The result is illustrated in Figs 7 and 8.

(15) Factor Γ

The inter-dust distance is

$$\Delta = (4\pi n_d/3)^{-1/3}\,. \tag{51}$$

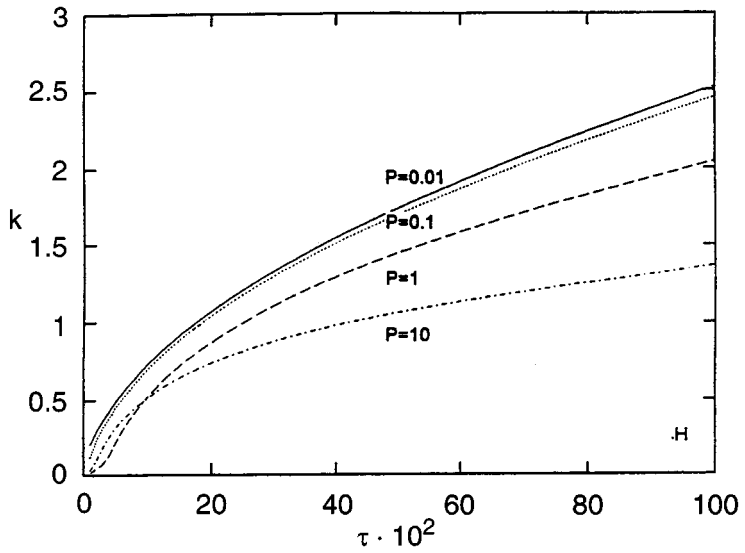

Fig. 5. Dependence of the critical wave number k_{cr} ($k = k_{cr} d_i^2 \sqrt{\tau}/a$) on the temperature ratio τ for different dust densities for hydrogen plasmas [A].

Useful is the relation

$$\frac{a_0}{\Delta} = \left(\frac{P\tau}{(1+P)3z} \right)^{\frac{1}{3}} a^{\frac{2}{3}} . \tag{52}$$

Then we get

$$\tau_d^2 = \frac{a^{\frac{2}{3}}}{\Gamma} \left(\frac{(1+P)3z}{P\tau} \right)^{\frac{2}{3}} . \tag{53}$$

The ratio of potential Coulomb energy to thermal dust energy is denoted as

$$\Gamma = \frac{e^2 Z_d^2}{\Delta T_d} . \tag{54}$$

(16) Estimate of Γ for transition to crystal state

The estimate can be made using the relation

$$\frac{U_m}{T_d} \approx 1 ,$$

where U_m is the minimum potential energy of the two dust particle interaction when the attractive and repulsive forces are equal (see Fig. 1). We find the critical value

$$\Gamma_{cr} = \left(\frac{L_f}{a_0} \right)^{\frac{5}{3}} \left(\frac{P\tau}{(1+P)3z} \right)^{\frac{1}{3}} \frac{(\eta_b + \eta_c)^2}{2\eta_r} . \tag{55}$$

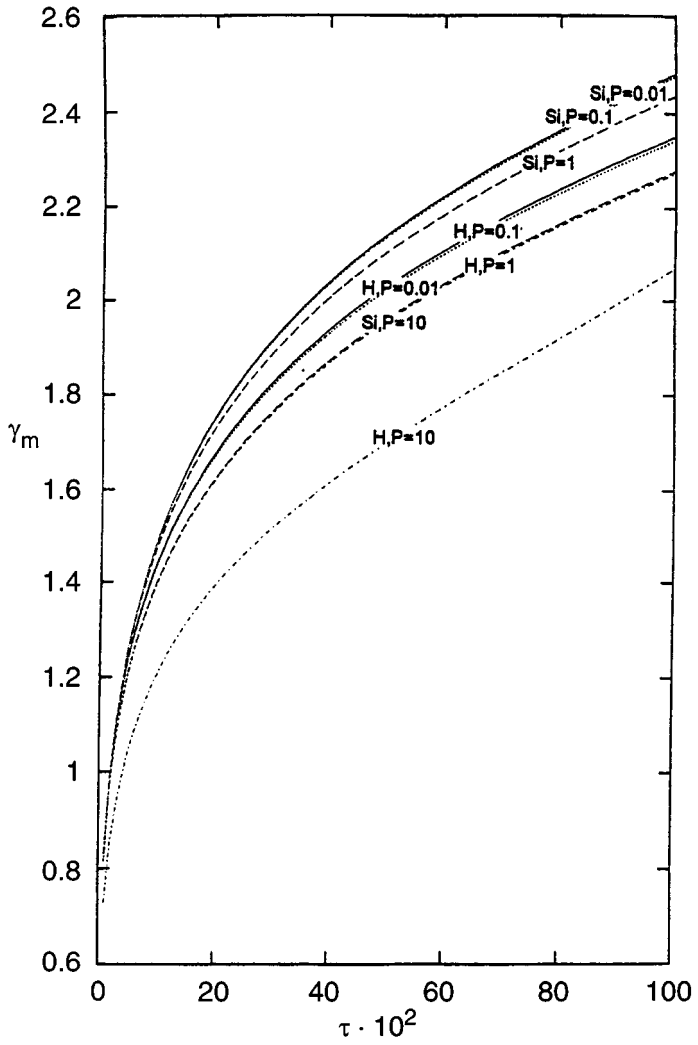

Fig. 6. Dependence of the maximum growth rate $\gamma_m = \gamma_{max} d_i \sqrt{\tau}/a\omega_{pd}$ on the temperature ratio for different dust densities and hydrogen and silicon plasmas [A].

The corresponding value of the critical dust temperature is given by

$$\tau_{cr}^2 = \frac{z}{\tau} \frac{1+P}{2P} \left(\frac{a_0}{L_f} \right)^2 . \tag{56}$$

Figs 9–10 show the numerical results for Γ_{cr}.

5. Some New Computational Results

The simulation was carried out using the code 'KARAT' (Khodataev *et al.* 1996*a*, 1996*b*). The simulation was performed for solitary dust particle charging,

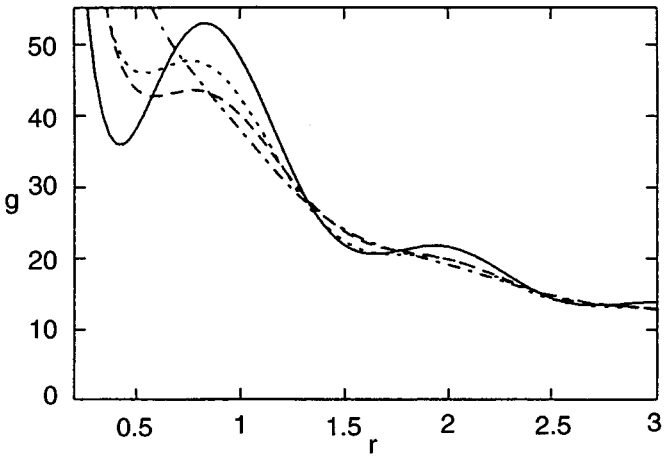

Fig. 7. Dependence of the total (molecular and long range type) binary correlation function h in a dusty plasma on the inter-dust distance r (normalised on L_f in the weakly correlated dusty plasma state). The solid and dotted lines correspond to $P = 1$ and the dashed and dash–dot lines to $P = 0{\cdot}1$. The solid and dashed lines correspond to $\tau = 2 \times 10^{-2}$ and the dotted and dash–dot lines to $\tau = 0{\cdot}1$. The curves were calculated numerically for hydrogen plasma [A].

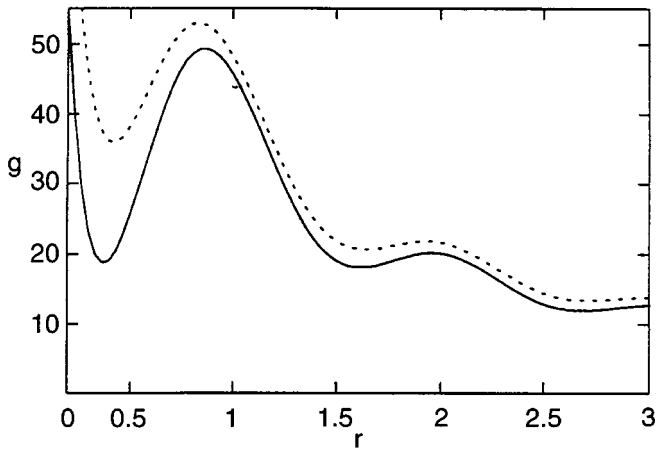

Fig. 8. Same as Fig 7, but for silicon plasma (solid line) and hydrogen plasma (dotted line) for $\tau = 2 \times 10^{-2}$ and $P = 1$.

for a dependence of the charges of two dust particles on the distance between them and for dust–dust attraction and repulsion for large inter-dust distances. The electric potential was calculated by means of Poisson's equation. The plasma was simulated by the $2\frac{1}{2}D$ PIC (particle in cell) method with three components of particle velocity. The dust grains were spherical and situated on the axis of the cylindrical geometry used. The calculation was performed on a rectangular uniform grid in the $r - z$ plane. The dimensions of the calculation grid were taken to be large enough for the distances between the grain surface and the

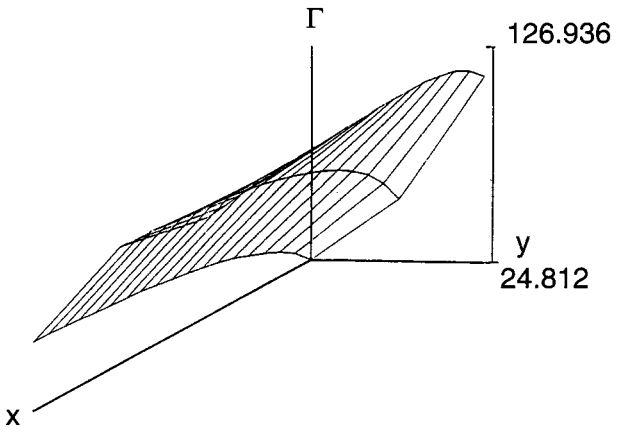

Fig. 9. Dependence of Γ (z axis) on the temperature ratio (y axis) varying from 10^{-2} up to 10^{-1} on the axis crossing and dust densities P (x axis) being $0{\cdot}1$ and $1{\cdot}10$ on the axis crossing.

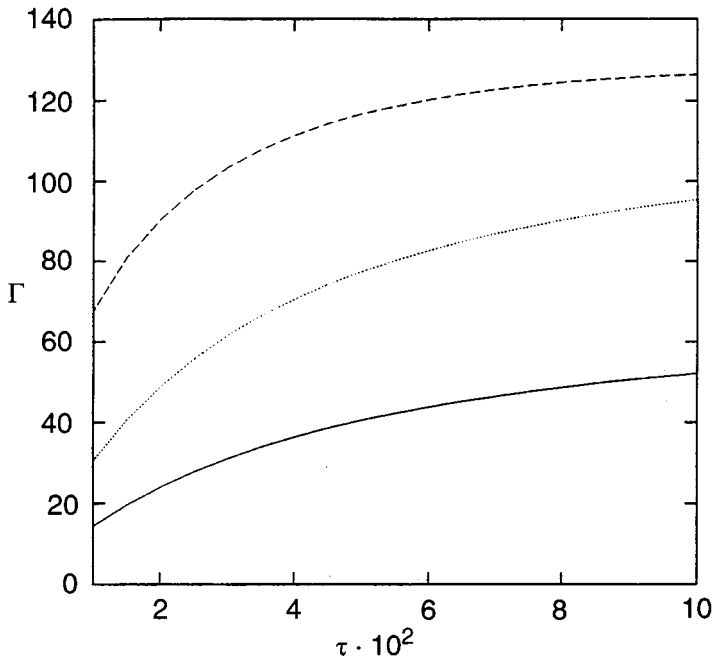

Fig. 10. Dependence of Γ on the temperature ratio $\tau \times 100$ from $\tau = 10^{-2}$ to $\tau = 10^{-1}$ for different gases. The solid line corresponds to $P = 0{\cdot}1$, the dotted line to $P = 1$ and the dashed line to $P = 10$.

boundary of the calculation area to be several times larger than the Debye length. The number of plasma superparticles was approximately $1{\cdot}5 \times 10^5$ for each plasma species. The time step was specified small enough for the distance travelled by the fastest electron during one time step to be several times less

than the size of the cell. The temperatures of the electrons and ions were chosen to be equal. The plasma particles attach to the grain surface as soon as they collide with the grain. As soon as a plasma particle attaches to the grain another plasma particle is injected from the surface of the calculation area with thermal distribution. As soon as the particle leaves the calculation area by approaching its boundaries, another thermal particle is injected from the surface of the calculation area. The mass ratio m_i/m_e was 18 and 30. The following results were obtained (Khodataev *et al.* 1996*b*):

- The results are not sensitive to the mass ratio. This is in accordance with the theory predicting only a logarithmic dependence of dust charge on the mass ratio.
- The charging process and plasma disappear as soon as the plasma particle injection is avoided. This is in agreement with general statements that a dusty plasma is an open system.
- The charging and dust interaction depend weakly on the type of plasma particle injection—volume or surface injection.
- The dynamics of charging a single dust grain is in good agreement with OML predictions up to grain sizes a of about d (the Debye length). The observed results are illustrated in Fig. 11. This statement is important since there exists a belief that the OML approach is valid (if at all) only for $a \ll d$. Both theory and experiment do not contradict the results.
- The non-Debye screening of a single grain was observed. The potential ϕ is found to be proportional to $1/r^2$ (see Fig. 2). This is in accordance with theoretical predictions given above, although the theory is more general since it covers also the case of unequal electron and ion temperatures and dust density dependence. These results confirm the existence of dust repulsion forces for distances larger than the Debye screening length.

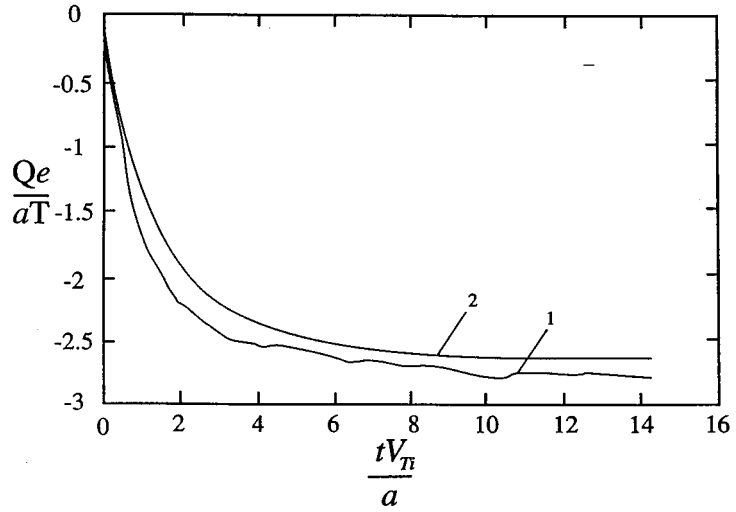

Fig. 11. Formation of equilibrium charge on dust particles in plasma. Asymptotic in time corresponds to the equilibrium charge. Line 1 corresponds to the results of numerical simulations, while line 2 corresponds to the OML theory (Khodataev *et al.* 1996).

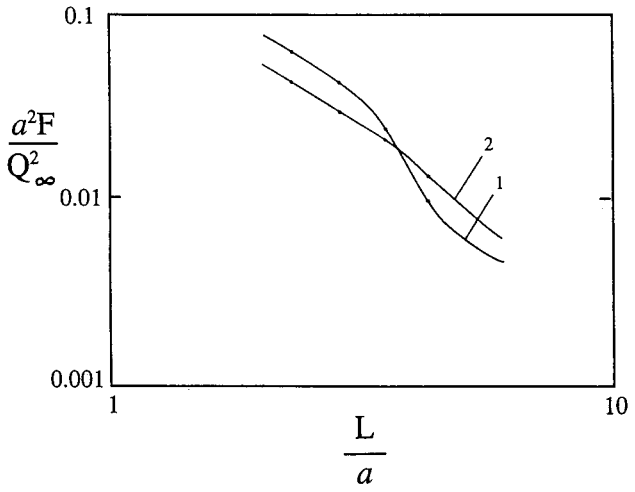

Fig. 12. Forces between two dust particles at large distances. Line 1 corresponds to non-Debye screening repulsion, while line 2 corresponds to bombardment attraction. The crossing point of the two lines corresponds to a balance of attraction and repulsion (Khodataev *et al.* 1996).

- The charges of two grains decrease with a decrease of inter-dust distance in accordance with the law keeping the potential on the surface of the grain constant. This is in accordance with the theoretical predictions (Tsytovich 1994) $Z_d = Z_{d,\infty}/(1 + a/r)$. The total electrostatic energy of two grains decreases with a decrease of the inter-dust distance in accordance with theoretical predictions. The result is important for the formulation of the variational principle in dusty plasmas as the minimum of the electrostatic energy with the restriction that the potential on the surface of each dust grain is constant.
- The attractive bombardment forces due to shadowing of plasma fluxes were found. The results are in agreement with the theoretically predicted $1/r^2$ dependence of the attractive forces with respect to inter-dust distances (Fig. 12), although the theoretical results are more general being valid for unequal electron and ion temperatures where direct bombardment and Coulomb scattering forces differ appreciably (for equal temperatures they are of the same order of magnitude).
- The possibility of dust–molecule formation was discovered. The binding molecular energy is in accordance with theoretical predictions given above, although the theoretical predictions are more general and valid for unequal electron and ion temperatures and describe the dependence on dust density. Fig. 13 shows the molecular-type of potential observed in numerical simulations.

With known forces between dust particles (attractive and repulsive forces), molecular dynamical simulations were performed. In addition friction forces were introduced to simulate the dust friction on the neutral plasma component. The interaction of dust particles was considered in binary approximation. The numerical 2D and 3D simulations where performed in the framework of the

PIC approach using the KARAT code in conditions where all three mentioned inequalities are satisfied (Khodataev *et al.* 1998; see also Vladimirov and Tsytovich 1998). The KARAT code allows input in physical units. In the 2D simulations

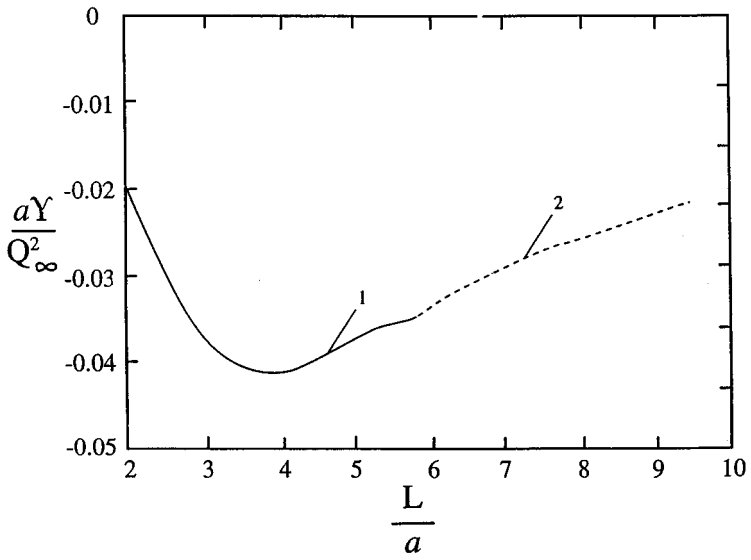

Fig. 13. Total potential of the two dust particle interaction. The solid line corresponds to the results of numerical simulations and the dotted line to the theoretical continuation of the simulation curve (Khodataev *et al.* 1996).

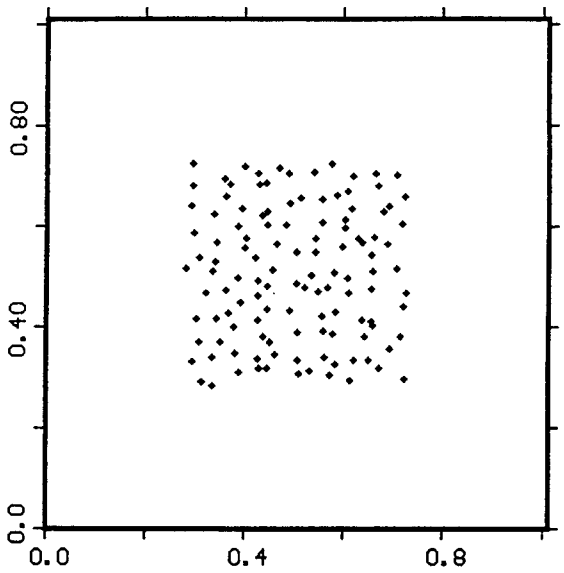

Fig. 14. Initial random dust particle distribution for starting the simulations (Khodataev *et al.* 1998*a*).

the number of dust particles was 50, while in 3D simulations it was 200. In both cases $T_i = 0{\cdot}03$ eV, $T_e = 3$ eV, $n_e = 3 \times 10^9$ cm^{-3}, $d_i = 23{\cdot}4\,\mu$m and $|\phi_0| = 3{\cdot}7$ V. The sizes of the dust particles were 2 and 1 μm, the dust molecular radii were 228 and 196 μm, and the charges of dust particles were 5200 and 2600, corresponding to the 2D and 3D cases. The starting distribution of the

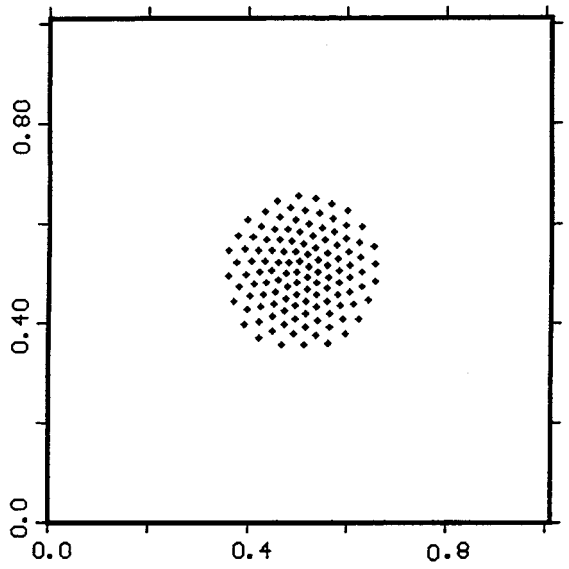

Fig. 15. Final crystal state in 2D case observed as the result of numerical simulations (Khodataev *et al.* 1998*b*).

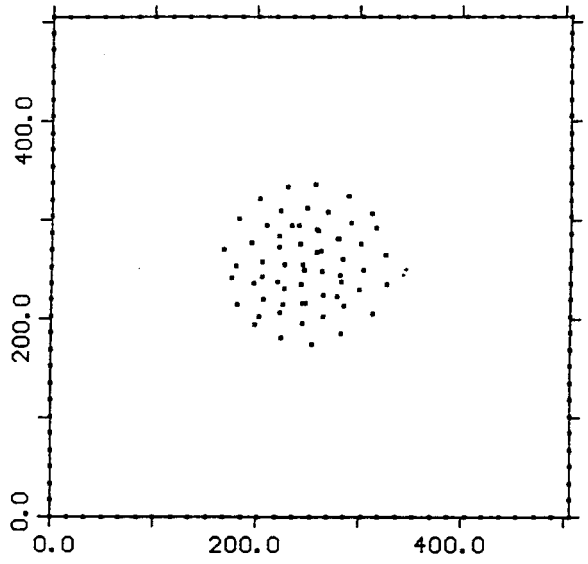

Fig. 16. Same as Fig. 15 but for the 3D case (Khodataev *et al.* 1998*b*).

dust particles was random (Fig. 14) . After beginning the simulations the initial stage in both 2D and 3D cases can be considered a collapse of the dust cloud, which is the result of attractive forces acting between grains. This initial stage is described analytically as a gravitation-like instability. A final equilibrium state is reached which cannot be described by the linear attraction instability. The final dust cloud is observed to have a spherical shape with a separation between dust particles of 500 μm in the 2D case and 300 μm in the 3D case, which is larger but of the order of the dust molecular radius. Figs 15 and 16 illustrate the structure of the cloud obtained the in 2D and 3D cases. Figs 17 and 18 represent the pair correlation $g(r)$ as a function of the inter-dust distance r for the 2D and 3D cases respectively. In the 3D case visualisation problems exist since different crystal plains are superimposed on each other, but it is clear that the regular structure appears with the size of the cell slightly increasing to the periphery. The dust cloud is in both cases a boundary-free crystal. In the central part of the structure each grain has six nearest neighbours which correspond to the structure of crystals observed in the laboratory. On the periphery of the structure the dislocations can be seen. The dust structure is inhomogeneous since in its central part the inter-dust distance is smaller than on the periphery. The presence of a very ordered state in the 3D case can be shown by plotting the pair correlation function $g(r)$ (Fig. 18). The space variation of the inter-dust distance results in a damping of the pair correlation function. In the 2D case, $g(r)$ corresponds to the presence of long-range order while due to inhomogeneity in 3D case the correlation function has only three peaks. The condition of applicability of the pair interaction in the final stage was checked and confirmed.

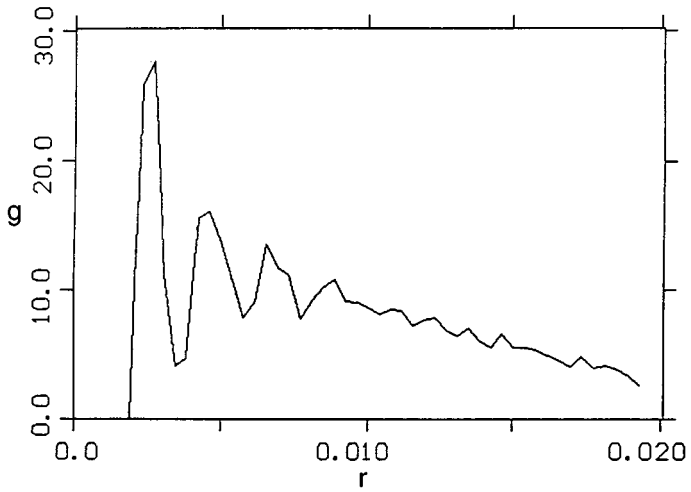

Fig. 17. Pair correlation function for dust particles observed in numerical simulations in the 2D case (Khodataev *et al.* 1998*b*).

The crystal structure appears only for low dust temperatures, while the gaseous structure can be found more often. In what follows we discuss only the gaseous self-organised structures. The question was already asked whether such a self-organised dust structure can exist in space since both dust and plasma are very common in space. In particular the question was raised as to whether in

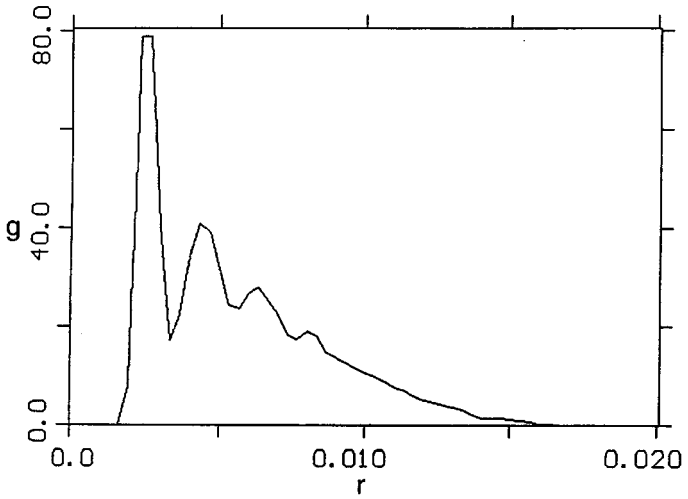

Fig. 18. Same as Fig. 17 but for the 3D case (Khodataev *et al.* 1998*b*).

space there already exist dust–plasma crystals. But the question is indeed more general since the variety of self-organised structure could be large and the type of interaction between them could be very complicated. The general questions arise about natural evolution of these structures in space and the possibility of natural storing of information in this evolution. In the case that questions can be answered positively it will be obvious that the present search for extra-terrestrial intelligence has been made in completely the wrong direction.

PART II. ONE-DIMENSIONAL STRUCTURES IN DUSTY PLASMAS

6. Introduction to Concept of Dust–Plasma Structures

Let us discuss a possible scenario for dust–plasma structure formation. Let us look at the plasma region where dust is absent. The plasma structures can be usually formed in the absence of dust. Any structures of this kind could have, for example, the electric potential inside the structure less (negative) or larger (positive) than the surrounding plasma potential. Suppose the difference in potential is negative and is ϕ. Then the trapping energy for the dust particle will be $Z_d\phi$. It is well known that dust particles are trapped in electrostatic potential wells due to their large charges. They are known to be trapped by floating potentials of the walls of the discharge chamber. The same is true for potential structures in plasma—they become traps for dust particles. In the case where many dust particles are trapped in the structure they start to change the structure itself. First of all they create an additional negative potential and absorb the plasma particles. The other plasma particles outside the structure will start to move toward the structure creating a flux of plasma to the structure. Soon the structure becomes a dissipative structure and should be treated as a self-organised structure. This scenario is only one of the possible ways to create

dust–plasma structures. In some laboratory experiments the plasma is created by volume ionisation, as it is in the case of etching experiments. These structures, after formation, will also be dust–plasma self-organised structures.

Another possibility for creation of a dust–plasma structure is an increase of instability caused by presence of dust particles in the plasma. Since dust introduces a high rate of dissipation, all negative energy modes such as drift modes or beam modes become much more unstable in the presence of dust in plasma.

As soon as the dust–plasma structure is formed it should create plasma fluxes toward the structure. One can foresee the consequences of the presence of these fluxes. First of all they transfer the energy, momentum and angular momentum to the structure. Conservation of angular momentum implies that the structure in general should be a vortex structure. The simplest of this kind is the 2D vortex structure. Therefore, one can expect that the formation of vortex structures in a dusty plasma should be a typical phenomenon.

In the case where the flux does not transfer the angular momentum we can ask the question on the fate of the particle fluxes and momentum fluxes. Current conservation implies that outside the structure the flux of electrons is equal to the flux of ions (for a stationary structure). The main part of the momentum is transferred to the structure by ions. In the case of symmetry (planar or spherical) the total momentum transferred could be zero (momentum fluxes transferred from different sides compensate each other). Then such a structure will be stationary in the sense that it does not receive the momentum. The ions and electrons are absorbed by dust particles. But the collision frequency for ion scattering by dust particles is larger than the collision frequency of their absorption by dust particles. This statement was illustrated above in detail. Thus the ions will be more often scattered than absorbed, which means that the ions will be accumulated in the dust structure. Thus the structure not only traps the dust particles but also accumulates the ions. The pressure of the accumulated ions will work against the ram ion pressure of the ion flux towards the structure. Thus, one can foresee that the dust–plasma structures will be not only structures where the dust exists, but also the structures where the plasma ion density is increased. Both dust and plasma densities should be distributed in the structure self-consistently. This structure can survive only in the presence of plasma outside the structure to create the fluxes toward the structure, or in the presence of a constant source of external ionisation in the structure. In both cases the plasma serves as 'food' for the structure.

We consider here the problem of a 1D dust–plasma structure which can be treated by a 1D set of nonlinear equations, and will consider the case where the plasma structure is exposed from both sides to equal and opposite-in-direction plasma fluxes (the direction of the fluxes being perpendicular to the plane of the 1D structure). It will be assumed that the plasma fluxes are subsonic. In the case where the plasma fluxes are supersonic, the shock waves will be formed. Similarly, for unequal fluxes the momentum transfer to the dust structure will eventually transfer it and form a shock. Thus our next attempt will be to consider the properties of shock waves in dusty plasmas (see Part III).

We also consider here the spherical 1D structures (supposing the dependence only on one spherical coordinate).

The problem of stability of 1D structures will not be considered here. Recent investigations show that the 1D structures are stable for disturbances along the direction perpendicular to the plane of the structure. The general statement of this kind was not proved but for many conditions the stability was found. The structure can be unstable for disturbances along the plane of the structure, i.e. for 2D disturbances, leading to convection of dust in the structure (Bouchoule *et al.* 1998). This convection does not change the global properties of the structure since the convective disturbances appear to be small on the nonlinear stage of convective instability. These are the only statements which can be made presently on the problem of stability of 1D dust structures. We will not touch on it here in detail. Also we will not consider here the 2D vortex structures in dusty plasmas including the drift, nor the interesting problem of combined vortices in which both the neutral component forms a vortex which through collisions of neutrals with dust particles is bounded to a drift vortex forming a new type of vortex (see ref. [C]).

Only the stationary dust–plasma structures can be described by a system of nonlinear equations which in the dimensionless form (when the sizes are measured in the fundamental length introduced above) has no small parameter and in this system describes universal structures. The latter fact can be foreseen from the mentioned properties of the dusty plasma where the plasma particle mean free path is equal to the fundamental length divided by the parameter P. Thus the stationary dust–plasma structures will be described by a universal equation not containing a small parameter (and in which all terms are of the same order of magnitude), if on average in the structure the parameter P is of the order of 1. We show with examples that we consider this is indeed the case.

To illustrate this statement we can write down the equation for change of dust momenta in dimensionless form, assuming that all variables depend only on the space variable $\mathbf{r}'/L_f$, denoted in the subsequent equation as $\mathbf{r}$. As an example, we can write the expression for the 1D momentum equation for dust particles taking into account only the attractive and repulsive forces in the dimensionless coordinates x inroduced here:

$$\frac{u_d}{\sqrt{2}v_{Ti}}\frac{\partial\left(\dfrac{u_d}{\sqrt{2}v_{Ti}}\right)}{\partial x} = \eta_a Z_d^2\frac{\partial U_a}{\partial x} + \eta_r Z_d^2\int_{-\infty}^{+\infty}\frac{n_d(x')}{n_{0,i}}\frac{1}{2(x-x')}dx'\,, \tag{57}$$

$$\frac{\partial^2}{\partial x^2}U_a = -\frac{n_d}{n_{0,i}}\,. \tag{58}$$

We have written this expression to show that the attractive forces can indeed be described by a Poisson-type of equation, that the additional repulsive forces (the electrostatic repulsion is not included in RHS of equation 57) always have an integral form, and that there are in the dimensional variables no small parameters—all the terms can be of the same order of magnitude.

One important point should be mentioned which is related to the dust attraction which causes the contraction of dust clouds and helps the formation of dust–plasma structures. This attraction will work on large time scales when the dust particles

will be able to respond to disturbances in their density. But for stationary dust structures the presence of this interaction is very important. In the case where the size of the structure is larger than the plasma particle, the mean free path the attraction creates a surface layer with a surface tension similar to some sort of 'skin' separating the structure from the surrounding plasma.

Of course the structures cannot be stationary or can be varying in time so fast that the dust particle will be not able to respond to these changes. Then these structures will not be universal. The point is that the dust in the plasma opens many new possibilities to form self-organised structures. The variety of these structures in dusty plasma is much larger than in other systems.

To illustrate the general properties of self-organised dust–plasma structures we start with the simplest case of 1D structures.

7. 1D Self-organised Dust–Plasma Structures

In a 1D model the plasma–dust cloud is a sheath with plasma fluxes from both sides which dissipate in the dust cloud. They bring the energy and momentum. In the case where the cloud is symmetric, both fluxes of plasma are equal in value and opposite in sign. The appearance of these fluxes is due to the processes similar to that of charging of a single dust particle: the thermal electrons move faster to the cloud, they charge it until the ion and electron fluxes become equal and zero net current is formed. As already mentioned, such self-organised dust clouds will have interesting properties for accumulating plasma in the cloud, since the rate of plasma recombination on the dust is less than the rate of momentum transfer. This occurs due the continuity of plasma fluxes since a decrease in average flux velocity leads to an increase of plasma particle density. But this density cannot go to infinity when the particle drift velocity becomes zero (in the centre of the cloud) since the absorption on the dust particle decreases their density. As a result some self-consistent distribution of dust and plasma will appear with an enhancement of ion density, depletion of electron density and an increase of dust density compared with the value close to zero on the periphery of the cloud to its maximum value in the centre of the cloud. The cloud is not overall neutral being negatively charged, but the electron density does not go to zero in the centre of the cloud, which one could expect by not treating all the processes self-consistently (for example considering only the charging process for which there seem to be less electrons available with the penetration inside the cloud). One needs to treat self-consistently and nonlinearly all the components: dust density, electron and ion density and dust charges. None of these components tends to large or infinite values in the case where the problem is treated self-consistently. The zero of electron density is excluded. Thus the electrons are always left to charge the dust particles, although the charges can decrease somewhat towards the centre of the cloud. Thus, by increasing the dust density inside the cloud the plasma drift velocity outside the cloud increases to make the charges of dust particles inside the cloud not very small.

Two actual structures can be related to the model of the plasma–dust cloud formulated here—one is planetary dust rings and the other is the plasma sheath. The physical processes in planetary rings are more complicated than those we will consider below due to Kepler motion of dust particles in the plane of the ring and bounding of the motion in the plane of the ring to the motion perpendicular

to the plane of the ring. Such a plasma theory of planetary rings does not exist at the present time and we will be not be able to consider this problem here, but leave it for future research and special representation. The plasma in the sheath is also more complicated than the model we consider due to the presence of the electric field and ion fluxes which do not decrease to the plate determining the boundary of the sheath. But, still, an analogy exists and we will keep it in mind.

The plane dust–plasma structures without the Keplerian motions, and without the ion drift and electric fields, can actually be found in the upper atmosphere and lower ionosphere and are known as noctilucent clouds. One of their amazing properties is the presence of sharp edges separating the regions where the dust exists and does not exist. We will try to find this property for the model of the 1D dust–plasma self-consistent structure.

There are free parameters in the self-consistent treatment which include the dust density per unit surface area integrated over the whole thickness of the cloud and the plasma density outside the cloud. However, the drift velocity, the charge on the dust, and the ion and electron densities distributions in the cloud are established as a result of the nonlinear interaction of all components of the dusty plasma.

The thickness of the cloud varies but not very much and is of the order of the dust fundamental length. This follows from the fact that nonlinear equations governing the structure of the dust cloud can be written in dimensionless variables containing as a measure of length the dust fundamental length.

We write the nonlinear hydrodynamic equations taking into account dust and plasma particle interactions in dimensionless variables, using the friction force between the dust and the ions as the main process of momentum exchange and assuming that all variables depend only on the space variable $\mathbf{r}'/L_f$, denoting it below as $\mathbf{r}$ to give the simplest notation for dimensionless variables.

We consider certain simplifications of the new hydrodynamic equations to describe the dust–plasma structure. These equations in dimensionless variables still depend on several dimensionless parameters. Among them is the parameter τ_d introduced above which is related to the dust pressure. For self-organised structures the simplest and most interesting case is $\tau_d \ll 1$. One can consider as well the opposite limit. We call the case where the dust pressure does not play any role as the *cold dust cloud case*, while the case where the dust pressure dominates is the *hot dust cloud case*. The main consideration is given for cold clouds keeping in mind that the dust temperature can also be low due to dust radiation cooling (see below).

In principle, we should not only consider the balance of the density of all particles (dust, electrons and ions) and the balance of their momenta, but also the balance of heat in the dust cloud. Here we arbitrarily exclude the heat transfer supposing the temperature of all particles to be constant. We will see that even in such a consideration many unexpected features of self-organisation arise.

The next simplification will be made by considering the type of dust–dust interactions. They are described by the coefficients η_a and η_r already mentioned. The complications in the mathematical treatment arise from the repulsion of dust particles described by η_r. This repulsion should be added to the Coulomb repulsion described by electrostatic interactions and it should be compared with

the attraction described by the coefficient η_a. The attractive forces present no difficulties for the mathematical description. The reason is that the new attractive forces are similar to gravity and thus can be described by equations similar to Newton's equation which is differential (the sign in the 'Poisson' equation should be changed to the opposite one). But the repulsive forces which depend on the inter-dust distance as $1/r^3$ can never be converted to a differential form. We consider only the case where the inter-dust distances are large enough to neglect the electrostatic repulsion of dust particles. The ratio of the coefficients η_a to η_r changes with the dust sizes and the smaller the dust particles the less important are the repulsion forces related to the coefficient η_r. Already for $a = a_0/d < 0{\cdot}01$ we can neglect the additional repulsive forces. This is the reason for simplifying the problem and for taking into account only the attractive forces and neglecting the additional repulsive forces.

We also introduce another dimensionless value to be able to describe the cloud nonlinearly. Since the main momentum transfer occurs between dust and ions it is useful to normalise all densities, not on the electron, but on the ion unperturbed density $n_{0,i}$. This is the ion density outside the dust cloud. In this section we use the notation n_e for $n_e/n_{0,i}$. Outside the dust cloud we have $n_{0,i} = n_{0,e}$. We also use the notation n for the dimensionless ion density $n_i/n_{0,i}$ and the notation P for $n_d Z_d/n_{0,e} = n_d Z_d/n_{0,i}$, i.e. we relate the dust charge density not to the local electron density, but to the electron density outside the cloud. For the ion drift velocity we use the notation $\mathbf{u} = \mathbf{u}_i/\sqrt{2}v_{Ti}$.

We write the nonlinear hydrodynamic equations taking into account dust and plasma particle interactions in dimensionless variables, assuming that the drift dust velocity is zero, and that all variables depend only on the space variable $x = x'/L_f$. The electrons to a good approximation can be treated by the Boltzman distribution

$$\frac{1}{n_e}\frac{\partial n_e}{\partial x} = \frac{\partial}{\partial x}\left(\frac{e\phi}{T_e}\right). \tag{59}$$

In this expression x can have any dimension and we use for x the introduced dimensionless coordinate measured in the L_f. Similarly for the electron density, since the left side of (59) contains n_e in both the numerator and denominator.

From the continuity equation for electrons and ions we find in dimensional variables $n_e u_e = nu$ which is a consequence of zero currents on dust particles. Thus one finds directly the electron drift velocity for known ion drift and ion density and this ensures that the total plasma current on dust cloud is zero.

In the momentum equation for ions we take into account the electric force, the pressure force, the momentum converted in collisions with the dust (from the conservation law of momentum exchange between dust and ions) and the attractive and repulsive forces. We start first by writing this equation in dimensional variables dividing the dust momentum equation by $n_{0,i}T_e$:

$$\frac{n_d u_d}{n_{0,i}T_e}\frac{\partial u_d}{\partial x} = \frac{n_d Z_d}{n_{0,i}}\frac{\partial}{\partial x}\left(\frac{e\phi}{T_e}\right) - \frac{T_d}{n_{0,i}}\frac{\partial n_d}{\partial x}$$

$$+ \tilde{\nu}_i m_i (u_i - u_d)\frac{n_i}{n_{0,i}} - \eta_a \frac{n_d Z_d}{n_{0,i}}\frac{\partial}{\partial x}\frac{U_a}{T_e}, \tag{60}$$

where the attraction potential is

$$U_a = -e^2 \frac{a^2}{d_i^2} \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} Z_d n_d(\mathbf{r}') d\mathbf{r}' \,. \tag{61}$$

For investigation of the stationary slab we assume that the dust drift velocity is small and neglect the terms containing it. To proceed we need to discuss the charging process and the dependence of the dust charge on the coordinate x. We use the notation $z = z(x)$ for the dimensionless charge changing with distance x. Since we are considering the stationary distribution and the dust drift velocity is zero, the local equilibrium charges are reached and depend on x as a solution of the equation

$$\exp(-z(x)) = \sum_i (Z_i z(x) + \tau_i) \frac{n(x) Z_i}{n_e(x)} \sqrt{\frac{m_e}{m_i \tau_i}} \,. \tag{62}$$

We do not write further the argument of z. For the hydrogen plasma (62) gives

$$\exp(-z) = \frac{n}{n_e} \frac{1}{\sqrt{\tau m_p / m_e}} (z + \tau) \,. \tag{63}$$

Let us write next the momentum equations for dust and ions and the ion continuity equation in dimensional variables to show the starting equation in a convenient form, neglecting the additional repulsive forces between the dust particles:

$$n_d m_d u_d \frac{\partial u_d}{\partial x} = Z_d e \frac{\partial \phi}{\partial x} + \tilde{\nu} m_i n_i (u_i - u_d) + \frac{Z_d^2 e^2 \eta_a(\tau/z) a_2}{d_i^2} \frac{\partial U_a}{\partial x} - T_d \frac{\partial n_d}{\partial x} \,, \tag{64}$$

$$\frac{\partial^2 U_a}{\partial x^2} = -4\pi n_d \,, \tag{65}$$

$$n_i m_i u_i \frac{\partial u_i}{\partial x} = -e \frac{\partial \phi}{\partial x} - T_i \frac{\partial n_i}{\partial x} - \tilde{\nu}_i n_i m_i (u_i - u_d) \,, \tag{66}$$

$$\frac{\partial n_i u_i}{\partial x} = -\bar{\nu}_i u_i \,. \tag{67}$$

Note that the momentum transferred from ions to dust due to ion drag is equal to the momentum received by the dust due to the drag force. For the dust plane sheath where the plasma fluxes are acting from both sides of the sheaths we assume that the plasma conditions are the same from both sides of the sheath and thus in total no momentum is transferred to the dust sheath and thus $u_d = 0$.

It is easy to see that in the derivation of $\tilde{\nu}_i$ and $\bar{\nu}$ the z was arbitrary. We write the plasma–dust collision frequencies using the dimensionless quantities introduced above in the form:

$$\frac{\tilde{\nu}_i n_i m_i u_i}{T_e n_{0,i}} \approx \frac{1}{L_f} P n u z I_m(z/\tau)\,, \tag{68}$$

$$\frac{\bar{\nu}_i}{\sqrt{2} v_{Ti}} = P \frac{1}{L_f} I_c(\tau/z)\,, \tag{69}$$

where

$$I_m(x) = \frac{2}{3\sqrt{\pi}} \left(\ln\left(\frac{1}{a}\right) + 2x + 4x^2 \right)\,, \tag{70}$$

$$I_c(x) = \frac{1}{2\sqrt{\pi}}(1+x)\,. \tag{71}$$

Then the equations can be written by using the dimensionless variables and assuming that the dust density depends only on x. After integration with respect to $r_\perp$ the attractive force can be expressed through the 'Poisson' type equation, but the repulsive force can never be written in this form and will always lead to an integral interaction. Using introduced dimensionless variables we find that

$$P \frac{1}{n_e} \frac{\partial n_e}{\partial x} - \tau_d \frac{\partial}{\partial x} \frac{P}{z} + P z n u I_m(\tau/z) + \eta_a(\tau/z) P z \tau \frac{\partial U_a}{\partial x} = 0\,, \tag{72}$$

$$\frac{\partial^2 U_a}{\partial x^2} = -\frac{P}{z}\,, \tag{73}$$

$$\tau_d = \frac{e^2 T_d}{a T_e^2}\,. \tag{74}$$

For a cold dust cloud the parameter P is dropped from the equation:

$$\frac{1}{n_e} \frac{\partial n_e}{\partial x} + z n u I_m(\tau/z) + \eta_a(\tau/z) \tau \frac{\partial U_a}{\partial x} = 0\,. \tag{75}$$

The terms in (72) have a simple physical meaning: the first term represents the electric force (which according to the adiabaticity of electrons is determined by the inhomogeneity of the thermal distribution), the second term describes the dust pressure force, the third term describes the momentum which the dust particle receives due to the friction on ions (often this effect is called the ion drag force) and the fourth term represents the attractive force of dust particles due to the shadowing of the fluxes of plasma particles.

We should write then the momentum and continuity equations for ions in dimensionless variables. We find that

$$\tau \frac{\partial}{\partial x} u^2 = -\frac{1}{n_e} \frac{\partial n_e}{\partial x} - \frac{\tau}{n} \frac{\partial n}{\partial x} - PzuI_m(\tau/z)\,, \tag{76}$$

$$\frac{\partial nu}{\partial x} = -PnI_c(\tau/z)\,. \tag{77}$$

We can see from these equations that indeed in dimensionless variables for τ of the order of 1 there is no small parameter—all forces including the attractive forces are of the same order of magnitude (for τ of the order of 1). *This result is of great importance since even without any calculations one can conclude that the dust structures should have a size of the order of the fundamental dust length, and this fact is another reason to refer to L_f as the DFL.* For space research the qualitative conclusions are often the most important and one will able use the observations to either confirm or deny the conclusion that the dust clouds should have a size of the order of L_f. Such a comparison can be made for dusty planetary rings and the present data agree with this statement, at least for rings where the dust size distribution is well known. Although for tiny rings the dust size distribution is not established the dust sizes accepted at the present time do not contradict the statement that their thickness is comparable with the DFL.

Finally we write the Poisson equation in the dimensionless variables

$$\frac{\partial^2}{\partial x^2}\left(\frac{e\phi}{T_e}\right) = \frac{\partial}{\partial x}\left(\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) = \frac{\tau}{a^2}(n_e + P - n)\,. \tag{78}$$

For $a = a_0/d_i \ll 1$ the quasi-neutrality is valid and we are left with the statement that the only important length is the DFL. As soon as the quasi-neutrality is violated for some reason a new length d appears described by the Poisson equation (78). In some cases it can give a fine spatial structure in the dust cloud.

8. Numerical Results for 1D Dust Structures

Let us consider first the numerical results for 1D self-organised structures for cold ions. The structures we obtain from the nonlinear equations written above are rather different from the structures known as solitons or cavitons, since they can exist only in the presence of a constant plasma flux from the plasma surrounding these structures. Thus a certain flux of particles and energy should exist through the surface of the structure. This makes these structures different from the ones widely known. But in space conditions these fluxes exist all the time and it is natural to find these structures in space.

We consider first the structures neglecting both the dust pressure and the dust attraction. Since the attraction coefficient is large this is possible only in the case $\tau \ll 1$. Then we drop all terms containing τ in front of them, including the ion pressure and ion ram pressure terms. Then we find $P = n$, which does not mean $n_e = 0$ in the case where we do not suppose the local charge neutrality conditions to be valid and we need to assume the absence of charge neutrality in the case where we neglect all effects proportional to τ, including dust attraction. Thus

this case is not of great interest since it assumes $\tau \ll 1/\eta_a \ll 1$, but we can start with this simplest case to show that even in these conditions the self-organised structures can be created.

We obtain then the simplest equation which has the following meaning:

1. Balance of ion drag force and the electric force acting on dust particles:

$$\frac{1}{n_e}\frac{\partial n_e}{\partial x} = -PzuI_m\,. \tag{79}$$

2. The Poisson equation:

$$\frac{\partial}{\partial x}Pzu = -\frac{\tau}{a^2 I_m}n_e\,. \tag{80}$$

Although $\tau \ll 1$ the parameter a is very small and the factor τ/a^2 can have any value and thus we leave this term.

3. Continuity equation for ions:

$$\frac{\partial}{\partial x}Pu = -P^2 I_c\,. \tag{81}$$

4. Charging equation:

$$\frac{\partial z}{\partial x} = \frac{z}{1+z}\left(\frac{1}{n_e}\frac{\partial n_e}{\partial x} - \frac{1}{P}\frac{\partial P}{\partial x}\right)\,. \tag{82}$$

Although the charging equation is an algebraic one it is useful to calculate from it the derivative of the charge with respect to the coordinate x and use it as an initial (boundary) value to start with the solution of the charging equation for the boundary value of the parameters.

The written system of nonlinear equations is valid for $u \ll 1$, neglecting the dust pressure:

$$I_m \approx \frac{2}{3\sqrt{\pi}}\ln\left(\frac{1}{a}\right); \qquad I_c = \frac{1}{\sqrt{\pi}}\,. \tag{83}$$

The procedure for calculating a self-organised structure is the following: we start with some initial values at $x = 0$ for a particle flux Pu, an electron density n_e, and an ion density n, use the charge value corresponding to them and then proceed with the development of them for $x > 0$ according to the nonlinear equations, until $n_e = n$ which is the boundary of the dust slab. The point where $u = 0$ corresponds to the middle of the dust slab. Fig. 19 presents the results of such a numerical calculation. It is easy to see that the dust density has sharp edges (for density zero the derivative is not zero) and shows a qualitative difference with such structures in the absence of dust as solitons.

Another important approximation is the use of the condition of quasi-neutrality inside the structure. Let us give the numerical results for a quasi-neutral 1D

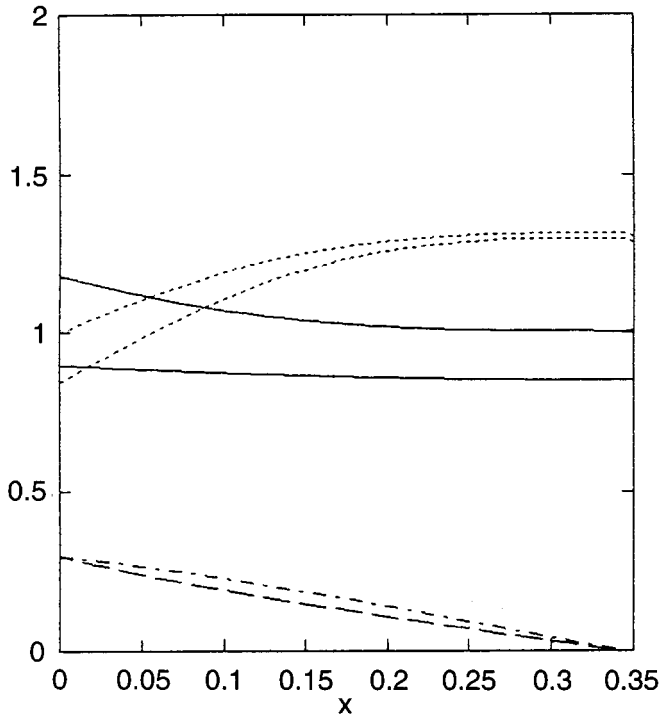

Fig. 19. Cold dust 1D structure obtained as a result of the numerical solution of nonlinear equations for the case of negligible attraction between the dust particles, taking into account the deviations of charge neutrality. The figure shows half the structure, and the flux of the plasma is from the left. The upper solid line represents the decrease of dust charge and the lower solid line the decrease of electron density with an increase of distance toward the centre of the structure. The upper dotted line represents the correspondending increase of the parameter P. The lower dotted line represents the increase of dust density, the dash–dot line the decrease of the ion flux Pu and the dashed line shows the decrease of the drift ion velocity.

dust slab. First of all we give the results for numerical computation neglecting the attractive forces. The quasi-neutrality condition $n = P + n_e$ can be used to describe the nonlinear system by n_e, P and u. Since for each step it is necessary to calculate the dust charge one can simplify the problem by introducing a new equation for dz/dx by differentiating the charging equation starting with the initial value of z by solving the charging equation for z corresponding to the initial values of the plasma parameters. In this way we can obtain the curve for z by solving the system of four ordinary nonlinear differential equations:

$$\frac{1}{n_e}\frac{dn_e}{x} = -(n_e + P)uzI\,, \tag{84}$$

$$\frac{dP}{dx} = u(n_e + P)\left[\left(1 + \frac{1}{\tau}\right)n_e zI + 2P\left(1 + \frac{\tau}{z}\right)\right]\,, \tag{85}$$

$$\frac{du}{dx} = -P\left(1+\frac{\tau}{z}\right), \tag{86}$$

$$\frac{dz}{dx} = -u\frac{\tau+z}{1+\tau+z}\left[\left(1+\frac{1}{\tau}\right)n_e I + \left(zI + \tfrac{4}{3}\left(2+\frac{\tau}{z}\right)\right)\right], \tag{87}$$

$$I = \tfrac{1}{3}\left[\ln\frac{1}{a\sqrt{4\pi(\tau+z)}} + \frac{2\tau}{z}\left(1+\frac{\tau}{z}\right)\right]. \tag{88}$$

After integration of this system of equations we obtain the curve for z and we can independently calculate from the data obtained the value of z for each point. A comparison between them is one possible check of the calculations. The results for numerical integration of this system of equations is presented in Figs 20–22 where this checking is also shown. The figures show the presence of ion enhancement and electron depletion in the dust structure, and the decrease of the dust charge and the drift velocity. Note that in the centre of the slab the drift velocity is zero since the slab is irradiated by plasma fluxes from both sides—on the half of the slab shown the flux velocity is from the right and on the other half it is from the left.

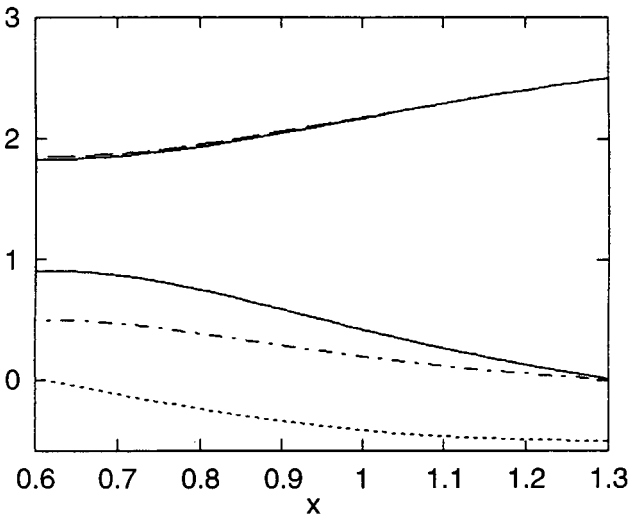

Fig. 20. Result of numerical computation of the 1D dust structure when the dust attraction is not taken into account. The local quasi-neutrality condition is used. The dashed line and the upper solid line represent the decrease of z calculated by the two methods described in the text. The coincidence is very good. The lower solid line shows the increase of P in the dust structure. The dotted line represents the decrease of drift velocity u and the dash–dot line shows the increase of dust density n_d, for $a = 0{\cdot}1$ and $\tau = 1$.

We then consider the case of a quasi-neutral slab with τ of the order of 1, but assuming the quasi-neutrality conditions to be valid and also taking into account the dust attractive forces. Then $P = n - n_e$ and the system of nonlinear equations describing the dust slab will have the form:

$$\frac{1}{n_e}\frac{\partial n_e}{\partial x} + zunI_m + \eta_a \tau z g = 0\,, \tag{89}$$

$$\frac{\partial nu}{\partial x} = -nI_c(n - n_e)\,, \tag{90}$$

$$\frac{1}{n_e}\frac{\partial n_e}{\partial x} + \frac{\tau}{n}\frac{\partial n}{\partial x} = -uI_m z(n - n_e)\,, \tag{91}$$

$$\frac{\partial g}{\partial x} = \frac{(n - n_e)}{z}\,, \tag{92}$$

$$\frac{\partial z}{\partial x} = \frac{z + \tau}{z + \tau + 1}\left(\frac{1}{n_e}\frac{\partial n_e}{\partial x} - \frac{1}{n}\frac{\partial n}{\partial x}\right)\,. \tag{93}$$

In these equations I_m and I_c are full expressions as functions of τ and z determined by the expressions for the collision frequencies given above:

$$I_m = \frac{2}{3\sqrt{\pi}}\left(\ln\left(\frac{1}{a}\right) + 2\frac{\tau}{z} + 4\frac{\tau^2}{z^2}\right)\,, \tag{94}$$

$$I_c(x) = \frac{1}{2\sqrt{\pi}}\left(1 + \frac{\tau}{z}\right)\,. \tag{95}$$

Since we need to fix the initial value for g it is necessary to explain its meaning. We start with some value of g (see equation 92) and then find the end and middle point of the cloud. According to the equations the negative value of the integral of the derivative of g with respect to x from the middle point of the cloud to the edge of the cloud gives the number of dust particles per unit surface of the slab. This integral is determined by the initial value of the parameter g at $x = 0$, which can then be directly related to the number of dust particles per unit area which is a free parameter of the cloud. But the value of the ion drag velocity with which we started has a certain value on the edge of the cloud which corresponds to the value of the flux needed to support the self-organised dust structure. The solution of these equations obtained by the procedure described is shown in Fig. 23.

Let us then give the results of numerical calculation of the charged dust slab. It is easy to see that by substitution of the value on $(1/n_e)\partial n_e/\partial x$ in the left-hand side of the Poisson equation we find an algebraic equation for P, which means that there will be no regions of sharp gradient of the order of the Debye length in the structure of the dust cloud. In the case where the ratio of dust size to the Debye length is small, the corrections to the quasi-neutrality will be small. Nevertheless, it is of interest to find the deviation from quasi-neutrality and the average charge of the slab per unit area.

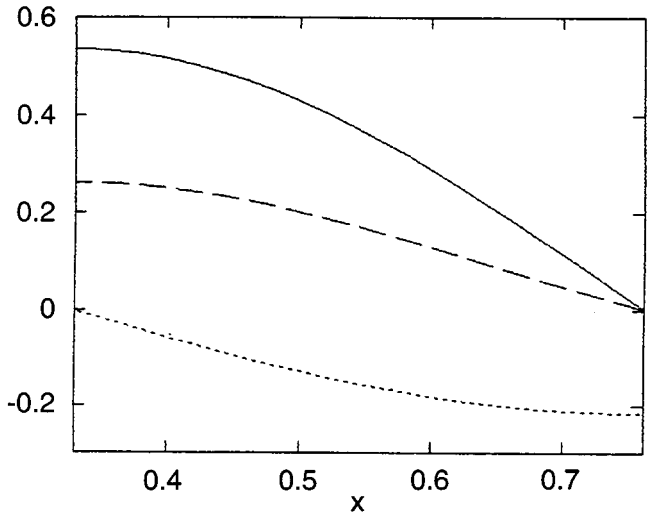

Fig. 21. Same as Fig. 20, but for $a = 0{\cdot}001$ and $\tau = 1$. The solid line gives the dependence on P in the structure, the dotted line gives u (the drift velocity is directed from the right), and the dashed line gives n_d.

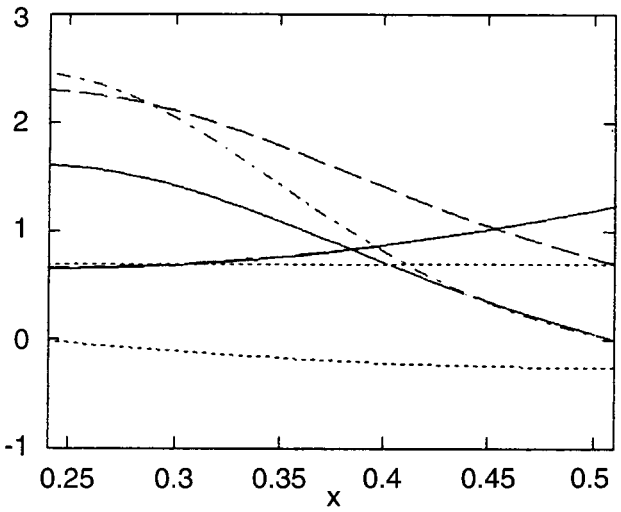

Fig. 22. Same as Fig. 20, but for $a = 0{\cdot}1$ and $\tau = 0{\cdot}01$. The upper left solid line shows the increase of P, the lower left line shows the decrease of z, the upper dotted line shows the decrease of n_e, the dashed line shows the increase of n_i, and the dash–dot line shows the increase of n_d.

The parameter P can then be expressed through the two functions

$$\eta_a' = -\frac{\partial \eta_a z}{\partial z}\,,$$

$$I_m' = -\frac{\partial I_m z}{\partial z}\,, \tag{96}$$

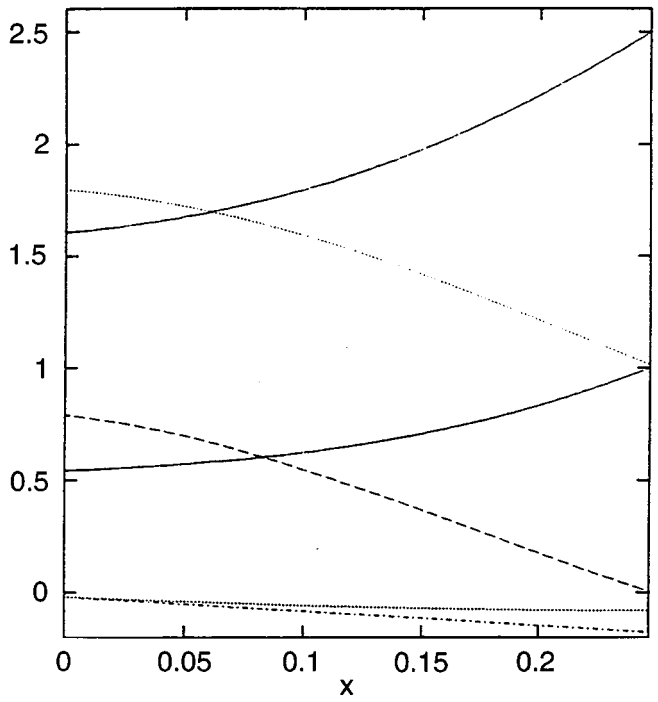

Fig. 23. Results of numerical solution of the system of nonlinear equations taking into account the dust attraction $\tau = 1, u = 0{\cdot}02, a = 0{\cdot}01$. The upper solid line shows the decrease of the dust charges z in the structure, the lower solid line shows n_e, the upper dotted line shows n_i, the dashed line shows n_d, the lower dotted line shows g and the dash–dot line shows the results for drift velocity u.

and then

$$P = \frac{n - n_e - \dfrac{a^2}{\tau} L_1(n, u, z, g)}{1 - \dfrac{a^2}{\tau} L_2(n, u, z, g)}\,, \tag{97}$$

where

$$L_1(n, u, z, g) = (nuI'_m + \tau g \eta'_a) \frac{z + \tau}{\tau(1 + \tau + z)} (nuzI_m + \tau \eta_a z g)\,, \tag{98}$$

$$L_2(n, u, z, g) = I_m I_c n z + \eta_a \tau + (nuI'_m + \tau g \eta'_a) z u I_m \frac{\tau + z}{\tau(1 + \tau + z)}\,. \tag{99}$$

The solution of the nonlinear equation in which the charge neutrality condition is violated is shown in Fig. 24.

The general feature of all structures is an enhancement of ion density in the centre of the cloud. This effect is due to accumulation of the ion flux. The excess of pressure due to this accumulation can balance the attraction forces.

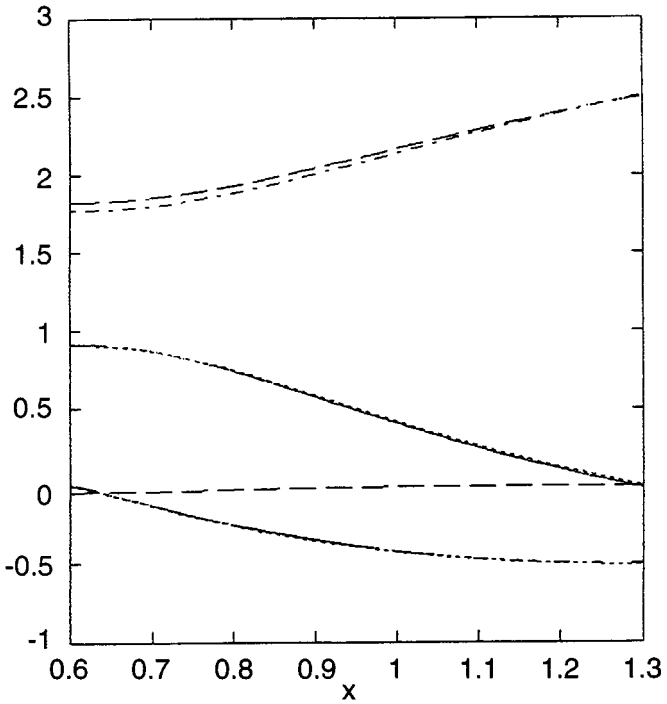

Fig. 24. Results of numerical calculation of the 1D dust structure taking into account the deviations of charge neutrality for $a = 0 \cdot 1$ and $\tau = 1$. The lower solid and dotted lines are the results for the drift velocity u with the assumption of charge neutrality and without the assumption of charge quasi-neutrality respectively. The upper solid and dotted lines are the same for P; the upper dashed and dash–dot lines are the same for z; and the lower dashed line is for the charge density $\rho = n - n_e - P$. The close overlap of curves for u and P shows that even for $a = 0 \cdot 1$ the quasi-neutrality assumption gives a good approximation to the more exact solutions. By taking into account the deviations from quasi-neutrality it is possible to see that the dust is somewhat less charged and that the whole dust structure is charged negatively (the curve for ρ in the centre of the structure).

Partially they are balanced due also to electrostatic repulsion of dust particles from the electric field, the appearance of which is due to thermal inhomogeneities in the electron distribution. Obviously the balance takes into account all the forces considered. The main point is that this balance indeed can be established throughout the dust cloud. This balance does not need an external gravity field or ion pressure—the only effects considered previously for example in the theory of the planetary ring thickness. Applications of the given results to the problems of planetary rings is only one among other possible applications.

9. Spherical Self-organised Structures

The spherical self-organised structures are of special interest, since in astrophysical applications there are some new objects different from stars, planets and comets, where the enhancement of plasma and dust density in them can provide a contribution to the so-called missing matter in the universe. On the other hand, another aim is to give an explanation of such phenomenon as ball lightning by developing the theory of self-organised spherical dust plasma structures.

Usually stars and planets are formed by gravitational forces. Dust in plasma has another type of attractive force which can provide dark spherical structure with the accumulation of both plasma and dust and they can in principle serve as an accumulation of mass in astrophysical conditions. The problem is what could be the enhanced density or total mass of the structure, what could be the total charge of the structure and for what sizes does the self-gravity start to play a substantial role. Stars can be formed in the case where self-gravity dominates. Thus this problem is related also to the problem of star formation. In stars, when already formed, nuclear reactions play an important role; the structures where the gravity is important but nuclear reactions have not started to be important are called protostars. It was once thought that in astrophysics the only stationary spherical structures are stars, and that the protostars are nonstationary self-contacting spherical dust structures where the radiation already plays an important role. At the present time we know much more about the dust properties and can formulate the problem of protostars more precisely. The main point is nevertheless that there can exist stationary self-organised dust structures which will not turn to stars. It is not possible to call these protostars. They are new objects which may not give light emission—and thus can be dark matter. These objects had not been considered previously and the problem is to estimate how much mass can be concentrated in such an object, what is their lifetime and evolution, what kind of measurements can be proposed to detect them, and finally what can be the physics of these objects. In other words, the question is then what kind of properties one can expect for the simplest structures of this kind and how to detect such structures.

Another application is the possible understanding of such phenomenon as ball-lightning, or at least to provide some new possibilities for such an understanding (see ref. [A]; Smirnov 1993). We give more details in Part IV below.

In the spherical case the flux of plasma exists towards the centre of the structure, is dissipated there and the ion density should be enhanced inside the structure. In a sense the spherical structures are also one-dimensional but all variables depend only on the radius from the centre of the structure. Let us write the nonlinear equations for spherical self-organised structures.

For spherical structures the modification of the equations are simple: the continuity equation for ions will be changed to have an additional term in the RHS:

$$\frac{\partial nu}{\partial r} = -\frac{2nu}{r} - PuI_c\,. \tag{100}$$

The Poisson equation describing the deviation from quasi-neutrality will change the relation between P and other variables having have an additional term I_r in the numerator:

$$I_r = -\frac{a^2}{\tau r} I_m z n u\,. \tag{101}$$

The equation for the attractive force will have also an additional term in the RHS:

$$\frac{\partial g}{\partial r} = -\frac{2g}{r} - \frac{P}{z}\,. \tag{102}$$

The total number of dust particles in the spherical structure will be determined by the value of g on its surface:

$$N_d \propto \int_0^R 4\pi r^2 \frac{P}{z} dr = -R^2 g(R)\,, \tag{103}$$

where R is the radius of the self-organised spherical structure.

The self-gravity can be considered and then gR represents the total attractive force on the surface of the spherical structure (total gravity). To include self-gravity we add to the attraction coefficient η_a the gravity coefficient η_G:

$$\eta_t = \eta_a + \eta_G\,, \tag{104}$$

where using the dimensionless coordinates introduced above we have from the Newtonian gravitational law, taking into account only the dust gravity (although it is easy to also include the gravity of plasma accumulated in the spherical structure), the following formula

$$\eta_G = G m_d^2 \frac{e^2 d_i^2}{a^4 T_e^2}\,. \tag{105}$$

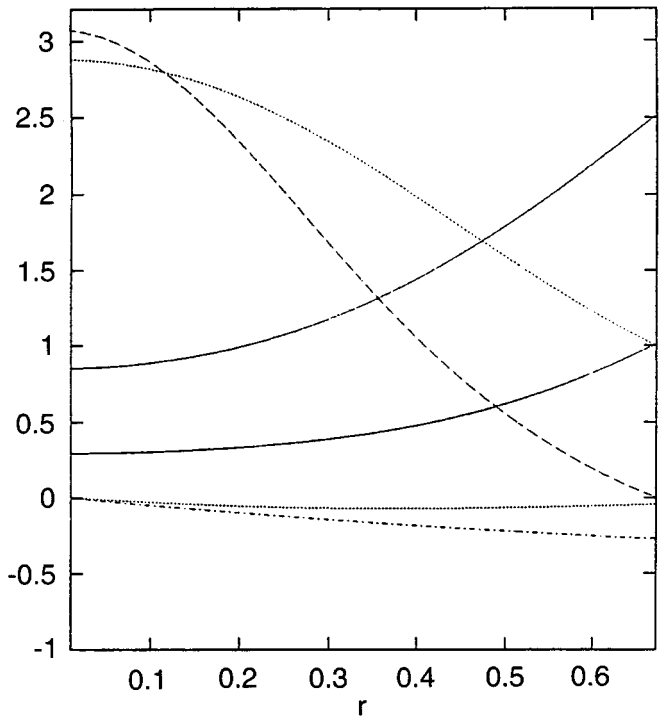

Fig. 25. Results of numerical solutions for the 3D spherical structure. The upper solid line shows the decrease of the charge z in the structure, the lower solid line shows the decrease of electron density n_e, the upper dotted line shows the increase of ion density n, the dashed line shows the increase of dust density n_d, the dash–dot line shows the decrease of the drift velocity u and the lower dotted line shows the change in the attractive potential g.

Since the mass of the dust particles $m_d \propto a^3$ the effect of self-gravity increases with dust sizes as a^2. For micron size dust particles and electron temperature of the order of eV, self-gravity is small compared with the dust attraction by several orders of magnitude. But for the low temperatures found in dust molecular clouds self-gravity can be rather important.

Let us turn to some results for the numerical calculation of spherical self-organised structures using the equations given above.

Fig. 25 shows the results of numerical calculation where the concentration of ion density in the cloud in the spherical case is larger than in the planar case—the ion density increases about three times for the example considered. This effect of larger plasma accumulation in the dust cloud is expected. The parameter P in the dust cloud increases from a value of 0 at the edge of the cloud up to a value larger than 3. The total mass of the spherical cloud can be estimated as

$$M \approx \left(\frac{d_i^2}{a}\right)^3 \frac{e^2 m_d n_{0,i}}{aT_e} \propto \frac{T^2}{a} n_{0,i}^2 \,. \tag{106}$$

For dust molecular clouds with $n_{0,i} \approx 10^{-3}$, $T \approx 100$ K, $n_{0,i} \approx 10^{-3}$ cm^{-3} and $a_0 \approx 10^{-4}$ cm we get an estimate that $M \approx 10^{23}$ g, that the total charge of the structure is approximately 10^{29} electron charges, and that the energy which the particle is driven from the surface of the cloud to infinity will be of the order of 10–20 GeV, i.e. an average cosmic ray energy.

10. Some Problems of Dust Structures

The presentation of material in this part of the paper has been made in a form where the reader can start an investigation of the problems mentioned in the text. Nevertheless, the restrictions made in consideration can be avoided in future investigations.

- The attractive and repulsive forces were calculated in the absence of magnetic fields. Although the condition where these forces can be used in the presence of magnetic fields can be written in a simple manner it is desirable to have an investigation of these forces in magnetic fields, including strong magnetic fields where the dependence of these forces on the field strength becomes important.
- In Part I a general theoretical approach for dusty plasmas was proposed with a new kinetic equation for dust particles where the dust charge was a new independent variable. This approach leads to new collision integrals, including not only dust–dust collisions but also the dust–plasma charging particle collisions. The first one in a certain approximation leads to the attractive and repulsive forces already discussed above. This indeed was checked by calculations by the author. But the generalisation of these forces can additionally appear to the effects of collective screening of these interactions (similar to the Landau–Balescu integral). This screening will be different in the presence of attraction (similar to the gravitational effect where the screening is absent). Thus an investigation of collective effects in particle collisions in a dusty plasma is one of the important general problems of dusty plasma theory.

- In the investigation of dust–plasma structures and the dust attraction instability only the binary dust–dust interactions were taken into account. It is easy to estimate that the restriction for these conditions to be appropriate is that the size of the structure should be less than the plasma particle mean free path. For larger structures either the many particle dust collisions will be important or the binary collisions form a surface layer of the structure with a certain surface tension introduced by binary dust–dust interactions. An investigation of this alternative is important for problems such as the explanation of the phenomenon of ball-lightning as a dust–plasma structure proposed by the author (see references below).
- The problem of the presence of many dust–plasma structures has not yet started to be investigated, but it is obvious that it can give unexpected results especially concerning the interaction of the structures and the competition for the plasma fluxes as a kind of 'food' for them. Also, similar to the vortex 2D structures in hydrodynamics, the larger dust–plasma structures can merge ('eat') the smaller dust–plasma structures. For drift waves in the absence of dust this process was established both analytically and numerically, and in air motions vortex merging has been observed experimentally. The difference between these vortices and the dust–plasma structures is the presence in the latter case of plasma particle fluxes.
- The dust merging can explain the observed phenomenon of dust agglomeration in etching experiments. It can occur if the attractive forces are larger than the Coulomb repulsive forces at distances less than the Debye radius. The attraction caused by charged particle fluxes is too weak to be responsible for that, but the attraction caused by the flux of neutrals can be sufficient to produce the dust agglomeration. The attraction caused by neutrals appears in the case where the surface temperature of dust particles is less than the kinetic temperature of neutrals. Thus the dust radiative cooling can be of importance for this interaction. In many laboratory experiments the dust cooling is appreciable and it is certainly important in astrophysical conditions. The problem of dust–plasma structures is to take into account the dust cooling resulting in dust agglomeration. Some effects of this kind can appear in the shock waves discussed in the next part.

PART III. SHOCKS IN DUSTY PLASMAS

11. Introduction to Shock Physics in Dusty Plasmas

The problem of shocks is of major importance in dusty plasmas. As an example we mention that the accepted point of view is that stars are formed in the region where the initial density enhancement is due to the shock propagating mainly in the galaxy arms. In the case where this shock meets the molecular dust cloud it can create an initial stage for development of the gravitational collapse to form stars. Star formation is indeed observed on the edges of dust clouds. But from the point of view of our present understanding of the properties of dusty plasma, the initial density enhancement can occur in the dust component and should describe the formation of smaller structure due to non-gravitational dust attraction. Nevertheless, the problems of shock waves in dusty plasmas is one of

the important topics for future research in space and laboratory physics. The shocks can be considered as another type of self-organised structure in dusty plasmas.

The shocks can be described in a stationary medium by conservation laws for particle momentum and energy flux through the surface of the shock. In dusty plasmas the presence of such conservation laws is not trivial since the dusty plasma is an open system. Therefore we should formulate what kind of nonlinear waves propagating in the dusty plasma we call 'shock waves'. We call the wave a shock wave in the case where it is propagating so that at the front of the shock the dusty plasma is stationary and not moving. This means that there should be some external ionisation which keeps the density of plasma particles on a certain level while they recombine on dust particles. Such ionisation could be produced by electromagnetic radiation of the burning star, by cosmic rays or by other sources (electron beams etc.). We will not define the conditions behind the shock—they should appear as a result of the disturbances produced by the shock. But we expect that the same source of plasma with the same rate of electron/ion generation per second also exists behind the shock front. This of course is one of the possible types of nonlinear shock propagating in dusty plasmas. The dissipation needed for the shock to exist can be provided by plasma recombination on the grain and by dust charging. Only in the case where the dust density is small (precisely for $P \ll 1$) can one in the first approximation neglect the plasma particle recombination on the dust and take into account only the charging dissipation which, as was shown, does not depend on P. This kind of shock conserves in the first approximation the number of plasma particles and is similar to electrostatic shocks in the absence of dust—the dust charging process determines only the front structure of the shock. For all other cases the momentum, energy and number of plasma particles is not conserved on the shock front, but the sum of the dust and plasma particle momentum, mass density and energy density is conserved. The point is that the characteristic size of the shock front will be determined by the plasma particle recombination on dust and thus the conservation of the total values of mass density, momentum density and energy density cannot be used to find the structure of the shock—one needs to consider separately the equation for the plasma particles number, momentum and energy and the dust particle equation for mass density, momentum and energy density.

12. General Nonlinear Equations and Conservation of Total Mass, Total Momentum and Total Energy Densities

We use the continuity equation for ion mass density introducing the source of ions q_i and the direction of shock propagation which we denote as x. We have

$$\frac{\partial n_i m_i}{\partial t} + \frac{\partial n_i u_i m_i}{\partial x} = -\bar{\nu}_i n_i m_i + q_i \,. \tag{107}$$

Previously we wrote this equation for $q_i = 0$ dropping the factor m_i. In the shock waves, similar to the structures described above, the electrons behave adiabatically and the main consideration is related with ions. The recombination of ions on dust particles of course conserves the mass density since the ions being

attached to the dust particles will increase their mass. We neglected this effect in the previous consideration but to have a deeper analogy with usual shock waves in absence of dust we take into account this effect here. The continuity equation for dust particle mass density we write in the form corresponding to conservation of total dust and ion mass density in absence of source q_i:

$$\frac{\partial n_d m_d}{\partial t} + \frac{\partial n_d u_d m_d}{\partial x} = \bar{\nu}_i n_i m_i \,. \tag{108}$$

In the previous consideration $u_d = 0$, the second term on the LHS of (108) was zero and the time dependence introduced by the first term is very week due to the very small ratio m_i/m_d.

The source of plasma particles q_i in (107) can be expressed through the frequency of undisturbed plasma parameters in front of the wave since we assumed that it compensates the plasma losses due to recombination on dust particles. All values in the undisturbed dusty plasma we denote by the subscript 0 and behind the shock by the subscript 1:

$$q_i = \bar{\nu}_{i,0} n_{i,0} m_i \,. \tag{109}$$

The $\bar{\nu}_i$ as shown above is a function of n_d, n_i, Z_d and in the general case is a function of the relative ion to dust drift velocity. The latter in the region in front of the shock wave should be zero due to relaxation of it in ion–dust collisions. Thus the $\bar{\nu}_{i,0}$ depends on $n_{d,0}, n_{i,0}, z_0$ (the last is the dimensionless charge in the front of the wave).

Taking the sum of equations (107) and (108) and neglecting the electron mass density contribution due to their small masses, we find the total mass density conservation law

$$\frac{\partial (n_d m_d + n_i m_i)}{\partial t} + \frac{\partial (n_d m_d u_d + n_i u_i m_i)}{\partial x} = \bar{\nu}_{i,0} n_{i,0} m_i \,. \tag{110}$$

Again the change of dust mass is a very slow process and during wave propagation (during the particle passing the dust shock wave front structure in the frame of the shock) one can neglect the dust particle mass increase, i.e. neglect the term on the RHS of (110). Then we get the mass density conservation law relating the values in front of the shock and behind the shock in the conventional form

$$(n_{d,1} m_d + n_{i,1} m_i) u_1 = (n_{d,0} m_d + n_{i,0} m_i) u_0 \,. \tag{111}$$

The difference between a similar relation in conventional gas dynamics and the relation (111) is that the latter contains both the mass density of ions and the mass density of dust. We have taken into account in (111) that due to a relaxation process the dust and ions have the same velocities in the regions behind the shock and at the front of the shock. As an example in dust–molecular interstellar clouds, both mass density components (of dust and ions) are comparable.

Another relation between the parameters behind and in front of the shock can be found from the condition of stationarity of the plasma in the presence

of the source. Equation (107) is then satisfied due to the absence of the space dependence of the ion mass density both behind and in front of the shock:

$$\bar{\nu}_{i,1} n_{i,1} = \bar{\nu}_{i,0} n_{i,0} \,. \tag{112}$$

This equation is an addition to the conventional gas dynamical set of equations determining the parameters of the shocks.

We should say a few words concerning the continuity equations for electrons. The electrons and ions are created in pairs and thus the number of electrons created by the source should correspond to the number of ions created. Then, due to the quasi-neutrality condition behind and in front of the shock, the fulfillment of the stationary condition (109) for ions means automatically fulfillment for electrons. This can also be easily seen by comparison of $\bar{\nu}_e$ and $\bar{\nu}_i$ given in previous section. This means that the relation for electrons corresponding to (112) for ions is automatically fulfilled if (112) is valid.

In a description of the structure of the shock in dusty plasmas we should take into account that the dissipation due to the plasma particle–dust collisions described by $\bar{\nu}$ is the one of the most important dissipative processes. To describe the structure in the front of the shock wave we can, in a manner similar to the general conservation law, neglect in the dust mass equation the growth of the dust masses during the passing of the wave, i.e. neglect the term on the RHS of (108) and write in the frame of the shock

$$\frac{\partial n_d u_d}{\partial x} = 0 \,. \tag{113}$$

For the ion mass density the presence of the recombination is important for the description of the shock wave structure. From (107) we find in the frame of the shock that

$$\frac{\partial n_i u_i}{\partial x} = -(\bar{\nu}_i n_i - \bar{\nu}_{i,0} n_{i,0}) \,. \tag{114}$$

A similar equation is valid for electrons after substitution in (114) of the subscript e for subscript i. Although for the equilibrium charge $\bar{\nu}_{i,0} n_{i,0} = \bar{\nu}_{e,0} n_{e,0}$ (and this relation was used to ensure that the fulfillment of the condition for stationary ions means also the fulfillment of the stationary condition for electrons) in the general case for the non-equilibrium values of dust charges the corresponding values are not equal to each other. Thus, subtracting from equation (114) the corresponding equation for electrons, we will not get zero on the RHS. This can be seen also from the general charging equation, which is simply the condition of charge conservation in the system:

$$\frac{\partial Z_d}{\partial t} + \frac{\partial Z_d u_d}{\partial x} = \bar{\nu}_e n_e - \bar{\nu}_i n_i \,, \tag{115}$$

which in the frame of the shock has the first term on the LHS absent. For neutrals we have

$$\frac{\partial u_n n_n}{\partial x} = 0 \,, \tag{116}$$

and usually the neutrals are attached to the dust particles for a finite residence time and then leave. Thus their number is conserved but their temperature becomes close to the dust temperature since they leave the dust particles with the dust surface temperature. The latter will be used in the equations for momenta conservation.

Let us turn then to the conservation of dust and ion momenta at the front of the shock. We want to write this equation also for the total change of momenta density to have a relation similar to that of conventional gas dynamics.

These equations are the same as those already used to describe the dust structures, but additionally we take into account the possible changes in plasma and dust pressure in front of the shock, i.e. we take into account the spatial dependence of the ion and dust temperatures. We rewrite the equations in the form, denoting the ion temperature by T_i,

$$n_i \frac{\partial m_i u_i}{\partial x} + n_i m_i u_i \frac{\partial u_i}{\partial x} + \frac{\partial n_i T_i}{\partial x} = -e n_i \frac{\partial \phi}{\partial x} - \tilde{\nu} m_i (u_i - u_d) n_i \,, \qquad (117)$$

where we have not taken into account the momentum exchange between the ions and neutrals, but which can easily be done. A similar equation can be written for electrons (the electron temperature is denoted by T_e).

Let us write a similar equation for neutrals. We suppose, as already mentioned, that the neutral temperature is equal to the dust temperature T_d:

$$n_n \frac{\partial m_n n_n}{\partial x} + n_n m_n u_n \frac{\partial u_n}{\partial x} + \frac{\partial n_n T_d}{\partial x} = -\nu_n m_n (u_n - u_d) \,. \qquad (118)$$

For dust particles we should take into account the momenta transferred in collisions with ions, neutrals and electrons (the momenta transferred in collisions with electrons is negligibly small):

$$n_d \frac{\partial m_d u_d}{\partial x} + n_d m_d u_d \frac{\partial u_d}{\partial x} + \frac{\partial n_d T_d}{\partial x}$$

$$= Z_d n_d \frac{\partial \phi}{\partial x} + \tilde{\nu}_i m_i (u_d - u_i) n_i + \tilde{\nu}_e m_e (u_d - u_e) n_i + \nu_n (u_d - u_n) m_n n_n \,, \qquad (119)$$

where ϕ is the electrostatic potential. These equations are satisfied in the regions in front and behind the front of the shock due to the absence of the space variations and stationary conditions of all parameters and due to the equality of the dust and ion velocities. By adding equations (117) and (118) and the corresponding equation for electrons we can, using the condition of quasi-neutrality, cancel all the electric force terms and use also equation (110). Then we find that

$$\frac{\partial (n_i m_i u_i + n_d m_d u_d + n_n m_n u_n)}{\partial t}$$

$$+ \frac{\partial}{\partial x} \left(m_i n_i \frac{u_i^2}{2} + m_d n_d \frac{u_d^2}{2} + m_n n_n \frac{u_n^2}{2} + n_i T_i + n_e T_e + (n_d + n_i) T_d \right) = 0 \,, \qquad (120)$$

which gives a relation in front of the shock similar to that of conventional gas dynamics:

$$(m_i n_{i,0} + m_d n_{d,0} + m_n n_{n,0})\frac{u_0^2}{2} + n_{i,0}T_i + n_{e,0}T_{e,0} + (n_{d,0} + n_{n,0})T_{d,0}$$

$$= (m_i n_{i,1} + m_d n_{d,1} + n_n m_n)\frac{u_1^2}{2} + n_{i,1}T_{i,1} + n_{e,1}T_{e,1} + (n_{d,1} + n_{n,0})T_{d,1}\,. \quad (121)$$

The electrons contribute only in the pressure term. These equations differ from those of conventional gas dynamics by adding the momentum density of the dust component. For a description of the structure of the shock in dusty plasmas we can use in the front of the shock the stationary limit of equations (119) and (120). Contrary to the considerations made before for dust structures, in the present problem of the structure for shock waves we cannot neglect the terms with ram pressure for ions and dust and cannot assume that the velocity of ions is much larger than the dust velocity, since far behind or far ahead of the shock front they are equal in the frame of the shock.

Equation (121) is for the balance of the energy density which we do not use at all in describing the dust structures, assuming the temperature to be constant. For the shocks such an assumption cannot be made since the change of the temperature and entropy in the shock is one of their important features.

With the energy balance in an open system such as for dusty plasmas the consideration should be similar to the particle density balance—there exists the energy absorbed by dust particles which together with the heating and the cooling of the plasma should be in balance to have stationary pre-shock conditions. Such a balance is indeed established since the ionisation source usually also heats the plasma and there exists a cooling due to thermal radiation of dust particles. We denote these external heating sources by q_i^E, q_e^E and q_d^E. The energy transferred to electrons and ions by external sources in stationary conditions is equal to the energy transferred to the dust particles. The Coulomb collisions of electrons and ions with dust particles are elastic and thus do not contribute to the energy transfer to dust particles. Only the charging collisions contribute to this energy transfer. We denote the rate of this energy transfer as $\hat{\nu}_e n_e T_e$ and $\hat{\nu}_i n_i T_i$ introducing the frequencies of this transfer $\hat{\nu}_e$ and $\hat{\nu}_i$. The calculation with charging cross sections given above leads to the following expressions for them:

$$\hat{\nu}_i = \frac{2\sqrt{2}}{\pi} n_d v_{Ti} \pi a^2 \left(2 + \frac{z}{\tau}\right)\,, \quad (122)$$

$$\hat{\nu}_e = \frac{2\sqrt{2}}{\pi} n_d v_{Te} \pi a^2 (2 + z)\exp(-z). \quad (123)$$

In the regions behind and at the front of the shock due to the stationary conditions the rate of energy dissipation should be equal to the energy of heating:

$$\hat{\nu}_{e,0} n_{e,0} T_{e,0} = q_e^E; \qquad \hat{\nu}_{i,0} n_{i,0} T_{i,0} = q_i^E\,, \quad (124)$$

and there is a similar relation behind the shock. As a consequence of these relations the sources of energy heating can be expressed through the parameters of dusty plasma in the front of the shock. The ratio of ion to electron heating determines therefore the electron to ion temperature ratio. Using the condition of equilibrium dust charge we find that

$$\frac{q_e^E}{q_i^E} = \frac{(2+z_0)(\tau_0+z_0)}{\tau_0(2\tau_0+z_0)}\,. \tag{125}$$

The condition that the stationary take place both behind and at the front of the shock wave gives another equation in addition to the conventional gas dynamics relations for the shock waves:

$$\hat{\nu}_{i,0} n_{i,0} T_{i,0} = \hat{\nu}_{i,1} n_{i,1} T_{i,1}\,, \tag{126}$$

which in fact determines the change of the ion temperature in the shock. A similar equation can be written for electrons or one can see the equality of the expression (125) with the corresponding expression behind the shock. Thus, contrary to the stationary condition appearing from the continuity equation for electrons and ions where the relation for ions automatically satisfies the relation for electrons, there will be two relations of heat transfer and consequently of the energy transfer equations. Thus the total number of additional equations is now three. But there will be a fourth equation.

Before going further we need to make some important statements. First of all *the relation (126) is valid only in the case where the heating rate behind the shock is the same as the heating rate in front of the shock*. This means that the sources of energy q_e^E and q_i^E do not change with a change of the dusty plasma parameters such as plasma densities and temperatures. The latter is not always the case. Above, for the source of the particles the assumption that the ionisation source is independent of the plasma particle parameters is more natural since, for example, the ionisation rate can depend on the density of neutrals which in the case of a low degree of ionisation is practically not changed by changing the plasma parameters. For the heating process the rate of heating more probably depends on the plasma parameters. In the case such a dependence is important, the relation (126) will be different since the plasma parameters behind the shock will be different from the plasma parameters in front as the shock and one cannot use the condition that the heating rate is not changed from the region in front of the shock compared with the region behind the shock. *But this does not mean that there will be not be another relation instead of (126).* The only result will be that the relation (126) will be modified. The relations (125) and (126) will be valid in any case, but in equalising (125) with the corresponding relation behind the shock we should take into account the changes in plasma conditions. All these relations can be easily written if one specifies the process of heating, but these relations will be different for different sources of heating. Having in mind the illustration of general principles of shock waves in dusty plasmas as an open system, we continue with the assumption that the heating sources are independent of the plasma parameters to show that all equations which one writes will be sufficient to relate the parameters behind the shock with the parameters in the region in front of the shock.

By adding all equations for energy transfer for electrons, ions and dust we will be left with the global stationary condition at the front and behind the shock which describes the equality of the heating processes to the cooling processes. In space plasmas, usually the cooling is due to dust reradiating the heat transferred to it and the rate of radiation lost in optically thin plasma due to the Stefan law is be proportional to $T_d^4 n_d$, and thus for this case we have

$$n_{d,0} T_{d,0}^4 = n_{d,1} T_{d,1}^4 \,. \tag{127}$$

Again we should mention that this relation is valid in the case we assumed that the sources of heating are independent of plasma parameters including the dust density and dust temperature. In the case where the dust is heated by the radiation this will be not true since the source of dust heating will be proportional to the dust density. But there will also be the other heating mechanisms which will be independent of dust density and equation (121) should contain the heating appearing to the energy transferred to the dust particles by dust plasma particle collisions. This can modify the relation (127). We need nevertheless to emphasise that the relation (127) or its modification always should be taken into account. We continue with the analysis of the case of external independence of plasma parameter sources of heating.

In the equation for the total energy balance in the frame of the shock we can express the external heating sources through the cooling in stationary conditions and omit the time-dependent terms:

$$\frac{\partial}{\partial x}\left[n_i T_i u_i \left(\tfrac{5}{2} + \frac{u_i^2}{2v_{Ti}^2}\right) n_n T_n u_n \left(\tfrac{5}{2} + \frac{u_n^2}{2v_{Td}^2}\right) \right.$$

$$\left. + n_d T_d u_d \left(\tfrac{5}{2} + \frac{u_d^2}{2v_{Td}^2}\right) + \tfrac{5}{2} n_e T_e u_e \right] = -\pi a^2 \sigma (n_d T_d^4 - n_{d,0} T_{d,0}^4) \,, \tag{128}$$

where we assumed that $T_n = T_d$. We neglect the terms proportional to the square of the ratio of electron drift to the electron thermal velocity. The factors $\frac{5}{2}$ appear as a sum of the pressure term nT and the internal energy term $\frac{3}{2}nT$.

In the case where we neglect the radiation losses we return to the standard gas dynamics relations:

$$n_{i,0} T_{i,0} u_0 \left(\tfrac{5}{2} + \frac{u_0^2}{2v_{Ti,0}^2}\right) + n_{n,0} T_{d,0} u_0 \left(\tfrac{5}{2} + \frac{u_0^2}{2v_{Td,0}^2}\right)$$

$$+ n_{d,0} T_{d,0} u_0 \left(\tfrac{5}{2} + \frac{u_0^2}{2v_{Td,0}^2}\right) + \tfrac{5}{2} n_{e,0} T_{e,0} u_0 =$$

$$n_{i,1} T_{i,1} u_1 \left(\tfrac{5}{2} + \frac{u_1^2}{2v_{Ti,1}^2}\right) + n_{n,1} T_{d,1} u_0 \left(\tfrac{5}{2} + \frac{u_1^2}{2v_{Td,1}^2}\right)$$

$$+ n_{d,1} T_{d,1} u_1 \left(\tfrac{5}{2} + \frac{u_1^2}{2v_{Td,1}^2}\right) + \tfrac{5}{2} n_{e,1} T_{e,1} u_1 \,. \tag{129}$$

The difference between this equation and the conventional gas dynamics relation is that all components of the dusty plasma—dust, electrons and ions—are taken into account.

In the case where the radiation processes dominate we should leave in (129) only the RHS terms

$$n_d T_d^4 = n_{d,0} T_{d,0}^4 \,. \tag{130}$$

We emphasise once more that in the case where the heating mechanisms depend on plasma parameters this equation will be modified, but it is certain that in the case where the radiation losses dominate an equation substituting the conventional equation of gas dynamics is always possible to formulate.

We then illustrate that the number of equations relating the parameters behind the shock with parameters in the region at front of the shock is sufficient to determine the parameters behind the shock. In many applications we can neglect the dust pressure terms compared with the ion or electron pressure terms, but cannot neglect the electron pressure compared with ion pressure. The RHS of (128) is zero on both sides of the shock wave. It is also approximately zero on the whole front structure if the additional energy emitted during the passing of the shock is small compared with the energy of the shock.

13. Examples of the Solution of Generalised Gugoniot Relations

The relations we find are much richer than the Gugoniot relations in conventional gas dynamics. We can consider some examples. Let us assume that:

1. the radiation is dominant and one uses the equation of type (130) instead of the energy flux conservation equation;
2. the dust pressure is small compared with the neutral, electron and ion pressures;
3. the dust and neutrals are dominant in the mass density;
4. the temperatures of neutrals and dust are equal.

At the front of the shock we fix the parameters $T_{i,0}, T_{d,0} = T_{n,0} \equiv T_0$ and τ_0, the $n_{i,0}, n_{n,0} \equiv n_0$ and the parameter $P_0 = Z_{d,0} n_{d,0}/n_{e,0}$. This determines both the electron density $n_{e,0} = n_{i,0}/1 + P_0$ and the dust charge $z_0 = Z_{d,0} e^2 \tau_0 / a T_{i,0}$. We fix also the velocity of the shock wave u_0, which is the velocity of the wave propagation in the undisturbed plasma in the front of the wave. In the frame of the shock this velocity corresponds to the velocity of the plasma flux colliding with the shock. The thermal energy of the plasma in front of the shock is then

$$p_0 \equiv n_0 T_0 + n_{i,0} T_{i,0} \left(1 + \frac{1}{\tau_0 (1 + P_0)} \right) , \tag{131}$$

while the thermal energy behind the shock front is

$$p_0 \equiv n_1 T_1 + n_{i,1} T_{i,1} \left(1 + \frac{1}{\tau_1 (1 + P_1)} \right) . \tag{132}$$

We can then write the equations which determine the parameters behind the shock

1. Conservation of momenta:

$$M^2 = (n_{d,0}m_d + n_0 m_n)\frac{u_0^2}{p_0} = (n_{d,1}m_d + n_1 m_n)\frac{u_1^2}{p_0} + 2\frac{p_1}{p_0}\,. \tag{133}$$

In this equation it is easy to find when the ratio of the shock to dust sound velocity appears and when the ratio of the shock to sound velocity in the neutral gas appears. This ratio in both cases is called the *Mach number of the shock wave*. In the case where the dust dominates in mass density, but the thermal energy of the charged particles dominates the energy of the neutral particles, the LHS of (133) is equal to the square of the ratio of the shock to dust sound velocity, but in the case where the neutrals dominate both in mass density and in thermal energy, the LHS of (133) contains the square of the ratio of shock to sound velocity in the neutral gas. Equation (133) was written in this general form to illustrate the relation between the different sound velocities, but in the general case the LHS of (133) contains the square of the Mach number M defined as the ratio of the shock velocity to the general expression for the sound velocity. The radiative shocks, contrary to the usual shocks, can have a large increase in density behind the shock. This fact is widely used in astrophysical applications where usually the radiative cooling at the front of the shock is related to the atomic processes. In the presence of dust the radiative cooling can be effectively produced by thermal emission of dust particles. It is therefore worth while to write the expression (133) for the case $M \gg 1$. The velocity behind the shock should then be small and in the RHS of (133) we can leave only the term with the ratio of thermal energies which physically means that in the frame of the shock front the ram pressure of the in-flowing plasma is compensated behind the shock front by the thermal pressure, which thus should increase substantially behind the shock. To simplify this example we can consider the case where the neutral component dominates in pressure. Then we find that

$$M^2 = 2\frac{n_1}{n_0}\frac{T_1}{T_0}\,. \tag{134}$$

2. Continuity equations for neutrals and dust:

$$\frac{u_1}{u_0} = \frac{n_0}{n_1} = \frac{n_{d,0}}{n_{d,1}}\,. \tag{135}$$

3. Dust cooling condition:

Using (135) we can write the condition (127) in the form

$$\frac{T_1}{T_0} = \left(\frac{n_0}{n_1}\right)^{\frac{1}{4}}\,, \tag{136}$$

which gives

$$\frac{n_1}{n_0} = \frac{n_{d,1}}{n_{d,0}} = \frac{M^{\frac{8}{3}}}{2^{\frac{4}{3}}}\,, \tag{137}$$

$$\frac{T_1}{T_0} = \frac{2^{\frac{1}{3}}}{M^{\frac{2}{3}}}\,, \tag{138}$$

$$\frac{u_1}{u_0} = \frac{2^{\frac{4}{3}}}{M^{\frac{8}{3}}}\,. \tag{139}$$

4. Electron and ion temperature balance:

We use the relation (125) and the equality of sources behind the shock and in front of the shock to find:

$$\frac{(2+z_1)(\tau_1+z_1)}{\tau_1(\tau_1+z_1/2)} = \frac{(2+z_0)(\tau_0+z_0)}{\tau_0(\tau_0+z_0/2)}\,. \tag{140}$$

This equation allows one to find τ_1 as a function of z_1, τ_0 and z_0. The last two quantities are given and thus we have an analytical expression for τ_1 as a function of z_1.

5. Balance of ion and electron fluxes:

Equation (112) gives

$$\sqrt{T_{i,0}}n_{d,0}n_{i,0}\left(1+\frac{z_0}{\tau_0}\right) = \sqrt{T_{i,1}}n_{d,1}n_{i,1}\left(1+\frac{z_1}{\tau_1}\right)\,. \tag{141}$$

6. Balance in ion heating:

Equation (126) gives

$$T_{i,0}^{\frac{3}{2}}n_{d,0}n_{i,0}\left(2+\frac{z_0}{\tau_0}\right) = T_{i,1}^{\frac{3}{2}}n_{d,1}n_{i,1}\left(2+\frac{z_1}{\tau_1}\right)\,. \tag{142}$$

By dividing the last two equations we find

$$\frac{T_{i,1}}{T_{i,0}} = \frac{\tau_1(2+z_0)}{\tau_0(2+z_1)}\,, \tag{143}$$

and by substituting (143) into (142) we get

$$\frac{n_{i,1}}{n_{i,0}} = \frac{2^{\frac{4}{3}}}{M^{\frac{8}{3}}}\frac{\tau_0+z_0}{\tau_1+z_1}\sqrt{\frac{(2+z_1)\tau_1}{(2+z_0)\tau_0}}\,. \tag{144}$$

7. Dust charges behind the shock:

From the relation given one can express the dimensionless dust density $P_1' = Z_{d,1} n_{d,1}/n_{i,1}$ through P_0' [recall that $P' = P/(1+P)$ where $P = n_d Z_d/n_e$ and thus $P' < 1$]:

$$\frac{P_1'}{P_0'} = \frac{M^{\frac{16}{3}}}{2^{\frac{8}{3}}} \frac{(2+z_0)^{\frac{3}{2}}(\tau_1+z_1)}{(2+z_1)^{\frac{3}{2}}(\tau_0+z_0)} \sqrt{\frac{\tau_0}{\tau_1}} \frac{z_1}{z_0} \,. \tag{145}$$

For $P_0 \ll 1$ there exists a broad range of Mach numbers values $M \gg 1$ where the P_1' is still not close to 1. Then the given relations are sufficient to find the charges of dust particles behind the shock by solving the equation

$$\exp(-z_1) = \sqrt{\frac{m_e}{m_i \tau_1}} \frac{\tau_1 + z_1}{1 + P_1} \,. \tag{146}$$

The z_1 is then of the order of 2 and behind the shock the charges of the dust particles correspond to the value of the floating potential of the plasma behind the shock.

8. Critical value of the Mach number:

The critical value of the Mach number corresponds to the condition that behind the shock P' comes close to its maximum possible value of 1 [recall that the definition of P here is $P = n_d Z_d/n_i = P_0/(1+P_0); P_0 = n_d Z_d/n_e$]. Then the charges on the dust particles behind the shock should decrease rapidly with the increase of the shock Mach number. From (145) we find an estimate of the critical Mach number:

$$M_{cr} \approx (1/P_0)^{\frac{3}{16}} \,. \tag{147}$$

In fact for $M \gg M_{cr}$ we have $z_1 \ll 1$ and (147) can be used to find the dependence of z_1 on the Mach number by putting $P_1' = 1$:

$$z_1 = z_0 \frac{1}{M^{\frac{16}{3}}} \frac{2^{\frac{11}{3}}(\tau_0+z_0)^{\frac{3}{2}}}{\tau_0(2+z_0)\sqrt{\tau_0+z_0/2}} . \tag{148}$$

9. Thickness of the shock:

The thickness of the shock wave δ is determined by the plasma particle mean free path in collisions with dust:

$$\delta \approx \frac{d^2}{aP} \,. \tag{149}$$

Since for $P_0 \ll 1$ and $M \gg 1$, P increases rapidly inside the shock front from P_0 to reach the value P_1, the front structure should sharpen in direction behind the shock front. The characteristic gradients increase in this direction. The scales close to the front are determined by $\delta_0 = d_0^2/aP_0$, while close to the region behind the shock the gradients increase approximately $M^{\frac{8}{3}}$ times. But even δ_0 is much

less than the particle mean free path for binary plasma particle collisions. In this sense these shocks are collision-less, but in fact the thickness of the shock is determine by the collisions of the plasma particles with dust particles, which for P of the order of 1 are Z_d times more frequent than the binary plasma particle collisions.

14. Some Problems of Shocks in Dusty Space Plasma

The example we gave serves only as an illustration of how the generalised conditions in the shock in dusty plasmas can be solved. In reality there could be many other important processes and a more detailed picture of the shock could be used. We list here the problems which could be of importance in applications:

- Shocks in diluted dust.
  In the case of diluted dust $P \ll 1$ one can find the conditions where the dust density is not changing substantially in the shock, but since the temperature and density are changing the charge of the dust particle is changing. The mean free path of charging does not depend on P and is approximately equal to d^2/a. Thus in the case where P is decreasing, the mean free path of plasma–dust collisions increases, but not the charging mean free path which is independent of P. Then the charging mean free path can determine the thickness of the shock front. Such shock waves are in some sense also collision-less, but in fact the collisions of plasma particles with dust particles which are responsible for dust charging are important. These shocks in diluted dust can have a thickness much less than the binary plasma particle collision mean free path.

- Role of dust attraction.
  The main conclusion for the example given above is that in dusty plasmas there can occur a dust accumulation behind the shock front. There seems to be no limit on the Mach number in the framework of the consideration given above, but indeed there exists such a limitation. One of the effects is the dust attraction which was discussed above. The rate of attraction depends on P and increases with an increase of P approximately as P^2. The difficulty in the solution of the corresponding Gugoniot equations is that by introducing the attractive force g one meets the necessity to fix its value (or more exactly g^2) in the region at the front of the shock and behind the shock. But the system is unstable in the presence of attraction and probably self-organised structures develop. Thus the shock propagates in inhomogeneous media with coexisting structures and should have a thickness of the order of these structures. It is also not clear whether the binary attraction approximation will be a good one to describe the attraction in the case where the dust density increases so much in the front of the shock. Probably the front of the shock will be unstable for disturbances perpendicular to the direction of the front propagation. All these questions wait to be answered. The modeling of the shocks should be suited to concrete astrophysical or laboratory conditions.

- Role of dissipation in atomic collisions.

  In atomic collisions there appear many mechanisms of emission which can trigger the radiative dissipation of shocks in the presence of dust. They include the surface emission in lines, features which could be characteristic for the shocks and different from the radiation patterns in the absence of the shock and thus can serve as a diagnostic of the shock. A detailed investigation in this field is needed since the line emission is much easier to observe.

- Agglomeration of dust behind the shocks.

  The process of agglomeration was already described in physical terms. An increase of dust density behind the shock accompanied by a substantial dust cooling serves to increase the rate of dust agglomeration. Then the mass density does not change, but the other parameters such as dust size and dust density are changing and these parameters enter separately in the generalised Gugoniot equations, and thus will influence both the dust charging and the structure of the front.

- Possible formation of plasma dust crystals behind the shock.

  This process is an alternative to dust agglomeration and the shocks which at the front are in gaseous state and behind the front are in solid state are completely new self-organised structures.

- Shocks in presence of high power of radiation.

  This case corresponds to positively charged dust particles and the physics of such shocks will be quite different from the physics of shocks where the dust is charged by the plasma currents. It is of interest whether such shocks will also be able to condense the dust behind the shock as the current charged dust can do.

- Role of dust distribution in sizes.

  The agglomeration changes the dust distribution in sizes. But even without agglomeration the existing different distribution in dust sizes can produce different shock structures and lead to other consequences which can be probably related to observations.

- Star formation in shocks.

  It is believed that star formation is produced by shocks giving rise to the initial condensation which then develops according to the Jeans instability. The role of self-organised structures due to dust attraction has not been investigated. This problem is directly related to the observations, which show that the presence of dust is well correlated with star formation. There are now direct observations of star creation in dust clouds. The problem is the role of condensations of smaller sizes produced by dust attraction.

- Shocks in supernovae explosions.

  A wall of dust behind the supernovae shock is indeed observed and indicates that the picture described above of dust condensation behind the shock front corresponds to the main features of the observed shocks, although more sophisticated models can be developed to describe this phenomenon.

- Particle acceleration in shocks.

  Having a microstructure in the form of self-organised charged structures, the shocks in dusty plasmas can rise to Fermi electron and ion acceleration. Also the dust in the regions in the front of the shock and behind the shock can serve as scattering centres to return the fast particle to the shock front and perform subsequent acceleration. These dust centres of scattering could be much more effective than the turbulent scattering by MHD waves excited by fast particles. This mechanism of particle acceleration waits to be developed.

In conclusion we emphasise that the narrow width of the shocks in dusty plasmas is directly related to the general properties of dusty plasma as a highly dissipative system and the system in which the collisions of plasma particles with dust particles can dominate very often.

PART IV. HISTORICAL COMMENTS

So far this paper has been presented in a lecture style, which is why references in the text have been given without historical comment. The advantage of such a presentation is that the reader does not need to look to the original literature for a description of the problem but can use directly the text of the lecture. On the other hand, most parts of the review do not follow already published papers, and much of the material is both new and unpublished, though it has been discussed with many scientists both at meetings of the European network on colloidal plasma (another word used sometimes for dusty plasmas, though not completely appropriate) and at other laboratories and meetings outside this network. Therefore, it is necessary to give some historical comments and acknowledgments pointing out which parts of the material are based on published papers and which material is new and belongs to unpublished work of the present author or performed in collaboration with coauthors. It is also necessary to point out the material which comes from lectures or seminars which can also serve as a type of publication and therefore the acknowledgments should be given here. It should be pointed out how the published material is modified in these lectures, and mention the publications closely related to the material given but not discussed here in detail. This is necessary for the reader to proceed with research in this field.

First of all I want to acknowledge Professor Robert Dewar who read this review for his comments and discussions and also the very interesting discussions on the subject of this review with W. Horton, S. Hamberger, T. Sheridan and S. Vladimirov. Second I want to discuss three important research problems in this field.

15. Three Major Problems in Dusty Plasma Physics

Physicists are motivated to work on these problems because it becomes suddenly clear that these problems are indeed deep and general. However, I will not relate my personal motivation in this area, but try to give an explanation as to why and how the three problem discussed below have been driven to the level of problems which have general physical importance.

1. *Collective Scattering by Dust Particles in Plasmas*

My first attempt at solving some problems of dusty plasmas came from an Oxford seminar 1988 headed by J. Allen in which U. de Angelis described the possibility of using the probe theory in plasmas for the solution of problems of dust charging. It is interesting that at that time a sizeable literature on dust in plasmas already existed, as pointed out at this seminar by A. von Engel who provided us with hundreds of references, mentioning that even in the first Langmuir papers mention was made of distortion in the gas discharges introduced by dust. At that time a new effect of transition scattering was considered by V. L. Ginzburg and the author (Ginzburg and Tsytovich 1984) which occurs for heavy particles even with infinite mass, since the shielding cloud consisting of electrons can produce the scattering. An ideal example is the dust particle and therefore the question arises whether the dust can effectively scatter the radiation in a plasma. The investigation of this problem was my first interest in dusty plasma problems. It appears naturally that the scattering is effective if the wavelength of radiation is larger than the Debye screening length. Apparently the shielding of dust particles in plasmas is nonlinear. In a series of papers with coauthors U. de Angelis and R. Bingham we investigated this problem and showed that the cross section for scattering in the condition where the wavelength is much longer than the nonlinear shielding length is Z_d^2 larger than the cross section for scattering by electrons (Tsytovich *et al.* 1989; de Angelis *et al.* 1989; Bingham *et al.* 1991). These results were given at the school on plasma physics in Trieste in 1989 (Tsytovich 1992) and at the first Capri European conference on dusty plasmas organised by U. de Angelis, G. Morfill, R. Bingham and the author. When acquainted with these results A. von Engel noticed that the scattering in plasma was experimentally observed by Langmuir who mentioned that dust disturbs the measurements very much. At that time when the papers (Tsytovich *et al.* 1989; de Angelis *et al.* 1989; Bingham *et al.* 1991) were published, the AISCAT scattering measurements in the lower ionosphere gave only a first indication on anomalous scattering in the regions of noctilucent clouds (which are dust clouds at a height of $80 - -90$ km). The subsequent detailed observations have indeed proved that such scattering is produced by dust. The main argument is a very low Doppler shift which can be produced only by very slowly moving heavy particles. We have had opportunity to collaborate in the interpretation of the observations with O. Havnes during several visits to Tromso University, with K. Goertz during a meeting of Norwegian Physical Society in Tromso and with G. Morfill (Max Planck Institute for Extraterrestrial Physics, Garching, Germany). The result was the paper by Havnes *et al.* (1990).

At the present time this interpretation of the observations as a scattering on the dust in the plasma is the one accepted by the scientific community. The recent direct rocket detection of dust in the ionosphere was performed by the Havnes group and showed evidence for the presence of much more dust than was expected. The problem is also related to pollution. The author appreciates very much discussions on this problem with A. von Engel, J. Allen, K. Goertz, G. Morfill, O. Havnes, U. de Angelis, R. Bingham and their collaborators. At the present time discussions have started on the possibility of diagnostic dust in tokamaks by means of this collective scattering. This was my first personal milestone in dust–plasma research.

2. *Dust Attraction, Agglomeration and Crystal Formation*

The second problem was that of dust agglomeration observed in most etching experiments raised originally by R. Selwyn and coauthors (Selwyn *et al.* 1990*a*, 1990*b*) and presented by many research groups at the Toulouse conference in 1994 (ref. [C]). The author appreciates the first discussion of this problem with O. Havnes, who gave references to Selwyn's work published in a journal which few plasma physicists read. The author also appreciates the discussion with A. Bouchoule and his colleagues who made the first detailed investigation of the first stage of dust agglomeration (Boufendi and Bouchoule 1994). The main puzzle is that the dust particle agglomerate has very large electric charges. The larger the size, the larger the charge of dust particles, but nevertheless they agglomerate having a large Coulomb repulsion (ref. [C]). This effect was waiting for an explanation. The author (Tsytovich 1994) tried to prove that due to dust charge changes the total electrostatic energy of two dust particles decreases with a decrease in the distance between dust particles. This appears due to a decrease of the self-energy with the distance and this decrease is twice as large as the increase of the interaction electrostatic energy. Does this mean attraction? The problem is where the energy is changing in plasmas and at which time scales since the general conservation of energy is certainly valid. In the equilibrium state it is known from textbooks (see e.g. Landau and Lifshits 1983) that the change of energy is related to the work of external sources. But the point is that a dusty plasma is an open system and is not in an equilibrium state. Recent numerical simulations (Khodataev *et al.* 1996*a*, 1996*b*) confirm that the total electrostatic energy of two dust particles in plasma decreases with a decrease in the distance between the dust particles in the way predicted. Still, the question is not resolved and the answer depends on how the system is open. The search for possible attractive forces continues (Tsytovich 1994) and lead to an understanding of the forces at large distances described in the main text. Tsytovich (1994) also mentioned all small changes in energy including that produced by the forces due to plasma fluxes. They are indeed small at distances less than the Debye radius. At distances larger than the Debye radius the Coulomb forces (whatever their sign) are screened, but not the forces due to plasma fluxes (the conservation of flux is a very rigid constraint). This gives the picture described in this review. It is necessary to mention that the crucial part in proving the existence of these attractive forces was made by numerical simulation (Khodataev *et al.* 1996*a*, 1996*b*). The author appreciates very much the collaboration in this research with his colleagues Ya. Khodataev, V. Tarakanov and R. Bingham (see Khodataev *et al.* 1996*a*, 1996*b*, 1998; Tsytovich *et al.* 1996*c*). The dust attraction instability was investigated in collaboration with D. Resendes, R. Bingham and Ya. Khodataev (see Tsytovich *et al.* 1996; Khodataev *et al.* 1998; Resendes *et al.* 1996; Resendes and Tsytovich 1997). The problem of the boundary free crystal was discussed with these coauthors (Tsytovich *et al.* 1996*a*; Khodataev *et al.* 1998) and at conferences with coauthors A. Sitenko and A. Zagorodny (Sitenko *et al.* 1995, 1996). Still, all this progress did not solve the agglomeration problem, since for agglomeration it is necessary to have attractive forces larger than the Coulomb forces at distances less than the Debye screening distance. The solution came only after the bombardment forces due to neutrals were considered—they indeed can exceed the Coulomb forces. This research was

carried out in collaboration with G. Morfill and Ya. Khodataev (Khodataev *et al.* 1998*a*, 1998*b*; Tsytovich *et al.* 1998).

With the attractive forces there is the deeply related problem of the dust–plasma crystal, first proposed by Morfill (1992) to be created in micro-gravity. The author appreciates many discussions on his subject with G. Morfill, Ya. Khodataev, U. de Angelis, D. Resendes and R. Bingham. This was the second personal milestone in my research in dusty plasma problems.

3. *Dust in Fusion Plasmas*

The third problem is dust in CTR devices and in particular in tokamaks. At the time the author and S. Benkadda started to discuss it there was not big interest in the problem. Therefore we decided to write a provocative paper (Benkadda and Tsytovich 1995) to create interest in the problem and to encourage researchers to make a diagnostic of the dust. The situation at the present time has changed completely since the problem of the first wall in a CTR becomes one of the most important, and it was found that close to the wall in tokamaks the conditions are not very much different from that of etching plasmas where the creation of dust it usually observed. The material given here is based on lectures given by the author at Culham Laboratory (Tsytovich 1996) and later at the JET Laboratory (Tsytovich 1997). The author appreciates very much the discussions with K. Lashmore-Davis and members of Culham seminar, and with J. Jaquonout and A. Gibson, especially on the problems of dust in disruptions, the first wall in ITER, the methods of dust diagnostics in tokamaks, and the anomalous dust enhanced diffusion in the scrape-off layer. The author had the opportunity to discuss with J. Winter the direct and indirect evidence for the presence of dust in CTR devices during the dust workshop in Bad Honnef in January 1997, where Winter (1997) gave a paper on dust in fusion plasmas. A more detail description of the observations can be found in Winter (1996) and Tsytovich *et al.* (1998) in which there is also a discussion on the problem of dust agglomeration in CTR devices. The author appreciates very much the collaboration with J. Winter and the work with him on the review [B]. This was the third milestone in my research on dusty plasma problems.

16. Related Topics

It is desirable to give references to other work on the related topics as well as references to the papers in which the author has been involved and to which not much attention has been paid so far in this review. It is also desirable to mention the differences in approach used by different authors.

1. *Dust attraction*

After a paper (Tsytovich 1994) on diminishing the electrostatic dust–dust energy with an estimation of the role of fluxes on this interaction at distances less than the Debye distance, a paper by Ignatov (1995) appeared with the same results concerning the forces due to fluxes, but for distances larger than the Debye distance. These estimates in both papers were rough but valid to an order of magnitude (from simple pressure arguments), and were in agreement. The most important in this field are the numerical simulations which proved the existence of these forces (Khodataev *et al.* 1996) and the analytical expressions obtained in

Tsytovich *et al.* (1996*c*), which not only gave the numerical coefficient of order of 1 for $T_e = T_i$, but are able to describe the interactions for $T_e \gg T_i$, the case of most interest for experiments (the rough estimate does not work in this case since the coefficients appear to be much larger than 1 and the physical explanation for this effect was given). Thus comparing Tsytovich (1994) and Ignatov (1995) with Khodataev *et al.* (1996) and Tsytovich *et al.* (1996*c*), we conclude that the latter contain a more developed approach and results appropriate for comparison with experimental data. In Ignatov (1995) the repulsive force was omitted, but was taken into account in Khodataev *et al.* (1996) and Tsytovich *et al.* (1996*c*). Thus the molecular-type of potential was obtained only in the latter two references and the conclusion that a new state of matter can exist in dusty plasmas can be made only using the results from these two works. Recently the paper by Hamaguchi (1997) discussed the problem raised by Tsytovich (1994) and agreed with the conclusion that the electrostatic energy decreases when the distance between the dust particles decreases. Hamaguchi repeated the arguments (Landau and Lifshits 1983) valid only for thermal equilibrium, but the dusty plasma is an open and highly non-equilibrium state and these arguments do not work (Vladimirov and Tsytovich 1998).

The results concerning dust attraction due to neutral thermal fluxes were obtained also in Ignatov (1996). The difference between Khodataev *et al.* (1997, 1998) and Ignatov (1996) is that the latter contains a more mathematical treatment, valid also for distances between dust particles equal to their sizes ($r > 2a$), while in Khodataev *et al.* (1997, 1998) a simpler and physically clearer method was used, valid only for $r \gg a$. The most important difference between the results is that Khodataev *et al.* (1997, 1998) considered not only the case where the distance between the particle is less than the plasma particle mean free path (which was the only subject of consideration in Ignatov 1996), but also the case where the distance is much larger than the plasma particle mean free path. The latter case has a simple interpretation in the language of thermophoretic forces and thus the opposite case also has a physical interpretation as the kinetic analogy of thermophoretic forces. In terms of these forces, one of the dust particles lowers the surrounding plasma temperature while the other particles are in the field of the temperature gradient and the thermophoretic force acts to attract dust particles. It was proved also that in the case of equal dust surface and neutral gas temperatures no forces appear, and that in an equilibrium situation also no forces between dust particles caused by neutral bombardment will exist. This general theorem was proven by Ya. Khodataev (see Khodataev *et al.* 1997). It is similar to the statement by Landau and Lifshits (1983), and reproduced in (Hamaguchi 1996), for electrostatic forces. Thus we conclude that in equilibrium the electrostatic forces are repulsive and the neutral bombardment forces vanish. This statement is never true in a non-equilibrium system, such as the dusty plasma system. The problem of the openness of the dusty plasma system was strongly emphasised in the recent review [A] by the author. But the statement that the dusty plasma is an open system was clear even before Tsytovich (1994) was written and had been extensively discussed with O. Havnes, U. de Angelis, R. Bingham, S. Benkadda, P. Gabai and A. Verga (see Benkadda and Tsytovich 1995; Tsytovich and Havnes 1994; Benkadda *et al.* 1996). Also the problem of the dust–dust interaction as an interaction in an open system

was extensively discussed with S. Benkadda and A. Verga (University Marseille) in 1997. But at that time the dust–dust attraction was not investigated in detail and the condition to use the dust–dust interactions as binary interactions was not written up explicitly. Concerning the thermophoretic forces, one should mention the direction to investigate them close to the wall (a problem discussed by Havnes *et al.* 1994) and investigate the change of dust–dust neutral bombardment interaction close to the wall. Concerning the influence of the ion drift close to the wall on the dust–dust interaction, it is necessary to mention the work of Benkadda and Tsytovich (1997) on the dust in the sheath. The general kinetic theory of single dust close to the wall was formulated with A. Sitenko and A. Zagorodny (see Sitenko *et al.* 1995, 1996). The problem of dust attraction was the subject of investigation in several papers on low temperature plasma etching problems (for reviews see ref. [D]).

2. *Problems of dust–plasma crystals*

The first theoretical statement that a Coulomb crystal can be obtained in a dusty plasma was made by Ikezi (1986) using the strongly correlated plasma approach of Ishimaru (1973, 1982). The criterion for crystallisation was obtained by Ishimaru using Monte-Carlo simulation of strongly coupled systems and reads as $\Gamma > 170$. No other proof of this relation exists and it is still a major question as to whether this criteria can be applied in dusty plasmas. But at the present time this criterion is used in most experimental work because of the absence of anything better. In the present experiments in which the dust plasma crystal was obtained, the inter-dust distance is much larger than the Debye shielding length and this is not a Coulomb system. On other hand, in etching experiments where the inter-dust distance is much less than the Debye shielding length the crystals are not observed. It is also not possible to improve the relation (Ishimaru 1973, 1982) by taking into account only the shielding of the Coulomb potential (Robbins *et al.* 1988), but for distances larger than the Debye radius a new long range unshielded repulsion appears as well as the discussed attraction of dust particles. The criterion for crystallisation written in the first part of this review was obtained by finding the binding energy as a balance of repulsion and attraction at distances larger than the Debye radius and comparing it with the dust kinetic energy. This criterion seems to be more appropriate for interpretation of the existing experiments than the one by Ishimaru (1973, 1982) and Robbins *et al.* (1988).

The present experiments can be listed as those by Thomas and Morfill (1993, 1996*a*, 1996*b*), Morfill (1994), Thomas *et al.* (1994), Morfill and Thomas (1996), Chu Lin (1994), Melzer *et al.* (1994), Hayashi and Tachibana (1994), Fortov *et al.* (1996) and Allen *et al.* (1996). Most of them were performed in the plasma sheath where both electric field and ion drift exists. The investigation of dust behaviour in the sheath has only recently started (Benkadda and Tsytovich 1997). The author appreciates collaboration with S. Benkadda on this subject. The paper by Ishihara and Vladimirov (1997) discusses the appearance of attraction as a weak field behind the dust particle produced by ion drift in the sheath. This approach can be compared with Bolotovsky (1961) and Vladimirov and Tsytovich (1998). First of all, the appearance of a weak field is natural and trivial in the sense that it was investigated more than 20 years ago when the

particle longitudinal-wave Cerenkov emission losses in plasma were found (see the review by Bolotovsky 1961). In this sense Ishihara and Vladimirov (1997) contains no new results except to say that such a wake creates a crystal, but this statement was not proved since the many dust-particle problem (or even the two dust-particle interaction) was not considered. On the other hand, the main problems for dust in the sheath is to find the attraction in the plane parallel to the wall, since in the vertical direction to first approximation the gravitational and electric forces are balanced. In Benkadda and Tsytovich (1997) it was mentioned that the ions, according to observations by Bache *et al.* (1995), should have a large thermal spread along the drift which causes the attraction in the plane parallel to the wall. On the other hand, the shielding should be considered as nonlinear which is still to be considered. The author appreciates very much the discussion of these problems with S. Vladimirov and K. Sheridan. Concerning the priority of experimental discovery of the dust–plasma crystals, it should be given to G. Morfill, J. Goree and T. Thomas (see Thomas *et al.* 1994). The first papers on this subject were at the Toulouse conference (Thomas and Morfill 1993) and the lecture by G. Morfill in Bochum at the Conference on Phenomenon in Ionised Gases (Morfill 1994). The author appreciates that the information on obtaining the crystal and the proposal for formation of the crystal in space in microgravity was given before the publication by Morfill, and also for receiving the first experimental results by email before they were published by J. Goree, as well as the discussions with T. Thomas and A. Melanso. The first investigation of melting was made by Chu and Lin (1994), but the detailed investigation of this phenomenon was made by Thomas and Morfill (1996*a*, 1996*b*) and Morfill and Thomas (1996). The author acknowledges discussions with J. Allen and his colleagues on the new phenomena in dust crystals obtained at Oxford University (Allen *et al.* 1997) and the discussion with J. Piel on his measurements of dust charges in crystals (Melzer *et al.* 1994).

3. *General dusty plasma theory*

The general Klimontovich-type equation with charge as an independent variable was first discussed with O. Havnes and T. Aslaksen. The only publication at that time was the paper by Tsytovich and Havnes (1994) on linear properties of collective plasma modes in dusty plasmas, showing the new effects appearing for variable charges of dust particles. The general method of kinetic equations with variable charges was presented in this paper and it coincides with that given in this review. A further investigation started with T. Aslaksen, but not published dealt with fluctuations and the derivation of a new collision integral which takes into account the dust charge variations. Although the procedure is similar to the well known fluctuations in the absence of charge it is extremely cumbersome and the author continued them himself. It should lead not only to new dust–dust interactions, including new attractive and repulsive forces for dust particles, but also the changes introduced by the dust charge variation in plasma particle collisions. The final result is complicated. The only results obtained up to the present time are the appearance of dust–dust attraction and repulsion, the distribution of dust particles in charges which appear in certain cases to be Gaussian, and the limit of fast plasma particle collisions with dust particles where the usual coefficient of wave damping is proved to have the additional factor

PZ_d compared with the damping in the absence of dust (de Angelis *et al.* 1993). All other results are waiting to be analysed in a form where the physical sense of the different terms will be clear. This work has also been reexamined and recalculated recently by Tsytovich and de Angelis (1998). The author appreciates very much the collaboration with O. Havnes, T. Aslaksen and U. de Angelis in this field. Some results on fluctuations in plasma were already published in papers by Sitenko *et al.* (1995, 1996). The fluctuation of dust charges were also the subject of the paper by Cui and Goree (1994) and the results are very similar to those obtained by Goree (1994). The problem of very low frequency fluctuations is of special interest for the theory of the pair correlation function in dusty plasmas (Tsytovich *et al.* 1997). In the low frequency limit only the discretness of the dust component and the dust–dust attractive and repulsive forces are of interest. The other correlations such as electron density fluctuations or dust charge fluctuations in the low frequency limit are induced by dust fluctuations and follow them. It was shown in Tsytovich *et al.* (1997) that even in the gaseous state the weak correlations have a long range behaviour, although not sufficient for formation of crystal. Thus the dusty plasma, even in the gaseous state, is proved to be prepared to convert to the crystal state which apparently does not need strong correlations as in the case of the Coulomb crystal. The gaseous binary correlations have some properties similar to the crystal state such as charge density waves caused by dust attraction. The author appreciates very much working with U. de Angelis on this problem. Already in Tsytovich and Havnes (1994) it was mentioned that practically the charging frequency is rather large which was the first step for realising that it should exist as a broad range of frequencies less than the charging frequency for which a new hydrodynamics will be valid. But this conclusion was not emphasised in Tsytovich and Havnes (1994). The whole ideology and the equation of the new hydrodynamics were given in Benkadda *et al.* (1996) in collaboration with S. Benkadda, P. Gabai and A. Verga where all effective collision frequencies used above were calculated. The general new hydrodynamic dispersion equation and new plasma modes were investigated by Tsytovich *et al.* (1996*c*), Resendes *et al.* (1997) and Resendes and Tsytovich (1997).

4. *Dust in space*

The importance of dust in space was realised many years ago. The treatment was simple as single particles without a self-consistent treatment of all components and without taking into account the openness of the dusty plasma system (Spitzer 1975; Kaplan and Pikel'ner 1979). The standard theory of star formation was given in an excellent review by (Kaplan and Pikel'ner (1974) and, although written 20 years ago, it has not needed to be updated. The main astrophysical point of view is the same as at the present time in that the stars are borne in dust molecular clouds by shock wave enhancement of density to start the gravitational contraction. The dusty plasma aspect of the problem is still waiting to be developed including all dust–dust and dust–particle interactions described above. Part III of the present review dealing with shock waves is also relevant to these problems. In general, star formation should be described as a self-organised process in which both gravitational effects, and radiation and plasma effects contribute to the process. Some aspects of the present point of view were given

by the author in the review [A]. Concerning the planetary ring problem some of the effects related to the dust charging were already taken into account. The thickness of the ring was the subject of discussion in Havnes *et al.* (1984), Havnes and Morfill (1984), Melanso and Havnes (1991) and Aslaksen and Havnes (1992) as the balance of gravitational attraction and Coulomb repulsion of dust particles. The gravitational contraction to the plane of the ring around the central planet is proportional to the deviation from the plane of the ring and in this sense is an attraction to the plane of the ring. But any plane around the central planet has these properties and in fact a dust particle not undergoing collisions will move on any Kepler orbit. Thus this attractive force acts only on the scale of binary dust–dust collisions which are very rare. The time scale of Coulomb repulsion is shorter $\Gamma < 1$ and for this case the balance cannot be established. The gravitational aspect of planetary rings was developed in much more detail in the review by Gor'kavy and Fridman (1994). The author appreciates the discussion of this point of view with these authors, where the criterion of self-organisation as a rate of dissipation due to collisions of dust particles moving on their Kepler orbit is introduced. The 1D slab model allows us to calculate the flux of plasma particles toward the ring and the rate of plasma dissipation in the case where this flux (perpendicular to the plane of the ring) is absorbed by the ring dust particles. Thus, the problem is what the thickness of the ring is compared with the mean free path of the plasma particles. In the case where the thickness of the ring is of the order of, or larger than, the mean free path the flux will be absorbed. Using the observational data we find that, amazingly for most of the rings, the thickness is of the order of the mean free path. This first means that the flux should be absorbed and second that the 1D slab model of the self-organised dust–plasma sheath can be relevant to the problem of planetary rings, since it predicts the thickness to be of the order of the mean free path. The problem is still more complicated since the orbital motion in the plane of the ring ruled by the central planet should be taken into account and the distribution of dust particles in the plane and perpendicular to the plane of the ring should be related to each other. At least some indication of the role of plasma effects in the structure formation of the dust in the rings can be obtained by comparing the rate of dissipation of plasma fluxes with the rate of dissipation calculated in Gor'kavy and Fridman (1994). It appears that the first is approximately two orders of magnitude larger than that reported. This result indicates that the plasma effects in structure formation of the rings cannot be neglected and probably they can contribute to the amazing ring structures observed. The unknown at the present time is the rate of interaction between the motions in the plane of the ring with that perpendicular to the plane. But independent of any theory, the fact that the thickness of the ring coincides with the fundamental length introduced above cannot be just coincidence.

5. *Ball lightning*

The most developed theory accepted by most of researchers is that ball lightning has two features—there exists some degree of ionisation and there exists dust (see the detailed review by Smirnov 1993). Thus, a realistic model of ball lightning should be based on the modern concepts of dusty plasmas. Such a model was proposed in the author's review [A]. This model differs from Smirnov (1993)

and we can briefly compare them. There are two main difficulties in the Smirnov (1993) model: (1) the dust density is too high and the dust particles are too small to be accumulated from naturally existing dust in the air [in the model proposed by Smirnov (1993) the dust particles have nanometre sizes and densities of the order 10^{15} cm^3] and (2) there are difficulties with the explanation of surface tension to keep the structure spherical. The model in ref. [A] deals with the slight enchancement of dust densities already existing in air of the size of microns (which are supported already by small air motions). The spherical structures we considered in Part II can be a starting point to understand the phenomenon. First of all, they are self-organised structures with surface tension. The main problem is the energy source. In ref. [A] it is proposed that this energy source is related to the kinetic energy of the dust particles confined in the structure—the structure is naturally negatively charged. In the case where the dust mass density becomes of the order of the air mass density, the sound waves will change their velocity inside the spherical structure which is sufficient to accumulate the sound wave by total inner reflection. These sound waves will move the dust for frequencies less than the dust–neutral collision frequency—it is ultrasound frequencies. Cascades in these frequencies can create sound and it is well known that the ball lightning is a source of sounds (cracking from ball lightning is the often-mentioned phenomenon). The ultrasound waves can have sufficient energy to drive the dust particles and the structure can be feed from the surroundings by ultrasound waves. The size, the energy and the lifetime found in this model is in agreement with that observed.

17. New Material in this Review

Some material in this review is published for the first time with calculations performed by the author. We can identify this material:

- The list of elementary dusty plasma relations is new, although all the individual relations were previously published in the original papers.
- The theory of one-dimensional structures and the numerical computations are new.
- Shock waves in dusty plasmas. All the material is new but the high frequency shocks (the structure of the shocks where the dust is considered immovable and the charging of dust is the only dissipative process on the front of the shock) are not discussed in this review—they determine the short range dust shock structure. For an investigation of these problems see Popel *et al.* (1996).

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Manuscript received 12 January, accepted 20 July 1998