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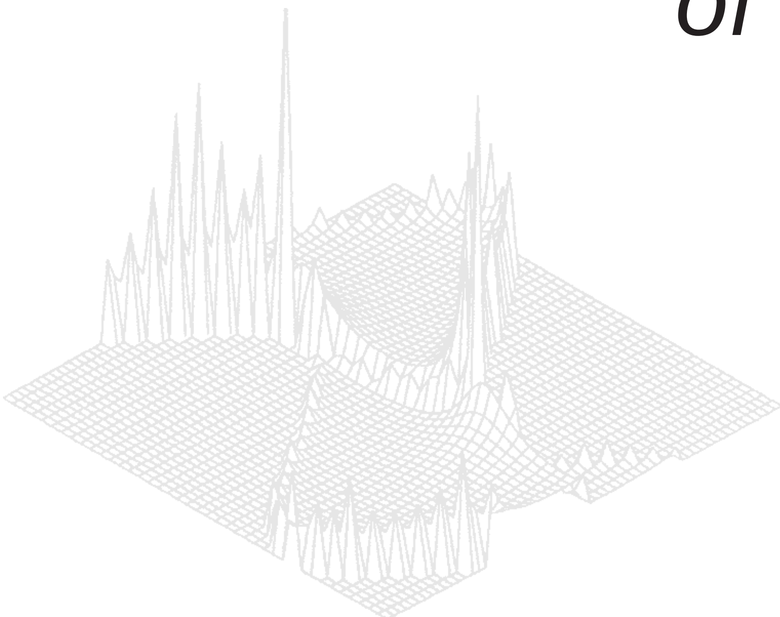

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Stability of Shear Kinetic Alfvén Waves in a Hot Relativistic Plasma

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Abstract

We have analysed the stability of a solitary shear kinetic wave in a hot relativistic plasma for oblique long wavelength perturbation following the method of Zakharov and Rubenchik. The Zakharov–Kuznetsov equation describing the wave propagation is deduced and the growth rate of instabilities due to large amplitude magnetic field perturbations is obtained as a function of the angle θ between the direction of propagation of the solitary wave and magnetic field, the streaming parameter v_0/c and the electron temperature σ.

1. Introduction

Large amplitude incompressible magnetic field perturbations have been observed in nature over a long period of time (Bdlecher and Davis 1971). One of the most important places for its occurrence is in the solar wind (Hada 1993). The existence of such kinds of waves cannot be explained by the usual MHD theory because exact large amplitude Alfvén waves are known to decay to ion acoustic waves over the order of astronomical distances (Yu and Shukla 1978; Shukla *et al*. 1982). Various authors have investigated this scenario in different situations either by use of a kinetic approach or hydrodynamic theory (Kalita and Kalita 1986). Hasegawa and Mima (1976, 1977) proved that solitary kinetic waves do exist and propagate in a direction oblique with respect to the external magnetic field. They assumed the dominance of electron pressure over electron inertia, whence the electrons were assumed to follow a Boltzmann distribution. Later the effects of ion inertia and electron thermal velocity were also incorporated. On the other hand, in recent years the importance of relativistic phenomena in a plasma has been realised (Das and Paul 1985). This is specially important for electrons in the astrophysical context. Many authors have shown that relativity does have a significant influence in the formation and on the propagation characteristics of solitary waves in plasma (Roy Chowdhury *et al*. 1988; Mukherjee and Roy Chowdhury 1992). In the light of the above observation we have studied the case of shear Alfvén waves in a hot relativistic plasma from the stability point of view (Infeld 1985; Infeld and Rowland 1978). We have derived the Zakharov–Kuznetsov equation and then analysed the stability of the solitary wave following the methodology of Zakharov and Rubenchik (1974). The growth rates are explicity

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calculated as functions of the angle θ between the propagation direction of the solitary wave and the external magnetic field, the streaming v_0/c and also as a function of the electron temperature σ. It is observed that the growth rate varies significantly due to the effect of streaming and ion temperature. While for the nonrelativistic situtation the value of the growth rate remained significant over a small range of θ $(0{\cdot}85 < \theta < 2{\cdot}85)$, in the relativisitic case the value remains considerable over wider values of θ. On the other hand, for a fixed θ the growth rate tends to a constant value as v_0/c becomes large.

2. Formation

Let us consider a plasma consisting of hot electrons in the presence of an external magnetic field B_0 directed along the z-axis. The magnetic field is assumed to be constant. The electrons are considered to be relativistic. For the three-dimensional problem the dynamics of the nonlinear slow shear wave in a homogeneous low β $(\beta \ll m_{\rm e}/m_{\rm i})$ plasma can be written as follows. Instead of writing the general three-dimensional forms of the equations of motion we here follow the strategy adopted by Shukla *et al*. (1982) of decomposing the electron motion into two parts, the drift part and the motion parallel to the z-axis. We simply generalise their equation to three dimensions, as originally formulated for the two-dimensional problem. Of course, in our case the motion in the z-direction is also considered to be relativistic. In the whole formulation it is furthermore assumed that variation with respect to transverse directions can be neglected.

The equation of continuity is

$$\frac{\partial n_{\rm e}}{\partial t} + \frac{\partial}{\partial z}(n_{\rm e} v_{{\rm e}z}) = 0\,, \tag{1a}$$

where $n_{\rm e}$ is the density of electrons and $v_{{\rm e}z}$ is the velocity in the z-direction. The corresponding momentum equation can be written as

$$\frac{\partial}{\partial t}(V_{\rm ez})_\alpha + V_{\rm ez}\frac{\partial}{\partial z}(V_{\rm ez})_\alpha = -\frac{e}{m_e}E_{\rm z} + \frac{e}{m_{\rm e}c}(\boldsymbol{V}_{\rm e} \times \boldsymbol{B}) - \frac{\sigma}{n_{\rm e}}\frac{\partial p_{\rm e}}{\partial z}\,, \tag{1b}$$

which is coupled with the pressure equations given as

$$\frac{\partial p_{\rm e}}{\partial t} + V_{\rm ez}\frac{\partial p_{\rm e}}{\partial z} + 3p_{\rm e}\frac{\partial}{\partial z}(V_{\rm ez})_\alpha = 0\,, \tag{1c}$$

whereas the governing equation for ion drift mode is

$$\frac{\partial n_{\rm i}}{\partial t} + \frac{c}{B_0\Omega_{\rm i}}\left(n_{\rm i}\frac{\partial E_x}{\partial t}\right) + \frac{c}{B_0\Omega_{\rm i}}\left(n_{\rm i}\frac{\partial E_y}{\partial t}\right)$$
$$+\frac{\partial}{\partial x}[n_{\rm i}(V_{Ex} + V_{{\rm p}x})] + \frac{\partial}{\partial y}[n_{\rm i}(V_{Ey} + V_{{\rm p}t})] = 0\,. \tag{1d}$$

On the other hand, one has Maxwell's equations

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{c}\frac{\partial B_y}{\partial t} \tag{1e}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{1}{c}\frac{\partial B_x}{\partial t}\,, \tag{1f}$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{4\pi e}{c} n_{\mathrm{e}} V_{\mathrm{e}z}\,, \tag{1g}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0\,. \tag{1h}$$

In the above equations $\boldsymbol{V}_E$ and $\boldsymbol{V}_{\mathrm{p}}$ are the velocity components due to $\boldsymbol{E} \times \boldsymbol{B}$ drift and polarisation drift. These have been included since the problem is three-dimensional, and are given as

$$\boldsymbol{V}_E = \frac{c}{B_0^2}(\boldsymbol{E} \times B_0\hat{\boldsymbol{z}})\,, \tag{2}$$

$$\boldsymbol{V}_{\mathrm{p}} = \frac{c}{B_0\Omega_{\mathrm{i}}}(\boldsymbol{V}_E\,.\,\boldsymbol{V})\boldsymbol{E}_\perp\,.$$

Here $\Omega_{\mathrm{i}} = eB_0/m_{\mathrm{i}}c$ and $(V_{\mathrm{e}z})_\alpha$ is the relativistic counterpart,

$$(V_{\mathrm{e}z})_\alpha = \frac{V_{\mathrm{e}z}}{(1 - V_{\mathrm{e}z}^2/c^2)^{1/2}}\,.$$

Further, n_e and n_{i} are respectively the electron and ion number density, $V_{\mathrm{e}z}$ is the z component of the electron velocity and Ω_{i} is the ion-cyclotron frequency. For a shear Alfvén wave there cannot be any magnetic field along the z-axis, which implies equation (2). Since V_{A}^2 (Alfvén speed) $\ll c^2$ we have assumed the quasineutrality condition to hold. Finally, the ion inertia and the displacement current have been neglected, since the wave frequency is assumed to be much smaller than Ω_{i}.

To derive the nonlinear wave equation we make the following stretching of spatial and temporal coordinates:

$$z' = \epsilon^{\frac{1}{2}}(z - t), \quad t' = \epsilon^{\frac{3}{2}}t, \quad x' = \epsilon^{\frac{1}{2}}x, \quad y' = \epsilon^{\frac{1}{2}}y\,. \tag{3}$$

We furthermore assume that the physical variables are expressed as

$$n_0 = n_0 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \ldots\,,$$

$$V_{\mathrm{e}z} = V_0 + \epsilon V_{\mathrm{e}z}^{(1)} + \epsilon^2 V_{\mathrm{e}z}^{(2)} + \ldots\,,$$

$$E = \epsilon^{\frac{1}{2}} E^{(1)} + \epsilon^{\frac{3}{2}} E^{(2)} + \dots ,$$

$$B_x = \epsilon^{\frac{1}{2}} B_x^{(1)} + \epsilon^{\frac{3}{2}} B_x^{(2)} + \dots ,$$

$$B_y = \epsilon^{\frac{1}{2}} B_y^{(1)} + \epsilon^{\frac{3}{2}} B_y^{(2)} + \dots ,$$

$$p_{\mathrm{e}} = 1 + \epsilon p_{\mathrm{e}}^{(1)} + \epsilon^{(2)} p_{\mathrm{e}}^{(2)} + \dots . \tag{4}$$

Substituting these in equations (1a) to (1h) and equating lowest powers of ϵ we get (to lowest order)

$$V_{\mathrm{e}z}^{(1)} = \frac{\lambda - V_0}{n_0} n^{(1)} ,$$

$$p_{\mathrm{e}}^{(1)} = \frac{3(1 + 3V_0^2/2c^2)}{n_0} n^{(1)} , \tag{5}$$

$$\frac{\partial E_x^{(1)}}{\partial x'} + \frac{\partial E_y^{(1)}}{\partial y'} = -\frac{B_0 \Omega_{\mathrm{i}}}{c n_0} n^{(1)} ,$$

$$\frac{E_y^{(1)}}{x} - \frac{E_x^{(1)}}{y} = 0 ,$$

$$E_z^{(1)} = 0, \quad B_y^{(1)} = \frac{c}{\lambda} E_x^{(1)}, \quad B_x^{(1)} = -\frac{c}{\lambda} E_y^{(1)} ,$$

along with the relation

$$\lambda^2 - V_0 \lambda - \frac{B_0^2}{4\pi m_{\mathrm{i}} n_0} = 0 . \tag{6}$$

Proceeding to the next order in ϵ and eliminating all the variables in favour of $n^{(1)}$ and $\phi^{(1)}$ we get

$$\frac{\partial n^{(1)}}{\partial t'} - A_{11} n^{(1)} \frac{\partial n^{(1)}}{\partial z'} + A_{31} \frac{\partial}{\partial z'} \left(\frac{\partial^2 n^{(1)}}{\partial x'^2} + \frac{\partial^2 n^{(1)}}{\partial y'^2} \right)$$
$$+ A_{21} \left(\frac{\partial n^{(1)}}{\partial x'} \frac{\partial^2 \phi^{(1)}}{\partial x' \partial z'} + \frac{\partial n^{(1)}}{\partial y'} \frac{\partial^2 \phi^{(1)}}{\partial y' \partial z'} \right) = 0 , \tag{7}$$

along with the relation

$$n^{(1)} = -\frac{cn_0}{B_0\Omega_i}\left(\frac{\partial^2\phi^{(1)}}{\partial {x'}^2} + \frac{\partial^2\phi^{(1)}}{\partial {y'}^2}\right),$$

$$E_x^{(1)} = +\frac{\partial\phi^{(1)}}{\partial x'}, \tag{8}$$

in accordance with equation (5). Here the coefficients A_{ij} are given as

$$A_{11} = \frac{\lambda(\lambda - V_0)}{n_0(2 - V_0)},$$

$$A_{21} = \frac{\lambda(\lambda - V_0)c}{B_0\Omega_{\mathrm{i}}(2\lambda - V_0)}, \tag{9}$$

$$A_{31} = -\frac{n_0\lambda^2(\lambda - V_0)}{B_0\Omega_{\mathrm{i}}(2\lambda - V_0)}\left[\left(1 + \frac{3V_0^2}{2c^2}\right)\frac{cm_{\mathrm{e}}}{e}\frac{\lambda - V_0}{n_0}\right.$$
$$\left.\times (V_0 - 1) + \frac{3cm_{\mathrm{e}}\sigma}{en_0}\left(1 + \frac{3V_0^2}{2c^2}\right)\right].$$

The set of equations (7) and (8) is the required Zakharov–Kuznetsov equation describing the propagation of the nonlinear wave. In the following, our motivation is to obtain a solution of this and to analyse its stability.

3. Solitary Wave Solution

In order to study the stability of a solitary wave that propagates in a direction making an angle α with the axis, we make a rotation of the $(x,\, z)$ axis and set

$$Z = z\cos\alpha - X\sin\alpha\,,$$

$$X = z\sin\alpha + x\cos\alpha\,. \tag{10}$$

In the new coordinate system equations (7) and (8) assume the following form:

$$\frac{\partial n^{(1)}}{\partial t'} + \beta n^{(1)}\frac{\partial n^{(1)}}{\partial z} + \gamma\frac{\partial^3 n^{(1)}}{\partial z^3} + \frac{\partial n^{(1)}}{\partial z}\frac{\partial E^{(1)}}{\partial z}\delta$$
$$+a_1 n^{(1)}\frac{\partial n^{(1)}}{\partial X} + b_1\frac{\partial n^{(1)}}{\partial Z}\frac{\partial E^{(1)}}{\partial X} + c_1\frac{\partial n^{(1)}}{\partial X}\frac{\partial E^{(1)}}{\partial Z}$$

$$+a_0 \frac{\partial n^{(1)}}{\partial X} \frac{\partial E^{(1)}}{\partial X} + b_0 \frac{\partial n^{(1)}}{\partial y'} \frac{\partial^2 \phi^{(1)}}{\partial Z \partial y'} + c_0 \frac{\partial n^{(1)}}{\partial y'} \frac{\partial^2 \phi^{(1)}}{\partial X \partial y'}$$

$$+d_1 \frac{\partial^3 n^{(1)}}{\partial Z^2 \partial X} + e_1 \frac{\partial^3 n^{(1)}}{\partial Z \partial X^2} + f_1 \frac{\partial^3 n^{(1)}}{\partial X^3} + h_1 \frac{\partial^3 n^{(1)}}{\partial X \partial {y'}^2} + g_1 \frac{\partial^3 n^{(1)}}{\partial Z \partial {y'}^2} = 0 \,, \quad (11)$$

$$n^{(1)} = \lambda_1 \frac{\partial E^{(1)}}{\partial Z} + \mu \frac{\partial E^{(1)}}{\partial X} + \gamma \frac{\partial^2 \phi^{(1)}}{\partial {y'}^2} \,,$$

$$E^{(1)} = \sin\alpha \, \frac{\partial \phi^{(1)}}{\partial Z} + \cos\alpha \, \frac{\partial \phi^{(1)}}{\partial X} \,. \quad (12)$$

The different coefficients occuring in the above equation are functions of the angle α and different plasma parameters. Their explicit form are given below:

$$\beta = -A_{11} \cos\alpha \,, \qquad b_1 = -A_{21} \sin^2\alpha \,,$$

$$\gamma = A_{31} \sin^2\alpha \cos\alpha \,, \qquad c_1 = A_{21} \cos^2\alpha \,,$$

$$\delta = A_{21} \sin\alpha \cos\alpha \,, \qquad a_0 = -A_{21} \sin\alpha \cos\alpha \,,$$

$$a_1 = A_{11} \sin\alpha \,, \qquad b_0 = A_{21} \cos\alpha \,,$$

$$f_1 = -A_{31} \sin\alpha \cos^2\alpha \,,$$

$$c_0 = -A_{21} \sin\alpha \,,$$

$$d_1 = A_{31} \sin\alpha \, (2\cos^2\alpha - \sin^2\alpha) \,,$$

$$c_1 = A_{31} \cos\alpha \, (\cos^2\alpha - 2\sin^2\alpha) \,,$$

$$g_1 = A_{31} \cos\alpha, \qquad h_1 = -A_{31} \sin\alpha \,,$$

$$\lambda_1 = -\frac{cn_0}{b_0 \Omega_{\rm i}} \sin\alpha, \qquad \mu = -\frac{cn_0}{B_0 \Omega_{\rm i}} \cos\alpha \,,$$

$$\gamma = -\frac{cn_0}{B_0 \Omega_{\rm i}} \,. \quad (13)$$

To construct the nonlinear wave we assume that $n^{(1)} = N_0(\bar{z})$, $\phi^{(1)} = \phi_0(\bar{z})$, $E^{(1)} = E_0(\bar{z})$, where $\bar{z} = z - u_0 t$ leading to

$$-u_0 \frac{\mathrm{d}N_0}{\mathrm{d}z} + kN_0 \frac{\mathrm{d}N_0}{\mathrm{d}z} + \gamma \frac{\mathrm{d}^3 N_0}{\mathrm{d}z^3} = 0 \,, \quad (14)$$

with $N_0 = \lambda_1\, \mathrm{d}E_0/\mathrm{d}z$, $E_0 = \sin\alpha\, \mathrm{d}\phi_0/\mathrm{d}z$ and $K = \beta + \delta/\lambda_1$. So we immediately obtain

$$N_0 = a\,\mathrm{sech}^2(pz)\,, \qquad a = \frac{3U_0}{k}, \qquad p = (u_0/4\gamma)^{1/2}\,,$$

$$E_0 = \frac{a}{p\lambda_{\mathrm{i}}}\tanh(pz)\,. \tag{15}$$

It may be noted that

$$a = \frac{3u_0}{\beta + \delta/\lambda_1} = -\frac{3u_0(2\lambda - V_{\mathrm{ez}}^{(0)})}{2\cos\alpha\lambda(\lambda - V_{\mathrm{ez}}^{(0)}} < 0\,,$$

as $\lambda > V_{\mathrm{ez}}^{(0)}$, so equation (15) represents a solution with a density depression.

4. Stability Analysis

To ascertain the stability of such a wave profile let us use the technique of Zakharov and Rubenchik (1974). We set

$$\begin{aligned} n^{(1)} &= N_0(\bar{z}) + q(\bar{z}, x, y', t')\,, \\ \phi^{(1)} &= \phi_0(\bar{z}) + \psi(\bar{z}, x, y', t')\,, \\ E^{(1)} &= E_0(\bar{z}) + F(\bar{z}, x, y', t')\,. \end{aligned} \tag{16}$$

Substituting in equation (12) and linearising with respect to q, ψ, F we get

$$\begin{aligned} -u_0\frac{\partial q}{\partial z} &+ \frac{\partial q}{\partial t'} + \beta N_0\frac{\partial q}{\partial z} + \gamma\frac{\partial^3 q}{\partial z^3} + \delta\frac{dN_0}{dz}\frac{\partial F}{\partial z} + \delta\frac{\partial E_0}{\partial z}\frac{\partial q}{\partial z} \\ &+ a_1 N_0\frac{\partial q}{\partial x} + b_1\frac{\partial N_0}{\partial z}\frac{\partial F}{\partial x} + c_1\frac{\partial E_0}{\partial z}\frac{\partial q}{\partial x} + d_1\frac{\partial^3 q}{\partial z^2\partial x} + e_1\frac{\partial^3 q}{\partial z\partial x^2} \\ &+ f_1\frac{\partial^3 q}{\partial x^3} + q_1\frac{\partial^3 q}{\partial z\partial {y'}^2} + h_1\frac{\partial^3 q}{\partial x\partial {y'}^2} = 0\,, \end{aligned} \tag{17}$$

$$\begin{aligned} q &= \lambda_1\frac{\partial F}{\partial z} + \mu\frac{\partial F}{\partial z} + \gamma\frac{\partial^2 F}{\partial {y'}^2}\,, \\ F &= \sin\alpha\frac{\partial \psi}{\partial z} + \cos\alpha\frac{\partial \psi}{\partial x}\,. \end{aligned} \tag{18}$$

We now assume that the perturbation terms (q, F, ψ) in the case of long wavelength perturbations in the direction (l, m, n) have the following forms. Note that (l, m, n) are the direction cosines which specify the directions

$$
\begin{aligned}
q &= \bar{q}\ (\overline{z}) \exp\left[\mathrm{i}k\ (lx + my' + nz) - \mathrm{i}\omega t'\right], \\
F &= \bar{F}\ (\overline{z}) \exp\left[\mathrm{i}k\ (lx + my' + nz) - \mathrm{i}\omega t'\right], \\
\psi &= \bar{\psi}\ (\overline{z}) \exp\left[\mathrm{i}k\ (lx + my' + nz) - \mathrm{i}\omega t'\right],
\end{aligned} \tag{19}
$$

where k is small and $l^2 + m^2 + n^2 = 1$.

We further assume that for small k we can expand the functions q, F and ψ as follows:

$$
\begin{aligned}
\bar{q}(\bar{z}) &= q_0(\bar{z}) + kq_1(\bar{z}) + k^2 q_2(\bar{z}) + \dots, \\
\bar{F}(\bar{z}) &= F_0(\bar{z}) + kF_1(\bar{z}) + k^2 F_2(\bar{z}) + \dots, \\
\overline{\psi}\ (\bar{z}) &= \psi_0(\bar{z}) + k\psi_1(\bar{z}) + k^2 \psi_2(\bar{z}) + \dots, \\
\omega &= k\omega_1 + k^2 \omega_2 + \dots .
\end{aligned} \tag{20}
$$

Our main interest is to obtain an expression for ω_1. On substituting these expressions in equations (17) and (18), we equate various powers of k. At the lowest order of K we get

$$
q_0 = \lambda_1 \frac{\mathrm{d}F_0}{\mathrm{d}z}, \qquad F_0 = \sin\alpha \frac{\mathrm{d}\psi_0}{\mathrm{d}z}, \tag{21}
$$

$$
-u_0 \frac{\mathrm{d}q_0}{\mathrm{d}z} + k \frac{\mathrm{d}}{\mathrm{d}z}(N_0 q_0) + \gamma \frac{\mathrm{d}^3 q_0}{\mathrm{d}z^3} = 0. \tag{22}
$$

Eliminating all other variables in favour of q_0 and integrating once one obtains

$$
[-1 + 2\mathrm{sech}^2(pz)]q_0 + \frac{1}{4p^2} \frac{\mathrm{d}^2 q_0}{\mathrm{d}z^2} = A, \tag{23}
$$

A being a constant. The two linearly independent solutions of equation (23) with the right-hand side equal to zero are

$$
f = N_{0z} \ \text{ and } \ g = N_{0z} \int^z \frac{\mathrm{d}z}{N_{0z}^2}, \tag{24}
$$

which leads to

$$
f = RS^2, \qquad g = pzRS^2 + \tfrac{2}{15}S^{-2} + \tfrac{1}{3} - S^2. \tag{25}
$$

Here $R = \tanh(pz)$, $S = \mathrm{sech}(pz)$, so the most general solution is

$$q_0 = A_1 f + A_2 g - f \int^z \frac{Ag}{W/4p^2} \mathrm{d}z + g \int^z \frac{Af}{W/4p^2} \mathrm{d}z\,, \tag{26}$$

where A_1, A_2 are two constants and W is nothing but the Wronskian of the two solutions, which being nonzero proves that the solutions are independent. After some simplification q_0 can be written as (Ince 1956)

$$\begin{aligned} q_0 &= A_1 RS^2 + A_2(pz\, RS^2 + \tfrac{2}{15} AS^{-2} + \tfrac{1}{3} - S^2) \\ &+ \frac{15A}{2}\left(\tfrac{1}{15} S^{-2} + \tfrac{1}{30}\, S^2 + \tfrac{1}{10} S^{-2} + \tfrac{1}{10} pz\, RS^2\right). \end{aligned} \tag{27}$$

Now the constants A_1, A_2, A are to be chosen in such a fashion that q_0 does not go to $\pm\infty$ as $|z| \to \infty$. Similarly F_0 and ψ are found to be

$$F_0 = -\frac{A_1}{2ap\lambda_1} N_0; \qquad \psi_0 = -\frac{A_1 R}{2p^2 \lambda_1 \sin\alpha}\,. \tag{28}$$

So we have explicitly determined q_0, F_0 and ψ_0 by equations (26), (27) and (28). We call this the zeroth order set. Next we pass over to the first order corrections, for getting information on q_1, F_1 and ψ_1.

First Order Equation

To first order in k we obtain

$$-u_0 \frac{\mathrm{d}q_1}{\mathrm{d}z} + k \frac{\mathrm{d}}{\mathrm{d}z}\,(N_0 q_1) + \gamma \frac{\mathrm{d}^3 q_1}{\mathrm{d}z^3} = \sigma\,,$$

$$\begin{aligned} \sigma = &- \frac{\mathrm{i}A_1}{2ap}\,(\omega_1 + V_0 N) \frac{\mathrm{d}N_0}{\mathrm{d}z} + \frac{\mathrm{i}A_1}{2ap}\,(3\gamma r + d_1 l) \frac{\mathrm{d}^3 N_0}{\mathrm{d}z^3} \\ &+ \frac{\mathrm{i}A_1}{2ap}\left[KN + \left(\frac{b_1}{\lambda_1} + \frac{c_1}{\lambda_1} + a_1 - \frac{\mu d}{\lambda_1^2}\right) l\right] N_0 \frac{\mathrm{d}N_0}{\mathrm{d}z}\,, \end{aligned} \tag{29}$$

$$q_1 = \lambda_1(\mathrm{i}nF_0 + \mathrm{d}F_1/\mathrm{d}z) + \mathrm{i}\mu l F_0\,. \tag{30}$$

Using the expressions for the zeroth order quantities we get

$$\begin{aligned} &\frac{1}{4p^2} \frac{\mathrm{d}^2 q_1}{\mathrm{d}z^2} + [-1 + 3\,\mathrm{sech}^2(p^2)] q_1 \\ &\qquad = B + \mathrm{i}A_1 a_2\, \mathrm{sech}^2(pz) + \mathrm{i}A_1 b_2\, \mathrm{sech}^2(pz) \tan^2(pz)\,, \end{aligned} \tag{31}$$

where a_2 and b_2 are new constants. The solution of (31) yields

$$q_1 = B_1\, RS^2 - \mathrm{i}A_1 pzRS^2(a_2+b_2) + \frac{\mathrm{i}A_1}{3}S^2(3a_2+b_2)\,, \tag{32}$$

from which corresponding expressions for F_1 and ψ_0 may be obtained.

Second Order Corrections

Proceeding now to terms of order of k^2 we get

$$-u_0\frac{\mathrm{d}q_2}{\mathrm{d}z} + K\frac{\mathrm{d}}{\mathrm{d}z}\,(N_0 q_2) + \gamma\frac{\mathrm{d}^3 q_2}{\mathrm{d}z^3}$$

$$= -\,\mathrm{i}(\omega_2 + h_1 l m^2)\; q_0 + \mathrm{i}a_3 q_1 + b_3\frac{\mathrm{d}q_0}{\mathrm{d}z} - \mathrm{i}\, C_3 N_0 q_1 - \mathrm{i}d_3\frac{\mathrm{d}^2 q_1}{\mathrm{d}z^2}$$

$$+\,\mathrm{i}l_3\frac{\mathrm{d}N_0}{\mathrm{d}z}F_1 + f_3\frac{\mathrm{d}N_0}{\mathrm{d}z}\psi_0\,,$$

$$q_2 = \lambda_1(\mathrm{i}n\; F_1 + \mathrm{d}F_2/\mathrm{d}z) + \mathrm{i}\mu(lF_1) + \nu m^2\psi_0\,, \tag{33}$$

where a_3, b_3, c_3 are some combinations of previous constants.

Now let us consider the equation adjoint to the homogeneous part of equation (33), which is

$$(-u_0 + kN_0)\mathrm{d}\bar{q}_2/\mathrm{d}z + \gamma\frac{\mathrm{d}^2}{\mathrm{d}z^2}\left(\frac{\mathrm{d}\bar{q}_2}{\mathrm{d}z}\right) = 0\,. \tag{34}$$

We look for a solution of this equation satisfying the same boundary conditions as required for equation (31). Whence we obtain

$$q_2 = A' S^2\,,$$

A being a constant, so that the kernel of the operator

$$(-u_0 + KN_0)\mathrm{d}/\mathrm{d}z + \mathrm{d}^2/\mathrm{d}z^2 \tag{36}$$

is S^2. So for the solution of equation (33) to exist the right-hand side of (33) should be orthogonal to this kernel, leading to the condition determining ω_1:

$$-\mathrm{i}(\omega_2 + h_1 l m^2)\int_{-\infty}^{\infty} q_0 S^2 \mathrm{d}z + \mathrm{i}a_3\int_{-\infty}^{\infty} q_1 S^2 \mathrm{d}z + b_3\int_{-\infty}^{\infty}\frac{\mathrm{d}q_0}{\mathrm{d}z}\;S^2 \mathrm{d}z$$

$$- \mathrm{i}c_3 \int_{-\infty}^{\infty} N_0 q_1 S^2 \mathrm{d}z - \mathrm{i}d_3 \int_{-\infty}^{\infty} \frac{\mathrm{d}^2 q_1}{\mathrm{d}z^2} S^2 \mathrm{d}z + \mathrm{i}l_3 \int_{-\infty}^{\infty} \frac{\mathrm{d}N_0}{\mathrm{d}z} F_1 S^2 \mathrm{d}z$$

$$+ f_3 \int_{-\infty}^{\infty} \frac{\mathrm{d}N_0}{\mathrm{d}z} \psi_0 S^2 \mathrm{d}z = 0 \,. \tag{37}$$

Substituting the previously obtained expressions for q_0, q_1 and ψ_0, F_1 etc., we get

$$\omega_1^2 + \overline{A}\omega_1 + \overline{B} = 0 \,, \tag{38}$$

where

$$\overline{A} = -\bar{a}_2 - \tfrac{1}{9}\bar{b}_2 - \tfrac{8}{9}C_3 a + \tfrac{4}{3}p^2 d_3 - \tfrac{4}{9}\frac{e_3 a}{\lambda_1} \,, \tag{39}$$

with a similar expression for B. The discriminant of equation (38) is $\Delta = \overline{A}^2 - 4\overline{B}$. So the condition for instability of a perturbation in the direction (l, m, n) is that

$$\overline{A}^2 - 4\overline{B} < 0 \,. \tag{40}$$

If this relation is satisfied then the growth rate g_{R} is given by

$$g_{\mathrm{R}}^2 = K^2(4\overline{B} - \overline{A}^2) \,. \tag{41}$$

For a perturbation in a plane through the z-axis making an angle θ with the (x, y) plane, the above expression for the growth rate is

$$g_{\mathrm{R}}^2 = (1 - n^2)K^2(4\overline{B} - \overline{A}^2) \,, \tag{42}$$

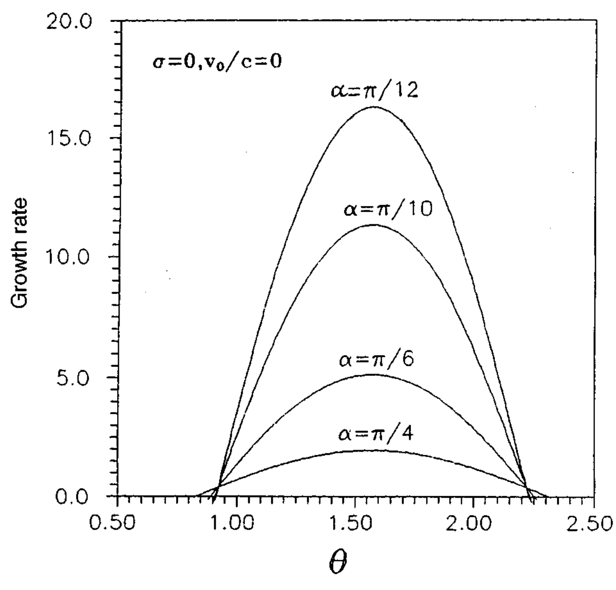

Fig. 1. Normalised growth rate (squared) versus the angle θ for various values of the angle α.

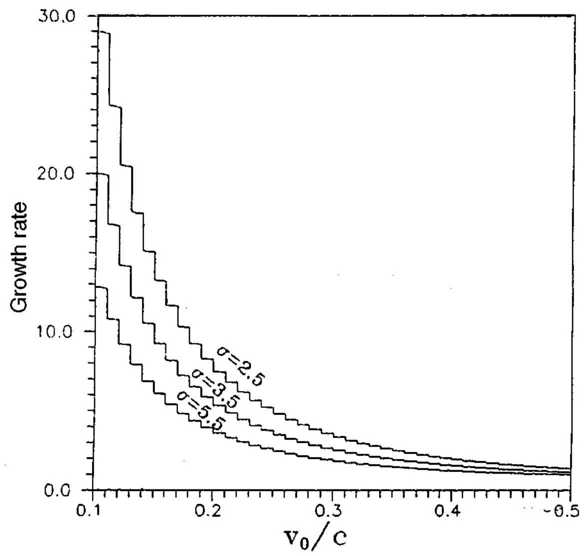

Fig. 2. Normalised growth rate (squared) versus v_0/c for various values of σ where $\alpha = \pi/8$ and $\theta = 1 \cdot 5$.

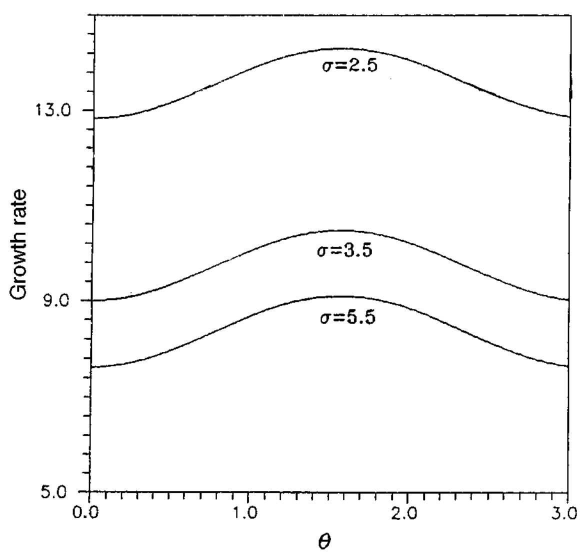

Fig. 3. Normalised growth rate (squared) versus θ for various values of σ where $\alpha = \pi/8$ and $v_0/c = 0 \cdot 8$.

which is a maximum for $n = 0$. Hence the maximum growth rate is attained for perturbation in the (x', y') plane, which is the plane of direction of propagation of the solitary wave. We have analysed the variation of this expression for g_R for different values of v_0/c, the electron temperature σ and the angle θ.

To start we consider the case $v_0/c = 0$ and $\sigma = 0$ and plot the growth rate as a function of θ for various α. This is shown in Fig. 1. It may be observed that our diagram exactly reproduces that of Das and Paul (1985). On the other hand, for finite but small values of v_0/c, we depict the growth rate as a function of v_0/c for fixed α and θ in Fig. 2. It is interesting to note that for various values of the electron temperature σ the growth rate reaches a constant value as v_0/c increases. On the other hand, in Fig. 3 we show the variation with θ for fixed v_0/c and α. It is actually comparable to the non-relativistic case given in Fig. 1. The trend has changed totally. The growth rate never becomes zero in this range of θ as it does in the non-relativistic situation. Also its value decreases with the temperature of the electron. Fig. 4 shows the variation of the growth rate as a function of θ, but for different v_0/c. The trend remains the same as in Fig. 3 and the growth rate again decreases with v_0/c.

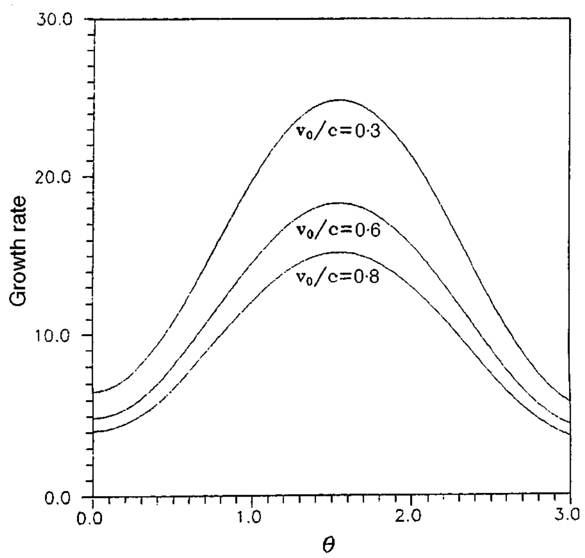

Fig. 4. Normalised growth rate (squared) versus θ for various values of v_0/c where $\alpha = \pi/8$ and $\sigma = 1 \cdot 5$.

5. Discussion

In our analysis of the stability of the shear kinetic solitary wave we have observed that the maximum value of the growth rate occurs for the perturbation in a plane perpendicular to the direction of motion of the solitary wave. This pattern was also there even in the non-relativistic case. On the other hand, there has been a significant change in the variational pattern of the growth rate with respect to the streaming and electron temperature. While in the non-relativistic case the profile of the growth rate was confined over smaller values of θ, in the present case it almost exists throughout the full range of θ and decreases with respect to both v_0/c and σ.

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