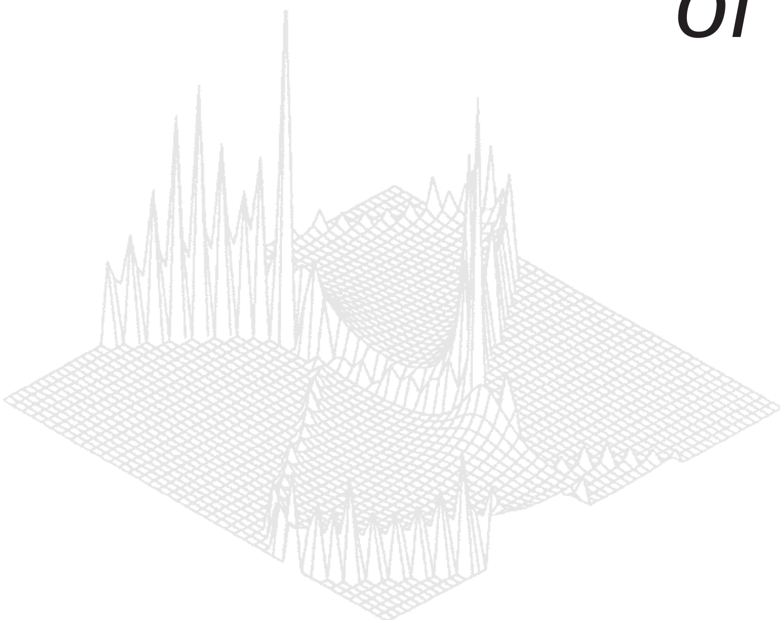
CSIRO PUBLISHING

Australian Journal of Physics

Volume 51, 1998



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood Telephone: 61 3 9662 7626
Vic. 3066 Facsimile: 61 3 9662 7611
Australia Email: peter.robertson@publish.csiro.au

Published by **CSIRO** PUBLISHING
for CSIRO and the
Australian Academy of Science

Aust. J. Phys., 1998, **51**, 935–41

Effect of Non-thermal Electrons on the Shock Wave in a Magnetised Plasma

K. Roy Chowdhury[A] *and A. Roy Chowdhury*[B]

[A] Physics Department, JCC College,
30 P. A. S. Road, Calcutta 70033, India.
[B] High Energy Physics Division, Department of Physics,
Jadavpur University, Calcutta 70032, India.

Abstract

We have studied the effect of non-thermal electrons on the structure of a shock wave in a magnetised plasma. Using the reductive perturbation technique we have derived the Zakharov–Kuznetsov equation, and also the modified version of it in the critical limit. The structure of the shock wave is then analysed as a function of the parameter β, which measures the deviation from the thermalised state. The corresponding behaviour of the maxima of the shock wave and its velocites are depicted graphically. Both comprehensive and rarefactive shocks are seen to be generated.

1. Introduction

Double layers in plasma have aroused considerable interest in recent years. Their presence in aural and magnetospheric plasmas has already been established by the S32-3 and Viking satellites (Temerin *et al.* 1982; Bostrom *et al.* 1988). The double layer or shock wave theory has successfully explained the mechanism of solar flares (Alfvén and Carlquist 1967), particle acceleration in space (Temerin *et al.* 1982; Carlquist 1986) and the ionosphere (Borovsky 1984). Besides astrophysical applications double layers have also been exploited for non-heating in linear turbulent heating devices (Saeki *et al.* 1980) and the confinement of plasma in tandem mirror devices. Attempts have also been made to discuss the effect of double layers in a magnetised plasma. Goswami and Bujurbaruah (1986) have studied the obliquely propagating ion acoustic double layer using the fluid equation and arbitrary equation of state for electrons. Bharuthram and Shukla (1986) studied multidimensional double layers in an unmagnetised plasma with cold ion and two temperature electrons. They did not use the quasi-neutrality condition. However, the propagation was considered to be along the magnetic field. Strong oblique double layers were analysed by Borovsky and Joyce (1983) in a numerical simulation experiment and they observed an increase in the thickness of the double layer with an increase of the obliqueness to the magnetic field. Ion acoustic double layers have also been studied in hot relativistic plasmas by a number of authors. Some have also considered the effect of electron inertia and finite geometry (Roy Chowdhury *et al.* 1994; Mondal *et al.* 1998). However, in all these papers a basic ingredient is the assumption of a Boltzmann-type distribution of electrons. On the other hand, the experimental observations of

 10.1071/PH98018 0004-9506/98/060935$05.00

the Freya satellite (Dovner *et al.* 1994) have brought to notice the fact that double layers are actually solitary structures with density depletion in plasmas where the electrons are non-thermal. Also it has been observed that in nature both the comprehensive and rarefactive type shocks are present, which is not the case with most of the theoretical formulations noted above. So here we consider an analysis of the shock-like structure in a magnetised plasma with hot ions and non-thermal electrons in multidimensions. Using the reductive perturbation procedure we deduce the Zakharov–Kuznetsov (ZK) (1974) equation and also its modified version in the critical case. The solution of such a system gives rise to a shock structure whose maxima and velocity is then analysed as functions of various plasma parameters. It is interesting to note that due to the presence of non-thermal electrons both comprehensive and rarefactive shocks are found to be present.

2. Formulation

We consider a collision-less fully ionised plasma with warm adiabatic ion and non-thermal electrons with density n and $n_{\rm c}$ in an external magnetic field $B = B_0 Z$. The dynamics of the plasma is then governed by the fluid equations

$$\frac{\partial n}{\partial t} + \nabla(n.v) = 0\,,$$
$$\frac{\partial v}{\partial t} + (V.\nabla)V = -\nabla\phi + (\Omega/\omega)V \times Z - \tfrac{5}{2}\sigma\nabla(n)^{\frac{2}{3}}\,,$$
$$\nabla^2\phi = n_{\rm e} - n\,. \tag{1}$$

The ion and electron densities are normalised to the unperturbed density n_0, V is the ion fluid velocity normalised to the ion fluid speed $C_{\rm s} = (T_{\rm e}/m)^{1/2}$, where $T_{\rm e}$ is the electron temperature in units of the Boltzmann constant, m is the ion mass, ϕ is the electrostatic potential normalised to $T_{\rm e}/e$, the spatial variables are normalised to the Debye length $\lambda_{\rm D} = (T_{\rm eff}\epsilon_0/n_0 e^2)^{1/2}$ and the time is normalised to the ion plasma period $\omega_{\rm p}^{-1} = (n_0 e^2/\epsilon_0 m_{\rm i})^{-1/2}$. Since the electrons are assumed to be non-thermalised with a population of fast particles, we can choose the distribution to be that given by Cairns *et al.* (1995), so that the electron density is

$$n_{\rm e} = (1 - \beta\phi + \beta\phi^2){\rm e}^{\phi}\,, \qquad \beta = 4\alpha/(1+3\alpha)\,. \tag{2}$$

The parameter α determines the presence of fast particles in the model; its value ranges from $0{\cdot}1$ to $0{\cdot}2$.

We now adopt a reducive perturbation procedure by setting

$$t' = \epsilon^{3/2}t, \quad y' = \epsilon^{1/2}y, \quad x' = \epsilon^{1/2}x, \quad z' = \epsilon^{1/2}(z - t)\,, \tag{3}$$

and expanding the dynamical quantities as

$$V_x = \epsilon^{3/2} V_x^{(1)} + \epsilon^2 V_x^{(2)} + \dots ,$$

$$V_y = \epsilon^{3/2} V_y^{(1)} + \epsilon^2 V_y^{(2)} + \dots ,$$

$$V_z = V_0 + \epsilon V_z^{(1)} + \epsilon V_z^{(2)} \dots ,$$

$$\varphi = \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \dots ,$$

$$n = 1 + \epsilon n^{(1)} + \epsilon^2 N^{(2)} \dots . \tag{4}$$

Substituting in the basic equation and collecting various powers of ϵ we get from terms of order $\epsilon^{3/2}$

$$(V_0 - \lambda) n^{(1)} = V_z^{(1)} , \tag{5}$$

$$[1 + \tfrac{5}{3}\sigma(1-\beta)]\partial/\partial x(\phi^{(1)}) = a V_y^{(1)} , \tag{6}$$

$$[1 + \tfrac{5}{3}\sigma(1-\beta)]\partial/\partial y(\phi^{(1)}) = a V_x^{(1)} , \tag{7}$$

$$(V_0 - \lambda)\partial/\partial z(V_z^{(1)}) + \tfrac{5}{3}\sigma\partial/\partial z(n^{(1)}) = -\partial/\partial z(\phi^{(1)}) , \tag{8}$$

$$(1-\beta)\phi^{(1)} = n^{(1)} , \tag{9}$$

and the dispersion relation is

$$1/(1-\beta) + [\tfrac{5}{3}\sigma - (V_0 - \lambda)^2] = 0 . \tag{10}$$

Proceeding to higher order in ϵ and eliminating all variables in favour of $\phi^{(1)}$ we get

$$\begin{aligned}(V_0 - \lambda)(1-\beta)^2 \partial/\partial t(\phi^{(1)}) - \phi_{zzz}^{(1)} \\ + \phi^{(1)}\partial/\partial z(\phi^{(1)})[1 - 3(V_0-\lambda)^2(1-\beta)^3 + \tfrac{5}{9}\sigma(1-\beta)^3] \\ - \partial/\partial z(\phi_{xx}^{(1)} + \phi_{yy}^{(1)})\{1 + (V_0-\lambda)^2/a^2[1 + \tfrac{5}{3}\sigma(1-\beta)]\} = 0 ,\end{aligned} \tag{11}$$

which is the ZK equation describing the propagation of the nonlinear wave in plasma. In the critical situation where the non-linearity in equation (11) vanishes, we get

$$(1-\beta)^3[\tfrac{5}{9}\sigma - 3(V_0-\lambda)^2] = 1 , \tag{12}$$

and the ZK equation is no longer valid. So we change the stretching parameters of the independent variables and set

$$x' = \epsilon x, \quad y' = \epsilon y, \quad z' = \epsilon(z - t), \quad t' = \epsilon^3 t. \tag{13}$$

Again proceeding in the same fashion we arrive at the modified ZK equation which is given as

$$\varphi_t - \varphi_{zzz}/[2(1-\beta^2)(V_0-\lambda)] + 3Q/[2(1-\beta)(V_0-\lambda)]\varphi^2\varphi_z$$

$$-P\partial/\partial_z(\varphi_{xx}+\varphi_{yy}) - C_2{}'(V_0-\lambda)\varphi_z + \tfrac{1}{2}a(\varphi_{xx}+\varphi_{yy}) = 0, \tag{14}$$

where

$$Q = \beta/(1-\beta) + 2(V_0-\lambda)^2(1-\beta)^2 - \tfrac{3}{2}(V_0-\lambda)^2(1-\beta) + (5\sigma/18)(1-\beta), \tag{15a}$$

$$P = 1/[2(1-\beta^2)(V_0-\lambda)]. \tag{15b}$$

But if the non-linear term in equation (11) is not identically zero but small, then the scaling (13) along with the expansions of the dependent variables leads to an equation which is a linear combination of both the ZK equation and the modified ZK equation, written as

$$\varphi_t + a\varphi\varphi_z + \tfrac{1}{2}b\varphi^2\varphi_z + \tfrac{1}{2}c\varphi_{zzz} + d\varphi_z + \tfrac{1}{2}r(\varphi_{xxz}+\varphi_{yyz}) + q(\varphi_x+\varphi_y) = 0, \tag{16}$$

where the coefficients a, b, c, d, r, q are given as follows:

$$a = 1/[(V_0-\lambda)(1-\beta^2)] - 3(V_0-\lambda)(1-\beta) + (5\sigma/9)(1-\beta)/(V_0-\lambda), \tag{17a}$$

$$b = [3/(1-\beta)(V_0-\lambda)][\beta/2(1-\beta) + 2(V_0-\lambda)^2(1-\beta)^3$$

$$- \tfrac{3}{2}(V_0-\lambda)^2(1-\beta) + (5\sigma/18)(1-\beta)], \tag{17b}$$

$$c = 1/[(V_0-\lambda)(1-\beta)^2], \tag{17c}$$

$$d = -C_2{}'(V_0-\lambda), \tag{17d}$$

$$r = [(V_0-\lambda)^4(1-\beta) + a'^2]/[(V_0-\lambda)(1-\beta)^2 a'^2], \tag{17e}$$

$$q = \tfrac{1}{2}a'; \quad a' = \omega_{\mathrm{i}}/\Omega_{\mathrm{i}}. \tag{17f}$$

We now search for a solution of this equation in the form

$$\varphi = \varphi(lx + my + nz - \omega t) = \varphi(\eta), \tag{18}$$

where η is the wavefront; here l, m, n are the direction cosines of the wave vector with respect to the x, y, z axes respectively. Under the assumption (18), equation (16) is finally reduced to an equation of the form $\varphi_{\eta\eta} + V(\eta) = 0$ or $\varphi_\eta = \chi$; $\chi_\eta + V(\eta) = 0$, where $V(\eta)$ satisfies conditions $g(0) = 0$ and $g'(0) < 0$.

Then the usual theory of dynamical systems predicts that the point (0,0) is a saddle point with local stable manifold and the unstable manifold through (0,0) coincides with the stable one, so that there is an orbit passing through (0,0) which is known as the homoclinic orbit. The existence of this orbit guarantees the existence of the type of solution we are searching for (Chow *et al.* 1980).

We proceed to search for a solution behaving like a shock wave. Such solutions do exist when the equation possesses a homoclinic orbit. The general structure of such a solution turns out to be

$$\varphi = (\varphi_{\mathrm{m}}/2)[1 - \tanh(\varphi_{\mathrm{m}}^2/2)\tfrac{1}{6}(-B)^{\frac{1}{2}}\eta]\,, \tag{19}$$

which is obtained by integration of the reduced form of (16) given as

$$(\partial\varphi/\partial\eta)^2 + \Psi(\varphi) = 0\,, \tag{20}$$

where

$$\psi = 2[A\varphi^2 + A_1\varphi^3/3 + B\varphi^4/12]/D\,, \tag{21}$$

with

$$A = (dn + pl + qm - u)\,, \tag{21a}$$

$$A_1 = an\,, \tag{21b}$$

$$B = bn\,, \tag{21c}$$

$$D = [cn^2 + p(l^2 + m^2)]n\,. \tag{21d}$$

In the above expression for the shock wave the maximum of the wave is

$$\varphi_{\mathrm{m}} = -2A_1/B \tag{22}$$

and the velocity is $u = A_1^2/3BD$. The width turns out to be $\Delta = 4(-6D/B)^{\frac{1}{2}}/|\varphi_{\mathrm{M}}|$. The width of the double layer is normalised by the Debye length and it is related to the depth d of the classical potential by the formula $\Delta = \Psi d^{-\frac{1}{2}}$. The latter follows from renormalising the 'energy law' by introducing $\phi(x) = \Psi\phi(x/\Delta)$ and $V = v(\phi(\psi))$, where $\phi(x)$ $(0 < \phi < 1)$ and $v(\phi)$ $(-1 < v < 0)$ are functions which vary on scales of order unity and depend on each other as $\phi'(\xi)^2/2 + v(\phi) = 0$. The depth d is uniquely determined by the parameters $\alpha = T_i/T_i$(trapped) of Ψ. This width depends on the value of n that is the angle between the magnetic field and the direction of propagation of the double layer.

3. Discussion

To understand the implications of these analytical results we have plotted the expressions for φ_{m} and φ (the shock wave itself) for various values of the plasma parameters, especially the new parameter β which is a measure of the non-thermalisation of the electrons. Remember that φ_{m} is the maximum value of the shock-like structure given by φ. We also depict the variation with respect to the phase velocity $(V_0-\lambda)$. In Figs 1a and 1b the variation of u and φ_m are exhibited as a function of the phase velocity. In each figure we give the case corresponding to $\beta = 0$, i.e. the purely thermalised situation. Whereas in the latter case the nature of u and φ_m shows a trend to saturate for large values of $(V_0-\lambda)$, the non-thermal case suggests a totally different behaviour. One should note that the trend of φ_m and u changes completely from the $\beta = 0$ case. Also, in the case of finite β we can get positive as well as negative values of φ_m, indicating the fact that both compressive and rarefactive shocks can be formed. Next in Figs 2a and 2b we exhibit the form of shocks with respect to η for $\beta = 0{\cdot}3$ and $0{\cdot}5$. It is quite clear that for low values of β we get rarefactive shocks, whereas for larger β values compressive shocks are generated. One should note that the presence of both compressive and refractive shocks is very important in the formation of aurora in the ionosphere. Here η actually stands for the wave front of the shock wave.

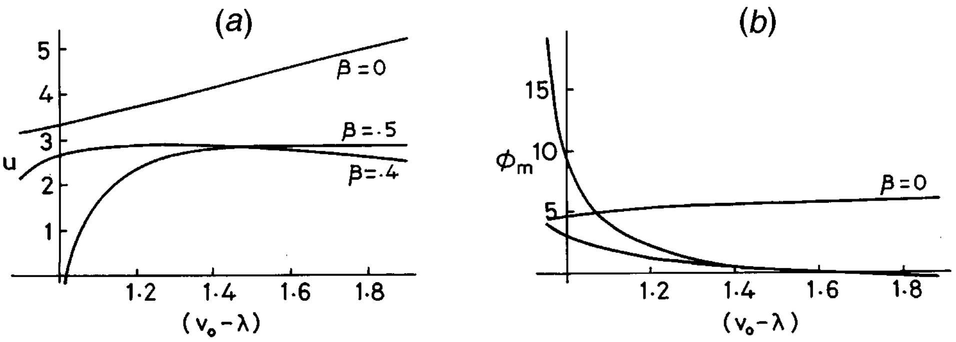


Fig. 1. Variation of (a) u and (b) φ_m with respect to $V_0-\lambda$ for various values of β.

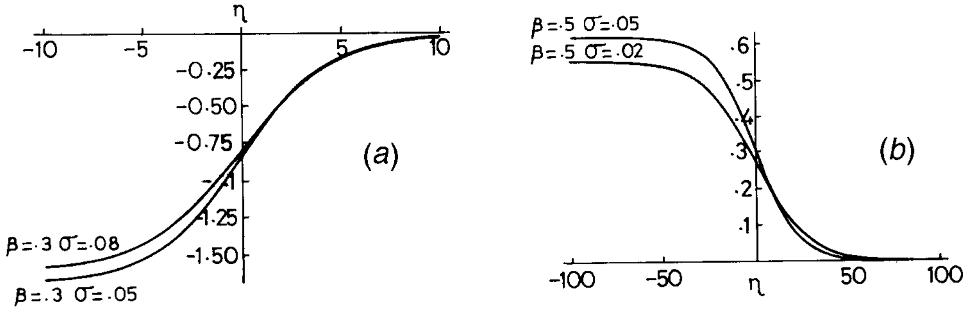


Fig. 2. Shock wave profile for (a) low β values and (b) high β values.

It may be mentioned that Mamun and Cairns (1996) did not observe any significant change in the behaviour of solitary waves due to the non-thermal nature of electrons. However, in the present situation of shock waves we find (a) the presence of both types of shocks; (b) their physical characteristics show prominent changes due to the variation of β; and (c) the effect of β actually signifies the presence of fast particles which are practically in abundance in free space. Under these circumstances it is quite natural to conclude that the effect of non-thermal electrons is to modify shock-like structures, though they do not influence stationary waves. The observations of the Freya satellite show density depletions which are due to the large population of energetic electrons. Here lies the importance of both the positive and negative potential. Our analysis actually takes care of this practical situation. The importance of such phenomena is apparent from the Freya observations as noted before. Studies have already been made on the effect of non-thermal electrons on the solitary waves (Mammun and Cairns 1996). So it seems quite relevant to study further the case of shock waves in plasmas. Our observations may help in the understanding of the origin and distribution of the parallel electric fields that accelerate electrons and produce visible aurora (Scammel 1982).

References

Alfvén, H., and Carlquist, P. (1967). *Solar Phys.* **1**, 220.
Bharuthram, R., and Shukla, P. K. (1986). *Phys. Scripta* **34**, 732.
Borovsky, J. E. (1984). *J. Geophys. Res.* **89**, 2251.
Borovsky, J. E., and Joyce, G. (1983). *J. Plasma Phys.* **29**, 45.
Bostrom, R., Gustaffson, G., Holback, B., Holmgren, G., Koskinen, H., and Kinter, P. (1988). *Phys. Rev. Lett.* **61**, 82.
Cairns, R. A., Mamun, A. A., Dendy, R., Bostren, R., and Shukla, P. K. (1995). *Geophys. Res. Lett.* **22**, 2709.
Carlqvist, P. (1986). *IEEE Trans. Plasma Sci.* **PS-14**, 794.
Chow, S. N., Hale, J. K., and Paret, J. (1980). *J. Differential Equation* **37**, 351.
Dovner, P. O., Ericson, N., Bostrom, R., and Holback, B. (1994). *Geophys. Res. Lett.* **21**, 1827.
Goswami, K. S., and Bujurbaruah, S. (1986). *Phys. Fluids* **29**, 714.
Mamun, A. A., and Cairns, R. A. (1966). *J. Plasma Phys.* **56**, 175.
Mondal, K. K., Paul, S. N., and Roy Chowdhury, A. (1998). *Aust. J. Phys.* **51**, 113.
Roy Chowdhury, K., Paul, S. N., and Roy Chowdhury, A. (1994). *Aust. J. Phys.* **47**, 785.
Saeki, K., Izuka, S., and Soto, N. (1980). *Phys. Rev. Lett.* **45**, 1983.
Scammel, H. (1982). The theory of double layers. Preprint Report No. Rio. R-472.
Temerin, M., Cerny, K., Lotko, W., and Mozer, F. S. (1982). *Phys. Rev. Lett.* **48**, 1175.
Zakharov, V. E., and Kuznetsov, E. A. (1974). *Sov. Phys. JETP* **39**, 285.

Manuscript received 20 February, accepted 24 August 1998