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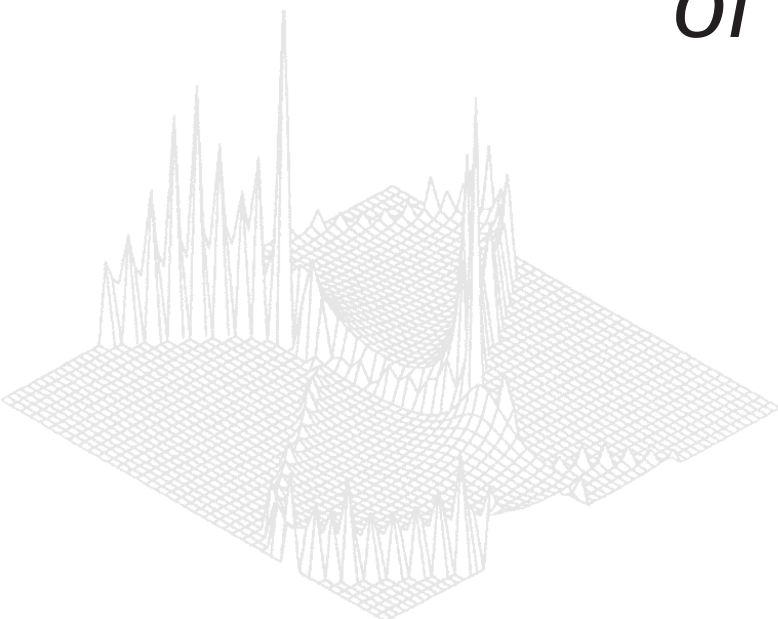

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Effect of the Motion of Ions on the Penetration of a Rotating Magnetic Field into a Plasma Cylinder

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Abstract

A simplified model for the rotating magnetic field (RMF) current drive in an infinitely long cylindrical plasma is considered. The model allows for motion of both the electron and ion fluids in the z and θ directions. It is assumed that equilibrium is satisfied on the average and hence the r components of the equations of motion are not considered. It is shown that the motion of the ion fluid does not introduce any significant modifications to the nonlinear mechanism for the penetration of the RMF into the plasma.

1. Introduction

The rotating magnetic field (RMF) current drive and its applications have been investigated in a number of laboratories over the the past two decades (Jones 1990, 1998; Ohnishi *et al*. 1998; Hoffman 1998). In this paper the effect of the motion of the ion fluid on the nonlinear mechanism for the penetration of the RMF into the plasma is reexamined.

It is assumed, for the purpose of the discussion here, that the angular frequency ω of the RMF, its magnitude B_ω and the electron–ion momentum transfer collision frequency ν_{ei} satisfy the conditions

$$\omega_{ci} \ll \omega \ll \omega_{ce}\,, \tag{1}$$

$$\nu_{ei} \ll \omega_{ce}\,, \tag{2}$$

where $\omega_{ce} = eB_\omega/m_e$ is the electron cyclotron frequency in the RMF, $\omega_{ci} = eB_\omega/m_i$ is the ion cyclotron frequency in the RMF, e is the electron charge, m_e is the electron mass and m_i is the ion mass. Under these conditions, the motion of the electron fluid is approximately flux preserving (Newcomb 1958; Hugrass 1988). For the plasma model under consideration, the only possible flux preserving motion is that of rotation at the angular frequency of the RMF.

The conditions (1) and (2) would not be satisfied inside the plasma unless the RMF penetrates into the plasma, contrary to predictions based on the classical skin effect. A nonlinear theory for the penetration of the RMF into the plasma

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was investigated in earlier studies (Jones and Hugrass 1981; Hugrass and Grimm 1981) neglecting the effect of the motion of the ion fluid. These investigations showed that the motion of the electron fluid is not exactly flux preserving, i.e. the angular velocity of the electron fluid is not exactly equal to ω. The slip between the electron fluid and the RMF $\omega - v_{e\theta}/r$ depends on the parameter ν_{ei}/ω_{ce}. The RMF induces an axial current J_z in the plasma. In turn this axial current tends to screen off the RMF (the skin effect). The effective skin depth $\delta^\star$ is equal to the skin depth calculated at the Doppler-shifted frequency of the RMF, as observed in a frame of reference rotating with the electron fluid,

$$\delta^\star = \sqrt{\frac{2\eta}{\mu_0(\omega - v_{e\theta}/r)}} = \sqrt{\frac{\omega}{\omega - v_{e\theta}/r}}\,\delta\,, \tag{3}$$

where

$$\eta = \frac{m_e \nu_{ei}}{ne^2} \tag{4}$$

is the resistivity, μ_0 is the permeability of free space and

$$\delta = \sqrt{\frac{2\eta}{\mu_0\omega}} \tag{5}$$

is the classical skin depth. Alternatively, one may consider the quantity

$$\eta^\star = \frac{\eta}{1 - v_{e\theta}/\omega r} \tag{6}$$

as the effective resistivity for the axial component of the current

$$J_z = \eta^\star E_z. \tag{7}$$

From this perspective, the penetration of the RMF into the plasma can be attributed to the large value of the effective resistivity $\eta^\star$, since the effective skin depth is given by

$$\delta^\star = \sqrt{\frac{2\eta^\star}{\mu_0\omega}}\,. \tag{8}$$

For the purpose of this work, it is more convenient to express the theory in terms of the effective resistivity because, when one allows for the motion of the ion fluid, there are two Doppler-shifted frequencies, one for the electron fluid and another for the ion fluid. It should also be noted that the penetration of the RMF into the plasma is a nonlinear phenomenon because $\eta^\star$ is much larger than η only when the electron fluid rotates almost synchronously with the RMF ($v_{e\theta} \simeq \omega r$), and this condition can only be achieved and maintained if $\omega_{ce} \gg \nu_{ei}$.

Before proceeding further, it is useful to mention here that if the effect of the electron inertia is included, the effective resistivity is complex:

$$\eta^\star = \frac{\eta}{1 - v_{e\theta}/\omega r} + j\frac{m_e\omega}{e^2 n}\,, \tag{9}$$

and the effective skin depth is given by

$$\frac{1}{\delta^\star} = \mathrm{Re}\left[\sqrt{\frac{j\omega\mu_0}{\eta^\star}}\right]. \tag{10}$$

Equation (10) reduces to (8) when the electron inertia term is negligible. This is a good approximation provided that

$$\omega \ll \frac{\nu_{ei}}{1 - v_{e\theta}/\omega r}\,. \tag{11}$$

In the limit of a collisionless plasma, the electron inertia term is dominant and the skin depth is known as the electron collisionless skin depth:

$$\delta_e = \sqrt{\frac{m_e}{\mu_0 n e^2}}\,. \tag{12}$$

The purpose of this work is to investigate the effect of the motion of the ion fluid on the above results which were obtained assuming that the ions are immobile. Intuitively, the effect of the motion of the ion fluid may be estimated as follows. The motion of the ion fluid is approximately described by the equation

$$n m_i \frac{dv_{iz}}{dt} = neE_z\,. \tag{13}$$

When the electron fluid rotates at the angular velocity of the RMF, the electrons do not contribute to the axial current and hence the screening current can be obtained from equation (13) as

$$J_z = nev_{iz} = \frac{ne^2}{j\omega m_i}E_z\,. \tag{14}$$

This reasoning leads to the conclusion that the RMF does not significantly penetrate into the plasma for distances greater than the ion collisionless skin depth

$$\delta_i = \sqrt{\frac{m_i}{\mu_0 n e^2}} \simeq \frac{10^8}{n^{\frac{1}{2}}}\,, \tag{15}$$

where n is the number density in m^{-3}. It will be shown in Section 3 that this intuitive approach is not consistent with the results obtained by formally solving the relevant equations of motion.

2. The Physical Model

An infinitely long cylindrical plasma is considered. The fields consist of a steady axial magnetic field $B_z(r)$ and a transverse rotating magnetic field

$$\mathbf{B} = B_r(r)\exp[j(\omega t-\theta)]\mathbf{a}_r + B_\theta(r)\exp[j(\omega t-\theta)]\mathbf{a}_\theta + B_z\mathbf{a}_z\,, \tag{16}$$

where $\mathbf{a}_r$, $\mathbf{a}_\theta$ and $\mathbf{a}_z$ are the unit vectors in the r, θ and z directions in a standard cylindrical system of coordinates. Both B_r and B_θ are in general complex. In the absence of the plasma B_r is constant and pure real [which means that $B_r(r,\theta,t) = B_\omega\cos(\omega t-\theta)$] and B_θ is constant and pure imaginary. In the presence of the plasma the magnitudes and phases of B_r and B_θ are functions of r.

For the purpose of simplifying the analysis the motion of the electron and ion fluids in the r direction will not be considered. Such motion takes place on a relatively longer time scale and its inclusion would complicate the analysis and obscure the physical picture.

It was shown in a previous paper (Hugrass 1982) that the physical quantities relevant to our physical model can be divided into two groups. The first group includes B_z and the transverse components of the electric field, the current density and fluid velocities. These quantities can be expressed as superpositions of constant parts (dc) and even harmonics of $(\omega t-\theta)$:

$$Q(r,\theta,t) = Q_0(r) + \Sigma_m Q_m(r)\exp[j2m(\omega t-\theta)]\,. \tag{17}$$

The second group includes B_r, B_θ and the z components of the electric field, the current density and fluid velocities. These quantities can be written as superpositions of odd harmonics of $(\omega t-\theta)$. The effects of the second and higher harmonics are in general small and will be neglected in the remainder of this paper. Quantities of the first group are assumed to depend only on r and quantities of the second group are assumed to vary as $f(r)\exp[j(\omega t-\theta)]$; hence

$$\frac{\partial}{\partial t} = j\omega, \qquad \frac{\partial}{\partial\theta} = -j \tag{18}$$

for quantities of the second group.

3. The Equations of Motion

The RMF satisfies Faraday's law

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}\,. \tag{19}$$

Using equations (18) and (19) one obtains

$$E_z(r) = \omega r B_r(r)\,. \tag{20}$$

The equations of motion for the electron and ion fluids are

$$m_e n\left(\frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e\cdot\nabla\mathbf{v}_e\right) = -en(\mathbf{E} + \mathbf{v}_e\times\mathbf{B}) - \nabla P_e + m_e n\nu_{ei}(\mathbf{v}_i - \mathbf{v}_e)\,, \tag{21}$$

$$m_i n\left(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i\cdot\nabla\mathbf{v}_i\right) = en(\mathbf{E} + \mathbf{v}_i\times\mathbf{B}) - \nabla P_i + m_e n\nu_{ei}(\mathbf{v}_e - \mathbf{v}_i)\,. \tag{22}$$

The z components of the equations of motion are

$$jm_e\omega n\left(1 - \frac{v_{e\theta}}{\omega r}\right)v_{ez} = -en\left(1 - \frac{v_{e\theta}}{\omega r}\right)E_z + \frac{m_e\nu_{ei}}{e}J_z\,, \tag{23}$$

$$jm_i\omega n\left(1 - \frac{v_{i\theta}}{\omega r}\right)v_{iz} = en\left(1 - \frac{v_{i\theta}}{\omega r}\right)E_z - \frac{m_e\nu_{ei}}{e}J_z\,, \tag{24}$$

where equation (20) has been used to write B_r in terms of E_z. Equations (23) and (24) can be written as

$$v_{ez} = \frac{-e}{j\omega m_e}E_z + \frac{e}{j\omega m_e}\frac{\eta}{1 - v_{e\theta}/(\omega r)}J_z\,, \tag{25}$$

$$v_{iz} = \frac{e}{j\omega m_i}E_z - \frac{e}{j\omega m_i}\frac{\eta}{1 - v_{i\theta}/(\omega r)}J_z\,, \tag{26}$$

Using equations (25) and (26) together with

$$J_z = en(v_{zi} - v_{ze})$$

one obtains

$$E_z = \eta^\dagger J_z\,, \tag{27}$$

where

$$\eta^\dagger = \frac{m_i}{m_i + m_e}\left(\frac{\eta}{1 - v_{e\theta}/(\omega r)} + \frac{m_e}{m_i}\frac{\eta}{1 - v_{i\theta}/(\omega r)} + j\frac{m_e\omega}{e^2 n}\right). \tag{28}$$

Note that in deriving equation (28) one does not need to assume that (1) and (2) are satisfied. When the magnitude of the RMF is such that $\omega_{ce} \gg \nu_{ei}$, the electron fluid rotates almost synchronously with the RMF, and $\eta^\dagger$ is much larger than η. The effective skin depth is, in this case, much larger than the classical skin depth and the RMF penetrates into the plasma accordingly.

4. Discussion and Conclusions

The effective resistivity in the z direction is given by equation (6) when the motion of the ion fluid is neglected, and by equation (28) when the motion of the ion fluid is taken into account. It is evident that the effect of the motion of the ion fluid is of order m_e/m_i and can in general be neglected. Equation (28) however seems a little paradoxical, as it is in sharp disagreement with the intuitive reasoning presented at the end of Section 1, and further discussion is in order. It is helpful to consider the form which this equation takes for the simpler (but perhaps of little interest) case where the electron and ion fluids rotate at the same angular velocity v_θ/r:

$$\eta^\dagger = \frac{\eta}{1 - v_\theta/\omega r} + j\frac{m_e m_i \omega}{(m_e + m_i)e^2 n}\,. \tag{29}$$

In this case the inertial (second) term is obtained, from its value for the fixed ion model, by replacing the electron mass m_e by the reduced mass $m_e m_i/(m_e + m_i)$. This modification is both plausible and consistent with the linear theory.

The collisional (first) term is the same as that obtained from the fixed ion model. This can be explained as follows. When the motion of the ion fluid is included, the ions contribute to the axial current. However, this contribution is cancelled out by the change in the axial component of the velocity of the electron fluid which results from a change in the collisional drag from $-m_e\nu_{ei}v_{ez}$ to $m_e\nu_{ei}(v_{iz} - v_{ez})$. The z component of the current density, everything else being the same, remains unchanged to order m_e/m_i and hence the effective resistivity is not changed to order m_e/m_i.

Finally, it should be stated that most of the results presented in this paper are implicitly stated in Hugrass (1982), assuming $\omega_{ce} \gg \nu_{ei}$. This assumption is shown here to be unnecessary.

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