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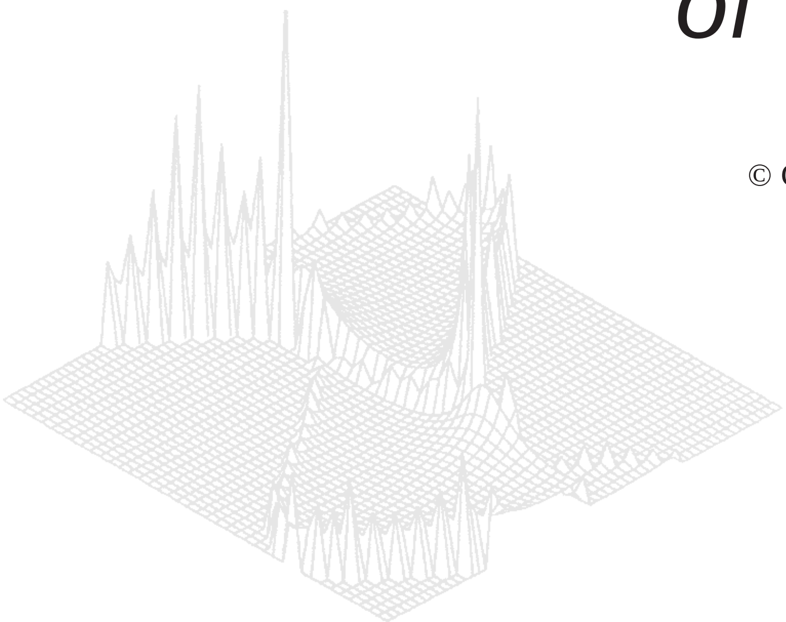

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Nonlinear Dynamics of the Dust Grain Charge and Dust Acoustic Waves in a Plasma with an Ion Beam

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Abstract

The temporal evolution of the dust grain charge is investigated for the first time in a dusty plasma with an ion beam. The grain charge attains to the stationary state after the fluctuation of short time. The dust charge increases as the ion beam temperature and the plasma ion density increase, but decreases as the beam density increases. The speed of the wave increases as the ion density and temperature increase, whereas it decreases as the dust charge increases. The variable dust mass-to-charge and ion-to-electron density ratios govern the existence of dust acoustic waves. The findings of this investigation may be useful in understanding laboratory plasma phenomena.

1. Introduction

There has been a growing interest in investigating new properties of dusty plasmas containing charged, micrometre-sized dust grains which have been observed not only in space environments (Hartquist *et al*. 1992; Tsytovich and Havnes 1993), but also in laboratory devices (Barkan *et al*. 1995; Bingham *et al*. 1991; Boufendi *et al*. 1992). The dust grains have variable charge and mass due to fragmentation and coalescence, and are charged due to the local electrons and ions. In actual situations, the variable grain charge and electrostatic waves have been studied over a range of frequencies in the laboratory (Chu *et al*. 1994; Thompson *et al*. 1997; Sugai *et al*. 1997). The ion beams in laboratory dusty plasmas have become indispensable in the field of materials processing such as etching chemical vapour deposition and surface modification (Sugai *et al*. 1997). Such circumstances in plasma applications and the ease of realising dusty plasmas on a laboratory scale have accelerated active studies on dust phenomena in plasmas. On the other hand, for low frequency nonlinear waves, the dust grains can be described as negative ions with large mass and large charge (Goertz and Morfill 1983; Rao *et al*. 1990). Ion and dust acoustic waves, as well as dust acoustic instabilities, in dusty plasmas have been studied theoretically by several authors (D'Angelo 1990; Bharuthram *et al*. 1992; Rosenberg 1993; Shukla 1995). Recently the topics of nonlinear grain charge variation and electrostatic ion waves (Nejoh 1997*a*, 1997*b*, 1997*c*, 1998*a*, 1998*b*) have been reported by regarding dust grains as point charges, where the Debye length is much larger than the inter-grain distance. Therefore, since the dust charge variation affects

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the characteristics of the collective motion of the plasma, the effect of variable charge dust grains is of crucial importance in understanding nonlinear waves excited in dusty plasmas. However, the dependence of the grain charge and dust acoustic waves on the ion beam velocity, density and temperature has not been investigated theoretically in dusty plasmas.

In this paper, we focus our attention on the characteristics of the grain charge and electrostatic dust acoustic waves in an unmagnetised dusty plasma with a positive ion beam. It is instructive to examine the variation of the grain charge and the effects of the beam velocity, density and temperature in dusty plasmas. We derive nonlinear equations for variable charge dust grains from a set of basic equations, and the Sagdeev potential, in Section 2. In Section 3, performing the numerical calculation of the nonlinear equations obtained in Section 2, we show the temporal evolution of grain charge, and the dependence of the dust charge on the dust mass-to-charge ratio, plasma potential, ion-to-electron temperature and density ratios, and ion-beam velocity and temperature in the stationary state. Our results show the nonlinearly variable dust charge and the existence of the dust acoustic waves. The last section is devoted to a concluding discussion.

2. Theory

We consider the nonlinear dust charge variation and the one-dimensional propagation of nonlinear dust acoustic waves in an unmagnetised dusty plasma. Our plasma model is as follows. Electrons and positive ions form the Boltzmann distribution. A positive ion beam flows at the uniform streaming velocity in the isothermal state. Since it is assumed that coalescence of electrons to dust grains occurs more frequently than the collision of beam ions and dust grains, we ignore the latter. The radius r of a spherical dust grain is assumed to be much less than the inter-grain distance, the electron Debye length λ_{D} and the wavelength of the waves. Thus the dust grains can be considered as heavy immobile point masses. It is assumed that the variation of the dust charge is due to the microscopic electron, ion and ion-beam currents flowing onto the grains because of the potential difference between the grain surface and the adjacent plasma. The plasma considered here consists of four components, i.e. Boltzmann-distributed electrons with a temperature T_{e}, warm positive ions having a temperature T_{i}, a positive ion beam with a temperature T_{b}, and a negatively-charged, heavy, cold dust fluid. The dynamics of the ion beam and dust fluids are governed by the continuity and momentum equations. We also assume that low frequency electrostatic waves propagate in this system.

The number densities of electrons and ions are assumed to be

$$n_{\mathrm{e}} = n_{\mathrm{e0}}\exp(e\phi/T_{\mathrm{e}})\,, \tag{1}$$

$$n_{\mathrm{i}} = n_{\mathrm{i0}}\exp(-e\phi/T_{\mathrm{i}})\,. \tag{2}$$

The continuity equation and the equation of motion for an ion beam are described by

$$\frac{\partial n_{\mathrm{b}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{b}}v_{\mathrm{b}}) = 0\,, \tag{3a}$$

$$\frac{\partial v_{\rm b}}{\partial t} + v_{\rm b}\frac{\partial v_{\rm b}}{\partial x} + \frac{T_{\rm b}}{m_{\rm b}n_{\rm b}}\,\frac{\partial n_{\rm b}}{\partial x} + \frac{e}{m_{\rm b}}\,\frac{\partial \phi}{\partial x} = 0\,, \tag{3b}$$

where we express the pressure term in (3b) by the isothermal equation of state. It is noted that the collision between the beam ions and dust grains is ignored. Here, $T_{\rm b}$ is the ion-beam temperature. The quantities $m_{\rm i}$ and e are ion mass and the magnitude of electron charge respectively.

For one-dimensional low frequency motions, we have the following two equations for cold dust grains:

$$\frac{\partial n_{\rm d}}{\partial t} + \frac{\partial}{\partial x}(n_{\rm d}v_{\rm d}) = 0\,, \tag{4a}$$

$$\left(\frac{\partial}{\partial t} + v_{\rm d}\frac{\partial}{\partial x}\right)v_{\rm d} - \frac{Q_{\rm d}}{m_{\rm d}}\,\frac{\partial \phi}{\partial x} = 0\,. \tag{4b}$$

Here $Q_{\rm d}$ ($= eZ_{\rm d}$) is the variable charge of dust grains, where $Z_{\rm d}$ is the charge number measured in units of e.

The Poisson equation is given as

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0}(n_{\rm e} - n_{\rm i} - n_{\rm b} + Z_{\rm d}n_{\rm d})\,, \tag{5}$$

where ϵ_0 denotes the permittivity of the vacuum. The variables $n_{\rm b}$, $n_{\rm d}$, $n_{\rm i}$, $v_{\rm b}$, $v_{\rm d}$ and ϕ refer to the ion-beam density, dust-grain density, ion density, ion-beam velocity, dust velocity and electrostatic potential respectively. We define the velocities of the electrons, positive ions, beam ions and dust grains in equilibrium as $v_{\rm e0}$, $v_{\rm i0}$, v_0 and $v_{\rm d0}$. We assume that $v_{\rm e,th} \gg v_{\rm ph} \gg v_{\rm i0}$, $v_0 \gg v_{\rm d,th}$, where $v_{\rm e,th}$ ($v_{\rm d,th}$) and $v_{\rm ph}$ are the electron (dust) thermal velocity and phase velocity of the dust acoustic waves respectively. The dust thermal velocity, in general, is much less than the wave phase velocity because of the massive dust grains. The ion-beam velocity v_0 is assumed to be less than the phase velocity $v_{\rm ph} \approx (Z_{\rm d}T_{\rm e}/m_{\rm d})^{1/2}$. Since the dust acoustic instability is brought about by the condition $v_0 > v_{\rm ph}$ (Rosenberg 1993), the dust acoustic instability does not occur in our system. It is also assumed that the ion-beam velocity is much less than the beam thermal velocity. We consider that $v_{\rm e0}$, $v_{\rm i0}$, $v_0 \neq 0$ in equilibrium and this imples the origin of the electron, ion and ion-beam currents. At infinity, $x \to \infty$, $v_{\rm e0}$, $v_{\rm i0}$, $v_{\rm d0} = 0$ and $v_0 \neq 0$ are assumed. Charge neutrality at equilibrium requires that $n_{\rm i0} + n_{\rm b0} = n_{\rm e0} + n_{\rm d0}Z_{\rm d}$, where $n_{\rm i0}$ ($n_{\rm b0}$, $n_{\rm d0}$) denotes the equilibrium ion (ion beam, dust grain) density.

We normalise all the physical quantities as follows. The densities, space coordinate x, time t, velocities and electrostatic potential ϕ are normalised by the background electron density $n_{\rm e0}$, the dust Debye length $\lambda_{\rm Dd} = (\epsilon_0 T_{\rm eff}/n_{\rm d0}e^2)^{1/2}$, the inverse dust plasma period $\omega_{\rm pd}^{-1} = (\epsilon_0 m_{\rm d}/n_0(eZ_{\rm d})^2)^{1/2}$, the dust acoustic velocity $v_{\rm DA} = (T_{\rm eff}/m_{\rm d})^{1/2}$ and $T_{\rm eff}/Q_{\rm d}$ respectively. The effective temperature is determined to $Z_{\rm d}n_{\rm d0}/T_{\rm eff} = n_{\rm e0}/T_{\rm e} + n_{\rm b0}/T_{\rm b} + n_{\rm i0}/T_{\rm i}$. Then, the basic equations described above can be written in the non-dimensional form

$$n_{\rm e} = \exp(\alpha_{\rm e}\phi)\,, \tag{6}$$

$$n_{\rm i} = \delta_{\rm i}\exp(-\alpha_{\rm i}\phi)\,, \tag{7}$$

$$\frac{\partial n_{\rm b}}{\partial t} + \frac{\partial}{\partial x}(n_{\rm b}v_{\rm b}) = 0\,, \tag{8a}$$

$$\frac{\partial v_{\rm b}}{\partial t} + v_{\rm b}\frac{\partial v_{\rm b}}{\partial x} + \frac{\tau_{\rm b}}{n_{\rm b}}\frac{\partial n_{\rm b}}{\partial x} + \frac{\mu_{\rm d}}{Z_{\rm d}}\frac{\partial \phi}{\partial x} = 0\,, \tag{8b}$$

$$\frac{\partial n_{\rm d}}{\partial t} + \frac{\partial}{\partial x}(n_{\rm d}v_{\rm d}) = 0\,, \tag{9a}$$

$$\frac{\partial v_{\rm d}}{\partial t} + v_{\rm d}\frac{\partial v_{\rm d}}{\partial x} - \frac{\partial \phi}{\partial x} = 0\,, \tag{9b}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\rm e} - n_{\rm i} - n_{\rm b} + Z_{\rm d}n_{\rm d}\,, \tag{10}$$

where $\tau_{\rm b} = T_{\rm b}/T_{\rm e}$, $\delta_{\rm i} = n_{\rm i0}/n_{\rm e0}$, $\alpha_{\rm e}\ (\alpha_{\rm i}) = T_{\rm eff}/Z_{\rm d}T_{\rm e}(T_{\rm eff}/Z_{\rm d}T_{\rm i})$ and $\mu_{\rm d} = m_{\rm d}/m_{\rm b}$.

In this system, the ordering $m_{\rm d} \gg m_{\rm i} \gg m_{\rm e}$ holds, as obtained in laboratory plasmas. Typical laboratory plasma frequencies are 10^2 Hz : 10^{5-6} Hz : 10^{9-10} Hz, and have roughly the same ordering as the mass ratios (Selwyn *et al.* 1990; Chu *et al.* 1994; Thompson *et al.* 1997; Sugai *et al.* 1997). Thus, the inclusion of the mass ratios is equal to considering the collective motion of dust grain particles (Nejoh 1997*b*, 1997*c*).

We assume that the charging of the dust grain particles arises from plasma currents due to the electrons, ions and an ion beam reaching the grain surface for spherical grains of radius r. In this case, the variable dust grain charge is determined by the charge current balance equation:

$$\frac{\rm d}{{\rm d}t}Q_{\rm d} = I_{\rm e} + I_{\rm i} + I_{\rm b}\,, \tag{11}$$

where

$$I_{\rm e} = -e\pi r^2\sqrt{\frac{8T_{\rm e}}{\pi m_{\rm e}}}n_{\rm e}(\phi)\exp\left(\frac{e\Phi_{\rm s}}{T_{\rm e}}\right), \tag{12}$$

$$I_{\rm i} = e\pi r^2\sqrt{\frac{8T_{\rm i}}{\pi m_{\rm i}}}n_{\rm i}(\phi,\tau_{\rm i})\left(1 - \frac{e\Phi_{\rm s}}{T_{\rm i}}\right), \tag{13}$$

$$I_{\rm b} = e\pi r^2\sqrt{\frac{8T_{\rm b}}{\pi m_{\rm b}}}n_{\rm b}(\phi,\tau_{\rm b})\,, \tag{14}$$

where I_e, I_i and I_b denote the electron, ion and ion beam current, and the dust grain surface potential is $\Phi \equiv Q_\mathrm{d}/4\pi\epsilon_0 r$ relative to the plasma potential. In order to derive the ion current I_i, we used the orbital limited motion theory (Goertz 1989). We normalise the electron, and ion-beam currents by $e\pi r^2(8T_\mathrm{e}/\pi m_\mathrm{e})^{1/2}n_\mathrm{e0} \approx e\pi r^2 v_\mathrm{e,th}n_\mathrm{e0}$. In order to express all the physical quantities in non-dimensional form, we use the definition $\beta = 1+\delta_\mathrm{i}/\tau_\mathrm{i}+\delta_\mathrm{b}/\tau_\mathrm{b}$, $\Delta = \delta_\mathrm{i}+\delta_\mathrm{b}-1$, where $\delta_\mathrm{b} = n_\mathrm{b0}/n_\mathrm{e0}$ and r is the average radius for spherical dust grains. Assuming that $\Phi_\mathrm{s} \equiv Q_\mathrm{d}/C_\mathrm{s}$, we can regard $C_\mathrm{s} = 4\pi\epsilon_0 r$ as the capacitance on the surface of the grains. Considering the assumptions of the orbital limited motion for plasma ions with the Boltzmann distribution and the constant flow velocity of beam ions, we have the normalised expressions for equations (12)–(14):

$$I_\mathrm{e} = -\, n_\mathrm{e}(\phi)\exp(aZ_\mathrm{d})\,, \tag{15}$$

$$I_\mathrm{i} = \sqrt{\frac{\tau_\mathrm{i}}{\mu_\mathrm{i}}}n_\mathrm{i}(\phi,\tau_\mathrm{i})\left(1-\frac{aZ_\mathrm{d}}{\tau_\mathrm{i}}\right), \tag{16}$$

$$I_\mathrm{b} = \sqrt{\frac{\tau_\mathrm{b}}{\mu_\mathrm{b}}}n_\mathrm{b}(\phi,\tau_\mathrm{b})\,, \tag{17}$$

where $a = e^2/4\pi\epsilon_0 rT_\mathrm{e}$, $\mu_\mathrm{i} = m_\mathrm{i}/m_\mathrm{e}$, $\mu_\mathrm{b} = m_\mathrm{b}/m_\mathrm{e}$ and $\tau_\mathrm{i} = T_\mathrm{i}/T_\mathrm{e}$.

In order to solve (1)–(5), we introduce the variable $\xi = x - Mt$, which is the moving frame with the velocity M normalised by the dust acoustic velocity v_DA. We study the one-dimensional propagation of the nonlinear dust acoustic wave in this system. For this purpose, deriving a set of equations in the moving frame, and using the boundary conditions, $\phi\to 0$, $n_\mathrm{d}\to\Delta/Z_\mathrm{d}$, $n_\mathrm{i}\to\delta_\mathrm{i}$, $n_\mathrm{b}\to\delta_\mathrm{b}$, $v_\mathrm{b}\to v_0$, $v_\mathrm{i}\to 0$ and $v_\mathrm{d}\to 0$ at $\xi\to\infty$, we obtain the ion-beam and dust-grain densities as

$$n_\mathrm{b} = \delta_\mathrm{b}\Big/\sqrt{1-\frac{2\mu_\mathrm{d}\phi}{Z_\mathrm{d}[(M-v_0)^2-\tau_\mathrm{b}]}}\,, \tag{18}$$

$$n_\mathrm{d} = \frac{\Delta}{Z_\mathrm{d}}\Big/\sqrt{1+\frac{2\phi}{M^2}}\,. \tag{19}$$

The derivation of the ion-beam density (18) is given in the Appendix.

In the non-stationary state, considering the charge current balance equation (11), we can derive a nonlinear equation for the variable charge of dust grains as

$$\begin{aligned}\frac{d(aZ_\mathrm{d})}{dt*} =& -\exp(\alpha_\mathrm{e}\phi + aZ_\mathrm{d}) + \delta_\mathrm{i}\sqrt{\frac{\tau_\mathrm{i}}{\mu_\mathrm{i}}}\exp(-\alpha_\mathrm{i}\phi)\left(1-\frac{aZ_\mathrm{d}}{\tau_i}\right)\\ &+\sqrt{\frac{\tau_\mathrm{b}}{\mu_\mathrm{b}}}\delta_\mathrm{b}\Big/\sqrt{1-\frac{\mu_\mathrm{d}}{Z_\mathrm{d}}\frac{2\phi}{[(M-v_0)^2-\tau_\mathrm{b}]}}\,,\end{aligned} \tag{20}$$

where $t* = t/\lambda_{\mathrm{Dd}} r^{-1} \omega_{\mathrm{pe}}^{-1}$. Here we note that $\alpha_{\mathrm{e}} = \Delta/\beta Z_{\mathrm{d}}$, $\alpha_{\mathrm{i}} = \Delta/\tau_{\mathrm{i}}\beta Z_{\mathrm{d}}$, $\beta = 1 + \delta_{\mathrm{i}}/\tau_{\mathrm{i}} + \delta_{\mathrm{b}}/\tau_{\mathrm{b}}$ and $\Delta = \delta_{\mathrm{i}} + \delta_{\mathrm{b}} - 1$. Equation (20) includes strongly nonlinear terms. Then, we can obtain the solution of (20) by numerical calculation in the next section. At the stationary state, from $\mathrm{d}Q_{\mathrm{d}}/\mathrm{d}t* = 0$, equation (20) reduces to

$$\exp(\alpha_{\mathrm{e}}\phi + aZ_{\mathrm{d}}) = \delta_{\mathrm{i}}\sqrt{\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{i}}}}\exp(-\alpha_{\mathrm{i}}\phi)\left(1 - \frac{aZ_{\mathrm{d}}}{\tau_{\mathrm{i}}}\right)$$

$$+ \sqrt{\frac{\tau_{\mathrm{b}}}{\mu_{\mathrm{b}}}}\delta_{\mathrm{b}} \Big/ \sqrt{1 - \frac{\mu_{\mathrm{d}}}{Z_{\mathrm{d}}}\frac{2\phi}{[(M - v_0)^2 - \tau_{\mathrm{b}}]}}\,. \tag{21}$$

Next, in order to confirm the possibility of the existence of dust acoustic waves, we derive the law of conservation of energy from (10). Integration of the Poission equation gives the *Energy Law*, $(\partial\phi/\partial\xi)^2/2 + V(\phi) = 0$, with

$$V(\phi) = \frac{1}{\alpha_{\mathrm{e}}}\{1 - \exp(\alpha_{\mathrm{e}}\phi)\} + \frac{\delta_{\mathrm{i}}}{\alpha_{\mathrm{i}}}\{1 - \exp(-\alpha_{\mathrm{i}}\phi)\} + \Delta M^2\left(1 - \sqrt{1 + \frac{2\phi}{M^2}}\right)$$

$$+ \delta_{\mathrm{b}}\frac{Z_{\mathrm{d}}[(M - v_0)^2 - \tau_{\mathrm{b}}]}{\mu_{\mathrm{d}}}\left\{1 - \sqrt{\frac{2\mu_{\mathrm{d}}\phi}{Z_{\mathrm{d}}[(M - v_0)^2 - \tau_{\mathrm{b}}]}}\right\}. \tag{22}$$

The oscillatory solution of the electrostatic dust acoustic waves exists when the following conditions are satisfied:

(i) If $V(\phi)$ satisfies the condition $\mathrm{d}^2V(\phi)/\mathrm{d}\phi^2 < 0$ at $\phi = 0$, the velocity of nonlinear waves propagating in this system can be determined. This condition gives rise to

$$-1 - \frac{\delta_{\mathrm{i}}}{\tau_{\mathrm{i}}} + \frac{\delta_{\mathrm{b}}}{\alpha_{\mathrm{e}}}\frac{1}{\{(M - v_0)^2 - \tau_{\mathrm{b}}\}} + \frac{\Delta}{\alpha_{\mathrm{e}}M^2} < 0\,. \tag{23}$$

(ii) Nonlinear dust acoustic waves exist only when $V(\phi_M) \geq 0$, where the maximum potential is determined by $\phi_{\mathrm{M}} = -M^2/2$. This implies that the following inequality holds:

$$\frac{1}{\alpha_{\mathrm{e}}}\{1 - \exp(-\alpha_{\mathrm{e}}M^2/2)\} + \frac{\delta_{\mathrm{i}}}{\alpha_{\mathrm{i}}}\{1 - \exp(\alpha_{\mathrm{i}}M^2/2)\} + \Delta M^2$$

$$+ \delta_{\mathrm{b}}\frac{Z_{\mathrm{d}}[(M - v_0)^2 - \tau_{\mathrm{b}}]}{\mu_{\mathrm{d}}}\left\{1 - \sqrt{\frac{\mu_{\mathrm{d}}M^2}{Z_{\mathrm{d}}[(M - v_0)^2 - \tau_{\mathrm{b}}]}}\right\} \geq 0\,. \tag{24}$$

We show the speed M of dust acoustic waves as a function of the ratio of the ion to electron density $\delta_{\rm i}$ in Fig. 1, from (23), in the case of $Z_{\rm d} = 100$ (solid lines) and 50 (dotted lines), where $r = 10^{-6}$ m, $a = 1\cdot4 \times 10^{-3}$, $\delta_{\rm b} = 1\cdot0$, $\tau_{\rm b} = 2\cdot0$, $\mu_{\rm i} = 1836$, $\mu_{\rm d} = 10^{11}$, $v_0 = 0\cdot1$ and $0\cdot05$. The thick and thin lines denote the ion temperature $\tau = 0\cdot2$ and $0\cdot1$ respectively. For example, a dust grain radius of 1 μm and mass density 2000 kg m^{-3} has a mass $\sim 5\times10^{-15}$ kg so that $\mu_{\rm d} \approx 10^{11}$. Using the Landau length $e^2/4\pi\epsilon_0 T_{\rm e} = 1\cdot4 \times 10^{-9}$ m for $T_{\rm e} = 1$ eV, we obtain $a = e^2/4\pi\epsilon_0 r T_{\rm e} = 1\cdot4 \times 10^{-3}$ for the gain radius $r = 10^{-6}$ m. It is noted that the parameters $a = 1\cdot4 \times 10^{-3}$, $\delta_{\rm b} = 1\cdot0$, $\tau_{\rm b} = 2\cdot0$, $\mu_{\rm i} = 1836$, $\mu_{\rm d} = 10^{11}$ and $v_0 = 0\cdot1$ are all dimensionless. When the ion-to-electron density ratio $\delta_{\rm i}$ becomes large, the speed of the dust acoustic waves increases. An increase of the ion temperature increases the wave speed.

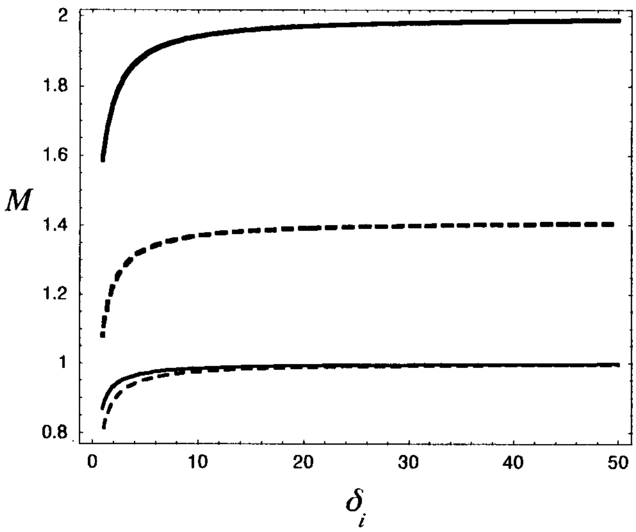

Fig. 1. Velocity (Mach number) of dust acoustic waves as a function of the ion-to-electron density ratio for the case of $Z_{\rm d} = 100$ (solid lines) and 50 (dotted lines), where $a = 1\cdot4\times10^{-3}$, $\delta_{\rm b} = 1\cdot0$, $\tau_{\rm b} = 2\cdot0$, $\mu_{\rm i} = 1836$, $\mu_{\rm d} = 10^{11}$, $\mu_{\rm b} = 39 \times 1836$ and $v_0 = 0\cdot1$. The thick and thin lines denote the ion temperature ratio $\tau_{\rm i} = 0\cdot2$ and $0\cdot1$ respectively.

3. Numerical Results

The aim of this section is to investigate non-stationary and stationary properties of the grain charge and Sagdeev potential for the existence of nonlinear waves. It is noted that, in the case where $m_{\rm d} = 2\cdot0 \times 10^{-16}$ kg, $Z_{\rm d} = 10^3$, $T_{\rm e} = 1\cdot0$ eV, $T_{\rm i} = 0\cdot4$ eV, the dust thermal velocity is $v_{\rm d,th} = 0\cdot004$ m s^{-1}, the dust acoustic velocity $v_{\rm DA} \approx 1\cdot0$ m s^{-1} and the wave phase velocity $v_{\rm ph} \approx (Z_{\rm d}T_{\rm e}/m_{\rm d})^{1/2} \approx 1$ m s^{-1}. If we assume K$^+$ ions as the ion-beam component, the beam thermal velocity is $\approx 11\cdot4$ m s^{-1}. When the beam velocity is $\approx 0\cdot1$ m s^{-1}, we obtain $v_0 = 0\cdot1$ because the velocity is normalised by the dust acoustic velocity $v_{\rm DA} = (T_{\rm eff}/m_{\rm d})^{1/2}$. Since the conditions for the velocities, which are stated after (5), are satisfied properly under these circumstances, we note that the dust acoustic instability due to the streaming ion beam does not occur in our system. In the following, in order to calculate equations (20)–(24), we use the parameters $a = 1 \cdot 4 \times 10^{-3}$, $\delta_{\rm b} = 1 \cdot 0$, $\tau_{\rm b} = 2 \cdot 0$, $\mu_{\rm i} = 1836$, $\mu_{\rm d} = 10^{11}$, $\mu_{\rm b} = 39 \times \mu_{\rm i}$ for K^+ ions, and $v_0 = 0 \cdot 1$.

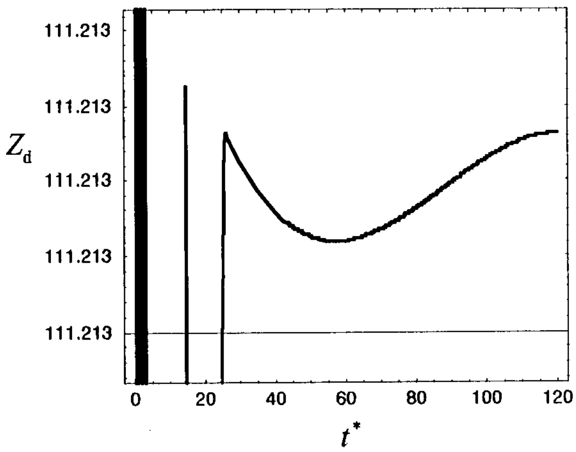

Fig. 2. A non-stationary property of the grain charge with $a = 1 \cdot 4 \times 10^{-3}$, $\delta_{\rm b} = 1 \cdot 0$, $\tau_{\rm b} = 2 \cdot 0$ and $v_0 = 0 \cdot 1$.

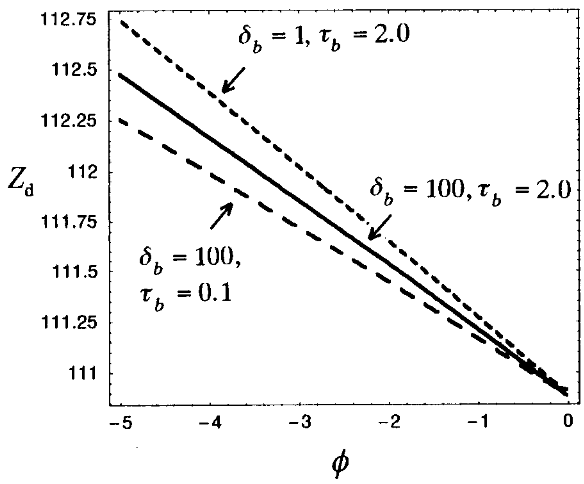

Fig. 3. A $Z_{\rm d}$–ϕ plane in the case of $\delta_{\rm b} = 1 \cdot 0$, $\tau_{\rm b} = 2 \cdot 0$ (dotted line) and $\delta_{\rm b} = 100$, $\tau_{\rm b} = 2 \cdot 0$ (solid line) and $\delta_{\rm b} = 100$, $\tau_{\rm b} = 0 \cdot 1$ (dashed line), where $a = 1 \cdot 4 \times 10^{-3}$, $\delta_{\rm i} = 500$, $\tau_{\rm i} = 0 \cdot 2$, $M = 0 \cdot 9$ and $v_0 = 0 \cdot 1$.

First, using (20) we show the temporal evolution of the grain charge $Z_{\rm d}$ in Fig. 2, where $\tau_{\rm i} = 0 \cdot 2$, $\delta_{\rm i} = 500$ and $M = 1 \cdot 6$. In this case, the grain charge attains to the stationary state after the fluctuation of a short time. The fluctuating time, as is seen from Fig. 2, is $t = 120 \times \lambda_{\rm D}(r\omega_{\rm pe})^{-1} = 0 \cdot 79$ s for

$r = 10^{-6}$ m, $n_e = 10^{10}$ m^{-3} and $T_e = 0\cdot4$ eV. In order to study the dependence of the dust charge on the potential, we illustrate a Z_d–ϕ plane in Fig. 3, in the case of $\delta_b = 1\cdot0$, $\tau_b = 2\cdot0$ (dotted line), $\delta_b = 100$, $\tau_b = 2\cdot0$ (solid line) and $\delta_b = 100$, $\tau_b = 0\cdot1$ (dashed line), where $\delta_i = 500$, $\tau_i = 0\cdot2$ and $M = 0\cdot9$. The dust grain charge increases as the beam temperature increases and beam density decreases. Fig. 4 illustrates the dependence of the dust charge number on the ion-to-electron density ratio δ_i, in the case of $\phi = -1\cdot0$ (solid line) and $-0\cdot1$ (dotted line), where $\tau_i = 0\cdot2$ and $M = 0\cdot8$. We find that the increase of the ion density increases the dust grain charge.

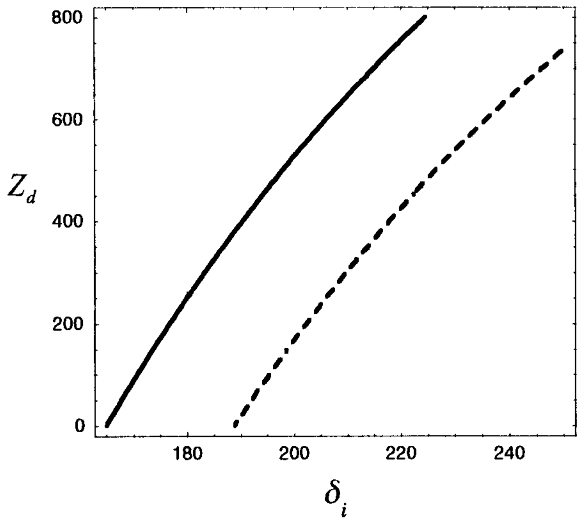

Fig. 4. Dependence of the dust charge number Z_d on the ion-to-electron density ratio δ_i in the case of $\phi = -1\cdot0$ (solid line) and $-0\cdot1$ (dotted line), where $a = 1\cdot4 \times 10^{-3}$, $\delta_b = 1\cdot0$, $\tau_i = 0\cdot2$, $\tau_b = 3\cdot0$, $M = 0\cdot8$ and $v_0 = 0\cdot1$.

Second, Fig. 5*a* shows a bird's-eye view of the potential $V(\phi)$ depending on the dust mass-to-charge ratio μ_d/Z_d, where $\tau_i = 0\cdot1$, $\delta_i = 50$ and $M = 0\cdot8$. We show a two-dimensional $V(\phi)$–ϕ plane for $\mu_d/Z_d = 5 \times 10^8$ in Fig. 5*b* in the case $\delta_i = 50$ (solid line) and 100 (dotted line). It turns out that the potential $V(\phi)$ strongly depends on the dust grain mass-to-charge and ion-to-electron density ratios, and that the amplitude of the dust acoustic waves grows when the ion density increases.

4. Discussion

We have investigated the effect of nonlinear dust charging on dust acoustic waves in a dusty plasma with an ion beam. We find that, unlike the ordinary electron–positive ion plasmas, the quite dense positive ion, the dust mass-to-charge ratio, the ion-beam density and temperature govern the collective motion of dust grains. We summarise the remarkable properties of the grain charge and dust acoustic waves as follows:

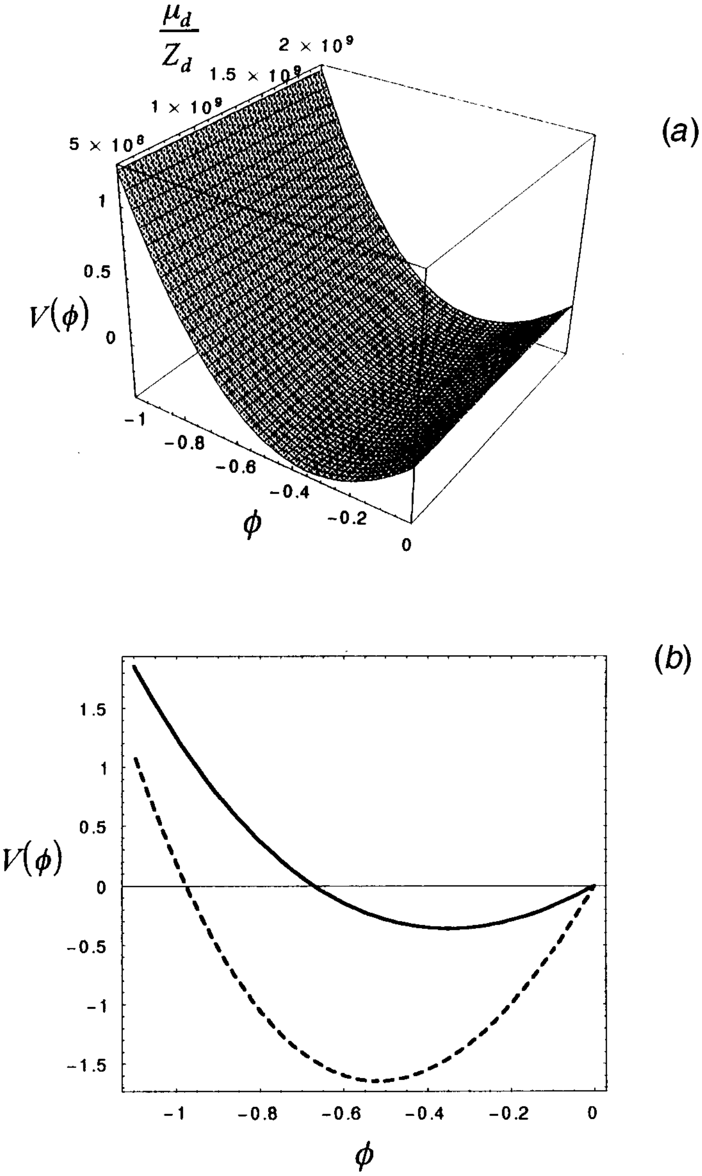

Fig. 5. (a) A bird's-eye view of the Sagdeev potential $V(\phi)$ and its dependence on the mass to charge ratio $\mu_{\rm d}/Z_{\rm d}$ and the electrostatic potential ϕ, where $a = 1{\cdot}4 \times 10^{-3}$, $\delta_{\rm b} = 1{\cdot}0$, $\tau_{\rm i} = 0{\cdot}1$, $\tau_{\rm b} = 3{\cdot}0$, $\delta_{\rm i} = 50$, $M = 0{\cdot}8$ and $v_0 = 0{\cdot}1$. (b) A $V(\phi)$–ϕ plane in the case of $\mu_{\rm d}/Z_{\rm d} = 5 \times 10^8$ in Fig. 5a, where the solid and dotted lines correspond to $\delta_{\rm i} = 50$ and 100 respectively.

(1) *Speed of the wave*: An increase in the ion density increases the speed of the wave. An increase of the ion temperature increases the wave velocity, but an increase of the dust charge decreases it.

(2) *Grain charge*: The temporal evolution of the grain charge is shown in a dusty plasma with an ion beam. The dust charge drastically fluctuates within $0{\cdot}79$ s from the commencement of the charging, and afterward it attains to the stationary state. In this state, an increase in the ion density increases the dust charge. The charge number increases as the beam density decreases and the beam temperature increases.

(3) *Existence of the waves*: We show the possibility for the propagation of dust acoustic waves by the calculation of the Sagdeev potential. It turns out that the amplitude of the wave increases as the ion density increases.

We point out that the model considered here is structurally unstable, in the sense that a small change in the parameters or inclusion of small additional effects will not produce just a small change in the solution, but completely change its nature, for example from a double layer to a solitary wave (Nejoh 1997*a*, 1998*a*). Considering such a viewpoint, the results presented here are highly suggestive in discussing the properties of ion-beam plasma systems with highly-charged, heavy, micrometre-sized dust grains. As possible examples, nonlinear dust acoustic waves are present in dusty plasmas, which have been observed in laboratory plasmas (Selwyn *et al*. 1990; Sugai *et al*. 1997). They may serve as a source of improvement in the etching rate of plasma processing. This investigation would be effective in understanding the properties of grain charging and dust acoustic waves with the positive ion beam.

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Appendix: Derivation of the Ion-beam Density (18)

It is necessary to derive the ion-beam density in the sense that the nonlinear variation of the dust charge depending on the ion-beam density is very important. The ion-beam density is required for investigation of the dust grain charge variation and the existence of nonlinear dust acoustic waves.

We consider the equation of continuity and the equation of motion with the ion-beam pressure gradient under the assumption of isothermal beam ions. The ion-beam pressure is expressed as $p_{\mathrm{b}} = k_{\mathrm{B}} T_{\mathrm{b}} n_{\mathrm{b}}$ in the isothermal process, where k_{B} denotes the Boltzmann constant. Using the beam pressure and (3b) and normalising, we obtain (8b) associated with the equation of continuity of ions (8a). In order to investigate the stationary collective motion, we introduce the moving frame $\xi = x - Mt$ with the velocity M in this system. Integrating (8a) and (8b) once and using the boundary conditions, we can obtain

$$\frac{n_{\mathrm{b}}}{n_{\mathrm{b0}}} = \frac{v_0 - M}{v_{\mathrm{b}} - M}\,, \tag{25}$$

$$(v_{\mathrm{b}} - M)^2 - (v_0 - M)^2 + 2\phi + 2\tau_{\mathrm{b}} \ln \frac{n_{\mathrm{b}}}{n_{\mathrm{b0}}} = 0\,. \tag{26}$$

We assume that the quasineutrality condition holds,

$$\left(\frac{n_{\mathrm{b}}}{n_{\mathrm{b0}}}\right)^2 = 1 + \delta\left(\frac{n_{\mathrm{b}}}{n_{\mathrm{b0}}}\right), \tag{27}$$

where $\delta(n_{\mathrm{b}}/n_{\mathrm{b0}})$ denotes the normalised perturbation of beam ions, and $1 \gg \delta(n_{\mathrm{b}}/n_{\mathrm{b0}})$. Substituting (25) into (26) and using (27), we reduce (25) to

$$\delta\left(\frac{n_{\mathrm{b}}}{n_{\mathrm{b0}}}\right) = \frac{2\phi}{(M - v_0)^2 - \tau_{\mathrm{b}} - 2\phi}\,,$$

where we neglect the higher-order terms of $n_{\mathrm{b}}/n_{\mathrm{b0}}$ in the expansion of the $\ln(n_{\mathrm{b}}/n_{\mathrm{b0}})$. Returning to the quasineutrality conditions, we can easily obtain the ion-beam density (18).

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