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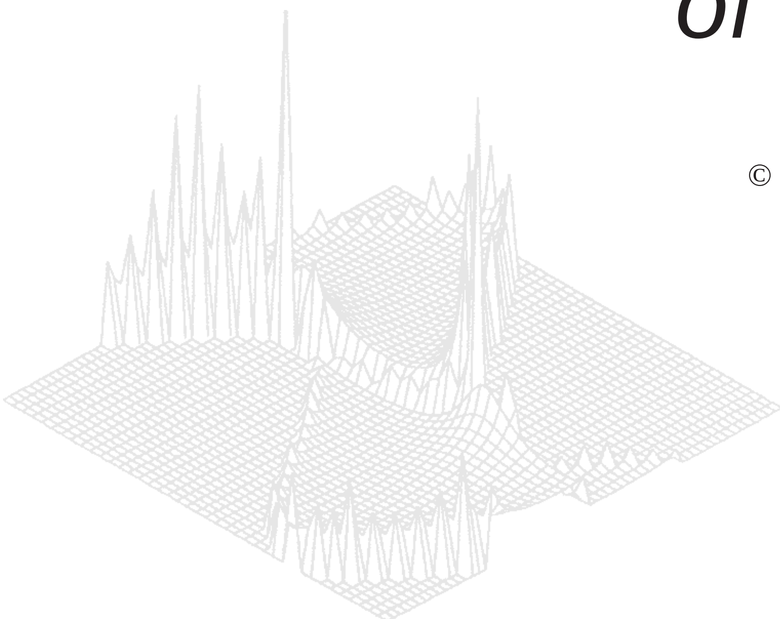
Australian Journal of Physics

Volume 52, 1999



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Aust. J. Phys., 1999, **52**, 49–58

Electron Acceleration by a Laser Pulse with a Rising Propagation Speed

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Abstract

The acceleration of an electron by a circularly polarised laser pulse propagating at a time-rising speed in a plasma is studied. The limitation on the electron energy gain due to dephasing may disappear. The characteristics of this non-dephasing direct acceleration scheme are analysed.

1. Introduction

The possibility of utilising the fields of an intense laser beam to accelerate particles to high energies has attracted a great deal of interest (Esarey *et al.* 1996; Sprangle *et al.* 1996). There is a basic difference between acceleration in a laser field which includes plasma effects (Esarey *et al.* 1996) and acceleration without a plasma. The latter is called 'acceleration in vacuum' (Hora 1988; Cicchitelli and Hora 1990; Scheid and Hora 1989; Häuser, Scheid and Hora 1994), or 'free-wave acceleration' (Woodworth *et al.* 1996). Electron acceleration by lasers without plasma effects has the advantage that well-known difficulties of instability and detuning due to plasma properties can be avoided. The first clear experimental demonstration of this type of acceleration was by Malka *et al.* (1997), where the identical theoretical formulations of that by Scheid and Hora (1989) were demonstrated. Recently, it has also been found by numerical simulation that the electron can be captured and violently accelerated by an extra-intense laser beam with $Q_0 \gtrsim 100$ ($Q_0 = eE_0/m_e\omega c$) (Wang *et al.* 1998), which is a breakthrough in laser-driven electron acceleration in a vacuum, and is of potential interest to far-field laser acceleration.

On the other hand, the ponderomotive force associated with a short laser pulse plays a dominant role in the dynamics of electrons in laser fields. The longitudinal component of the ponderomotive force is expected to be used to directly accelerate electrons. An electron can gain a large amount of longitudinal momentum at the peak of the pulse (Bardsley *et al.* 1989). However, it is difficult to extract the particle from the pulse before the pulse completely overtakes the particle. So, no net energy gain can be obtained because the deceleration effect of the back of the pulse offsets the acceleration effect of the leading edge of the pulse. Recently, Mckinstrie and Startsev (1996) studied this direct acceleration in a homogeneous plasma where the pulse propagation speed is less

 10.1071/PH98086 0004-9506/99/010049$05.00

than vacuum speed c, and showed that the pulse will be overtaken by the particle if the intensity is above a threshold which is dependent on the particle injection energy. Preaccelerated electrons can be accelerated to high energy and can be extracted easily. Although this direct acceleration scheme is less than ideal, because the pulse can generate a parasitic wake (Marqes *et al.* 1996; Siders *et al.* 1996), its simplicity is noteworthy. However, like many other laser-driven acceleration schemes (Esarey *et al.* 1996; Sprangle *et al.* 1996), dephasing between the particle and the pulse (Mckinstrie and Startsev 1997) always exists because the propagation speed of the pulse is assumed to be constant. Although the dephasing is essential to particle extraction, it causes two limitations. The first is that the intensity of the pulse must be above the threshold to keep the back of the pulse from catching the particle, or it will gain zero energy after it is completely overtaken by the pulse. The second is that even in the case in which the intensity of the pulse is above the threshold, the energy gain cannot be high if the intensity of the pulse and the particle injection energy are both low. In this paper we study the case where the pulse propagation speed is not constant but rises during the interaction time, and show that the above limitations may not exist. Thus it is possible to accelerate a particle to very high energy.

An identical question about laser acceleration with variation of the pulse speed has been discussed by Hora (1988), where two collinear laser beams are used and the laser intensity gradients are moved in phase with the accelerated particles by electro-optical modulation of the frequency and/or phase of the beams. A similar problem was also discussed by Cicchitelli and Hora (1990), where the intensity minima of the interference fields or of the standing wave fields are moved in a controlled way. In this paper, we concentrate on the direct acceleration of a charged particle by a short laser pulse where the variation of the pulse speed is due to the plasma inhomogeneity in the pulse propagation direction.

The outline of this paper is as follows. In Section 2, we study the motion of an electron in an electromagnetic field if is associated with a pulse moving through a plasma, divided by an interface into two regions which are different in density. We show that the energy gain can be largely improved due to the variation in the pulse speed. In Section 3 we further study the case in which the plasma density is smoothly varied spatially. The ideal density profile for speed-synchronised acceleration (in which there is no dephasing between the pulse and the particle) is derived. The main results of the paper are summarised in Section 4.

2. Particle Motion at a Plasma Interface

The motion of a particle of charge q and mass m in an electromagnetic field is governed by the equations (Mckinstrie and Startsev 1996)

$$d_\tau(\mathbf{v}+\mathbf{a}) = 0\,, \tag{1}$$

$$d_\tau u = -\partial_x(v^2/2)\,, \tag{2}$$

$$d_\tau \gamma = \partial_t(v^2/2)\,, \tag{3}$$

where $\mathbf{v}$, $\mathbf{u}$ and γ are the transverse momentum, the longitudinal momentum and the energy of the particle, respectively, and t and τ are the time mutiplied by c and the proper time mutiplied by c respectively. If the particle is initially in front of the pulse and not moving transversely, then

$$\mathbf{v} = -\mathbf{a}\,. \tag{4}$$

The above equations are for any density profile of the plasma medium. Consider the typical case in which the density profile is

$$n = n_1 \;\; (x < 0), \quad n = n_2 \;\; (x > 0)\,, \tag{5}$$

where $n_2 < n_1$. In a rarefied plasma the reflection of the interface can be neglected. A circularly polarised field is assumed:

$$a_n = \sqrt{\tfrac{1}{2}}(0, 0, a_n(\psi_n)\cos\phi_n, a_n(\psi_n)\sin\phi_n)\,, \tag{6}$$

where $n = 1, 2$ indicates region 1 and region 2, respectively, and

$$\phi_n = t - s_n x, \quad \psi_n = t - r_n x\,. \tag{7}$$

Similar to Mckinstrie and Startsev (1996), it follows from equations (2)–(4), (6) and (7) that

$$d_\tau(r_n\gamma - u) = 0\,. \tag{8}$$

We assume that the particle moves with momentum u_0, energy γ_0, along the x-direction at the initial time $\tau = 0$, and reaches the interface ($x = 0$) at time $\tau = \tau_I$. When $\tau < \tau_I$, combining equation (8) with the definition of γ, one can show that

$$\gamma = \frac{r_1(r_1\gamma_0 - u_0) \mp \omega_1}{r_1^2 - 1}\,, \tag{9}$$

$$u = \frac{r_1\gamma_0 - u_0 \mp r_1\omega_1}{r_1^2 - 1}\,, \tag{10}$$

where

$$\omega_1 = [(r_1\gamma_0 - u_0)^2 - (r_1^2 - 1)(1 + \mathbf{v}^2)]^{\frac{1}{2}}\,. \tag{11}$$

When $\tau > \tau_I$, γ and u are

$$\gamma = \frac{r_2(r_2\gamma_I - u_I) \mp \omega_2}{r_2^2 - 1}\,, \tag{12}$$

$$u = \frac{r_2\gamma_I - u_I \mp r_2\omega_2}{r_2^2 - 1}\,, \tag{13}$$

where

$$\omega_2 = [(r_2\gamma_I - u_I)^2 - (r_2^2 - 1)(1 + \mathbf{v}^2)]^{\frac{1}{2}}\,, \tag{14}$$

and where the subscript I indicates the interface. Here γ_I and u_I are the energy and longitudinal momentum associated with the particle at the interface. Note that $v \to 0$ as $\tau \to \infty$, following from (9)–(14), and u and γ are determined analytically by the injection energy γ_0 and the pulse intensity a_I^2, which is the intensity around the particle when the particle arrives at the interface. In (9), (10), (12) and (13), the minus signs apply to the cases in which $\gamma > ru$, which corresponds to a particle that is moving more slowly than the pulse, and the minus signs apply to the case in which $\gamma < u$, which corresponds to a particle that is moving more quickly than the pulse. The repel thresholds (Mckinstrie and Startsev 1996) a_B^2 (in region 1) and a_E^2 (in region 2) can be readily found from equations (11) and (14) by letting $\omega_1 = 0$ and $\omega_2 = 0$ respectively:

$$a_B^2 = 2[(r_1\gamma_0 - u_0)^2/(r_1^2 - 1) - 1]\,, \tag{15}$$

$$a_E^2 = 2[(r_2\gamma_I - u_I)^2/(r_2^2 - 1) - 1]\,. \tag{16}$$

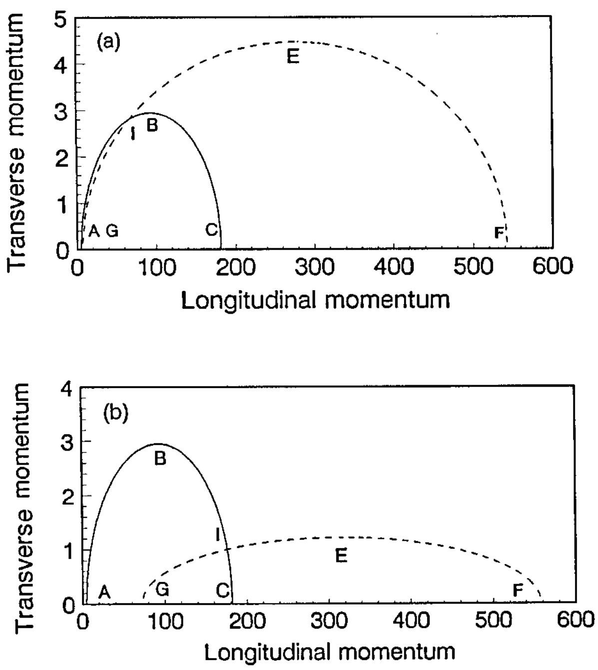

Fig. 1. Relation between the longitudinal and transverse momentum implied by equations (10)–(15). The solid curve corresponds to the motion of the particle in region 1 of the medium, while the broken curve corresponds to that in region 2 of the medium, where $\gamma_{p1} = 30$ and $\gamma_0 = 5$. Other parameters are: (*a*) $\gamma_{p2} = 60, v_I = 2{\cdot}8$ and (*b*) $\gamma_{p2} = 200, v_I = 1$. The momentum is measured in units of the particle rest mass.

The relationship between the longitudinal and transverse momentum is illustrated in Fig. 1, where a representative point moves along the solid curve initially and turns to the broken curve after the particle passes the interface. The final points are F and G, which correspond to the case in which the pulse intensity is above and below the repel threshold a_E^2 respectively.

If $u_I/\gamma_I < 1/r_1$ and the intensity of the pulse is below the threshold a_E^2, then the particle will be completely overtaken by the pulse at last. But the particle gains nonzero energy because the positive energy gain in the acceleration period will not be offset by the negative one in the deceleration period. The relationship between u and v is illustrated in Fig. 1*a*. This residual energy is almost zero if the particle is initially at rest, but it increases significantly with an increase of the injection energy, which is illustrated in Fig. 2.

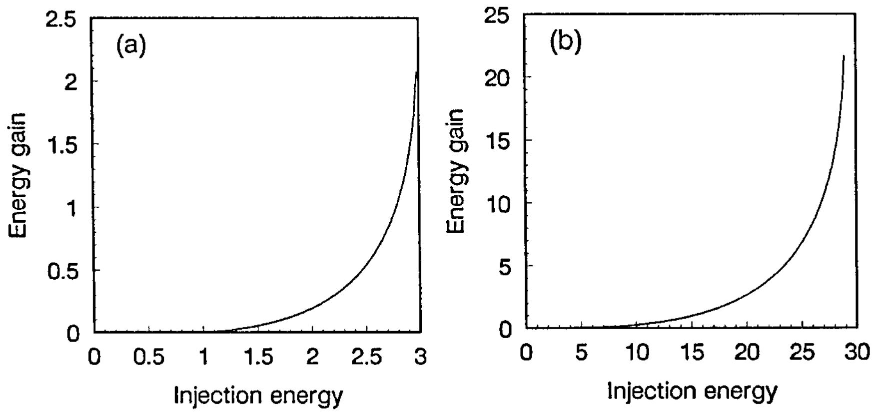

Fig. 2. Particle energy gain for the case in which the particle is overtaken by the pulse completely, where $a_I^2 = 5$. Other parameters are: (*a*) $\gamma_{p1} = 10, \gamma_{p2} = 100$ and (*b*) $\gamma_{p1} = 100, \gamma_{p2} = 1000$. The energy is measured in units of the particle rest mass.

If $1/r_2 > u_I/\gamma_I > 1/r_1$ and the intensity of the pulse is above the threshold a_E^2, then the particle will eventually overtake the pulse. The particle can gain a much greater energy than in the case where the pulse speed is constant. The relationship between u and v is illustrated in Fig. 1*b*. In this case if $v_I = 0$ (which corresponds to the case where the particle is out of the pulse field in the time between the two acceleration periods), according to equations (9) and (12), the energy gains $\delta\gamma_1$ (in region 1) and $\delta\gamma_2$ (in region 2) are

$$\delta\gamma_1 = 2({u_{p1}}^2\gamma_0 - \gamma_{p1}u_{p1}u_0)\,, \tag{17}$$

$$\delta\gamma_2 = 2({u_{p2}}^2\gamma_1 - \gamma_{p2}u_{p2}u_1)\,, \tag{18}$$

where $\gamma_1 = \gamma_0 + \delta\gamma_1$. The whole energy gain $\delta\gamma = \delta\gamma_1 + \delta\gamma_2$ depends on the injection energy γ_0, which is illustrated in Figs 3 and 4. In contrast to the case where the pulse speed is constant, the pulse intensity threshold and particle energy gain do not decrease monotonically as the injection energy increases, and the energy gain is considerable even in the case where the repel threshold is not very high.

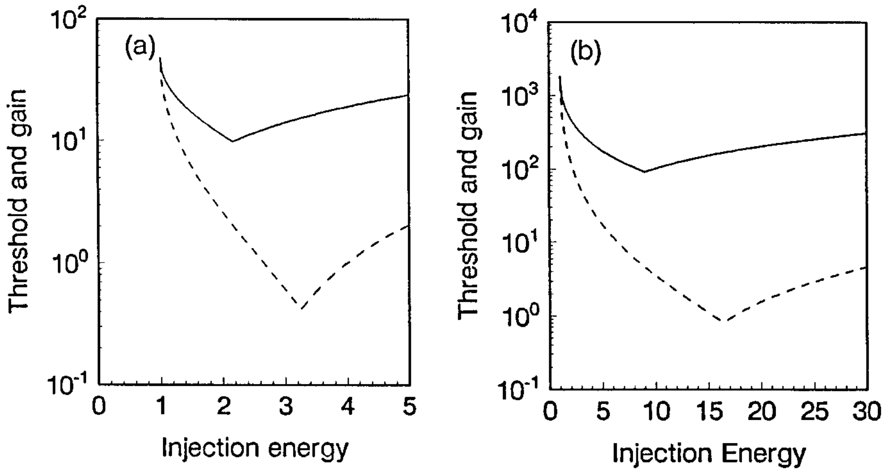

Fig. 3. Critical intensity a_{ct}^2 [$=\max(a_B^2, a_E^2)$] of the pulse (dashed curve) and the corresponding gain in particle energy (solid curve) plotted as function of the particle injection energy, where $a_I^2 = 0$. Other parameters are: (a) $\gamma_{p1} = 5, \gamma_{p2} = 12$ and (b) $\gamma_{p1} = 30, \gamma_{p2} = 200$. The energy is measured in units of the particle rest mass.

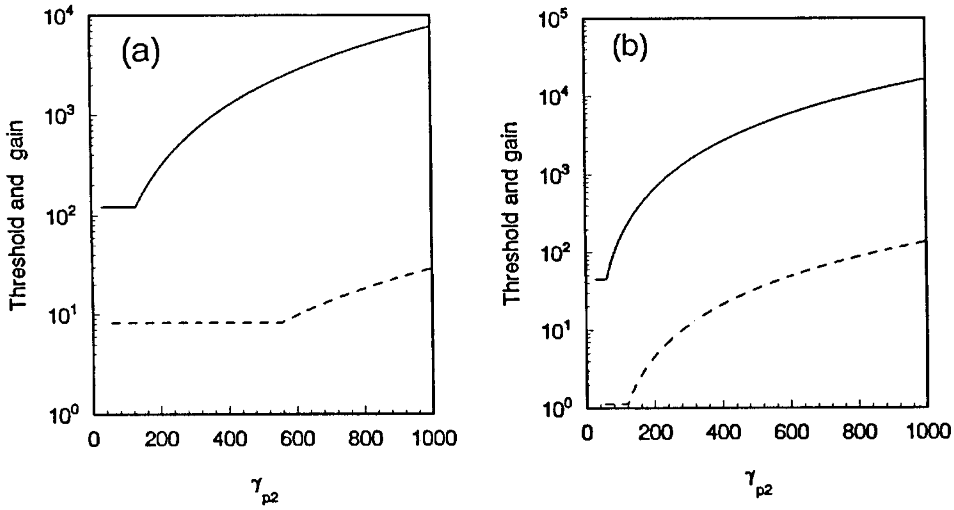

Fig. 4. Critical intensity a_{ct}^2 [$=\max(a_B^2, a_E^2)$] of the pulse (dashed curve) and the corresponding gain in particle energy (solid curve) plotted as a function of the Lorenz factor γ_{p2} associated with the pulse in region 2, where $\gamma_{p1} = 30$ and $a_I^2 = 0$: (a) the case of $\gamma_{p2} = 7$ and (b) the case of $\gamma_{p2} = 15$. The energy is measured in units of the particle rest mass.

3. Effects of Spatially Smooth Variations in Medium Density

In the previous section we studied the motion of a particle in an electromagnetic field in a plasma with the density profile (5). Although the energy gain could be improved greatly in this density profile, it is too simple to be ideal. In this section we study the case of a short, circularly polarised laser pulse propagating in a plasma with a smoothly varying density profile. We assume that $L\partial\epsilon/\partial x \ll 8\pi\epsilon$, where $\epsilon = 1 - \omega_p^2/\omega^2$ defines the dielectric function of the plasma medium, and L is the pulse length. It is obvious in this case that the condition for the WKB approximation (Kruer 1988, p. 30) is satisfied. So the field of the pulse can be written as

$$a = \sqrt{\tfrac{1}{2}}(0, 0, a(\psi)\cos\phi, a(\psi)\sin\phi)\,, \tag{19}$$

where

$$\phi = t - \int^x s(x')dx', \quad \psi = t - \int^x r(x')dx'\,. \tag{20}$$

It follows from (2)–(4), (19) and (20) that

$$rd_\tau\gamma - d_\tau u = 0\,. \tag{21}$$

Equation (21) is different from equation (5) in Mckinstrie and Startsev (1996) because r is not constant here. A general relationship between u and v cannot be given because the trajectory of the particle in momentum space (u, v) depends on the plasma density profile and the pulse profile.

Combining equations (21), (2) and (3), one can rewrite (2), (3) as

$$d_\tau u = r\frac{\partial}{\partial\psi}(v^2/2)\,, \tag{22}$$

$$d_\tau\gamma = \frac{\partial}{\partial\psi}(v^2/2)\,. \tag{23}$$

For convenience, we define the relativistic ponderomotive force by $f_p = \partial/\partial\psi(v^2/2)$, which is different from the usual force defined by the right-hand side of equation (22). However, the difference is unimportant because r is very close to 1. Note that v^2 is independent of ϕ, and we can see that f_p is only dependent on ψ. In general, f_p varies with τ due to ψ-slippage, but it is an important case in which f_p remains constant (does not decrease) during the interaction time. In this case, following from $d_\tau f_p = 0$, one can obtain $\partial f_p/\partial\psi = 0$ or $d_\tau\psi = 0$. In the former case $v^2 = \psi/L^2$ for $0 < \psi < L$, but f_p cannot remain nonzero out of this range because of ψ-slippage. In the latter case there is no dephasing and so $d_\tau v^2 = 0$, i.e. v is invariant, and f_p can be kept nonzero. It follows from (21) that $\gamma = ru$, i.e. the particle is speed-synchronised with the pulse. Note that $d_\tau = \gamma d_t$, and so from (23) one can obtain

$$d_t\gamma^2 = 2f_p\,. \tag{24}$$

We assume that the speed-synchronised acceleration begins at time $t = t_0$, when the particle energy is γ_0, longitudinal momentum is u_0, and transverse momentum is $\mathbf{v}_0$ $(= -\mathbf{a}_0)$. The energy of the particle γ at time t is

$$\gamma = \sqrt{\gamma_0^2 + 2f_p(t - t_0)}\,. \tag{25}$$

For the case of a linear group velocity (Decker and Mori 1995), the density profile $n(x)$ is determined by $n/n_c = 1 - 1/r^2$, where n_c is the plasma critical density which is defined by $\omega_0 = \sqrt{4\pi e^2 n_c/m_0}$. We assume that $x = 0$ at $t = t_0$. From $d_\tau x = u$, the acceleration distance x can be given by

$$x = \frac{1}{2f_p}\left(\gamma u - \gamma_0 u_0 - (1 + v_0^2)\ln\left(\frac{\gamma + u}{\gamma_0 + u_0}\right)\right). \tag{26}$$

The particle energy γ increases with x which is illustrated in Fig. 5*a*, and the density n decreases with x which is illustrated in Fig. 5*b*.

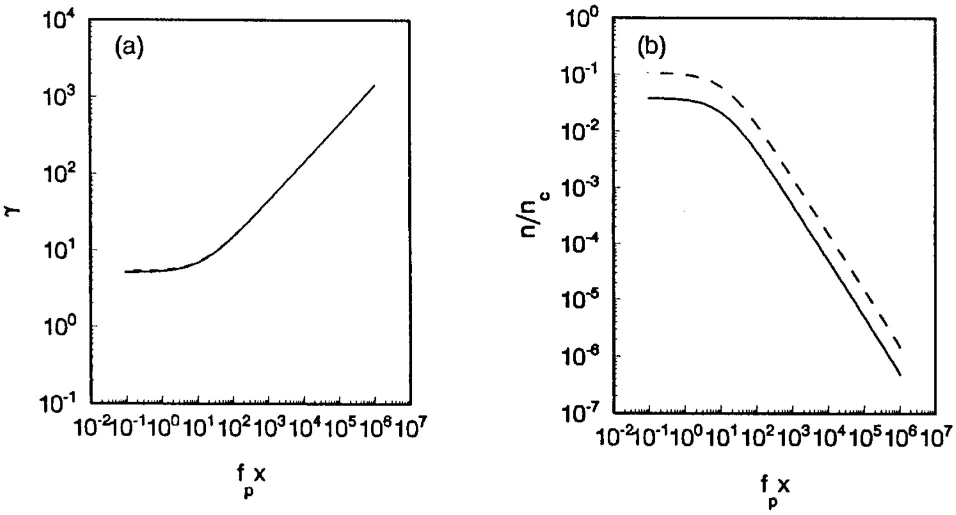

Fig. 5. (*a*) Particle energy γ versus normalised acceleration distance $f_p x$ and (*b*) the corresponding spatial density profile of the medium in speed-synchronised acceleration, where $\gamma_0 = 5$. The solid curve corresponds to the case of $v_0^2 = 0$ and the broken curve to the case of $v_0^2 = 2$. The energy is measured in units of the particle rest mass.

For the case of $1 + v_0^2 \ll \gamma_0^2$, the last term in equation (26) is much less than the others. Neglecting this term leads to $x = t - t_0$, and from (25) one can obtain

$$\gamma = \sqrt{\gamma_0^2 + 2f_p x}\,, \tag{27}$$

$$n(x)/n_c = \frac{1 + v_0^2}{\gamma_0^2 + 2f_p x}\,. \tag{28}$$

The particle will be speed-synchronised with the pulse in the case where the density profile is represented by equation (28), where no dephasing limits the particle energy gain. The particle can be accelerated to very high energy if the density profile is as ideal as possible. On the other hand, the acceleration gradient will significantly decrease when the particle is accelerated to very high energy, which is caused by the relativistic characteristic of the ponderomotive force associated with the pulse.

4. Conclusion and Discussion

In conclusion, we have studied the acceleration of an electron by a circularly polarised laser pulse propagating at a time-rising speed in a plasma. In contrast to the case of constant pulse propagation speed (Mckinstrie and Startsev 1996), the gain in electron energy is generally nonzero and, in the case where the electron eventually outruns the pulse, one can greatly improve the energy gain by choosing an appropriate density profile of the plasma. In the ideal case where the plasma density profile is choosen as that represented by equation (28), the electron can be speed-synchronised with the pulse where the electron energy γ increases with x as in equation (27).

We emphasise that in this direct acceleration scheme, a high intensity for the pulse is not essential, because there is no dephasing between the test particle and the pulse, but preacceleration is necessary. In our analysis the pulse is assumed to be short enough that the motion of ions can be neglected. In the absence of collective effects, after the pulse completely overtakes the particle which is initially at rest, the residual energy of the particle is almost zero, as illustrated in Fig. 2. In general, the pulse energy will not be lost in background electrons as long as we choose the density profile to be adaptable only to the acceleration of the injection electrons. In the presence of collective effects, however, the wake field of the pulse will influence the motion of the particle. One needs to choose the pulse duration judiciously (Mckinstrie and Startsev 1996) to eliminate the wake. In addition, in our analysis the pulse field is assumed to be a plane wave and a 1-D model is used. Considering that 3-D effects may be important in the case where the pulse field is not wide enough, a more detailed analysis should include effects of the transverse ponderomotive force associated with the pulse. This will be studied in future work.

As a basic problem, the increase in pulse propagation speed with time can cause great changes in the dynamics of electrons in a laser field. In this paper we have studied this acceleration mechanism in a stationary plasm. In general, the rise in the pulse propagation speed can be not only in a spatially inhomogeneous plasma, but also in a non-stationary plasma, such as an expanding plasma (Kruer 1988, p. 116). Because the majority of laboratory plasmas are typically inhomogeneous and non-stationary, this direct acceleration effect may play an important role in many laser–plasma interaction phenomena.

Acknowledgments

The authors are grateful to Dr Y. Cheng, Dr G. H. Zeng and Dr Z. Y. Chen for their many very useful and helpful discussions.

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Manuscript received 15 October 1998, accepted 28 January 1999