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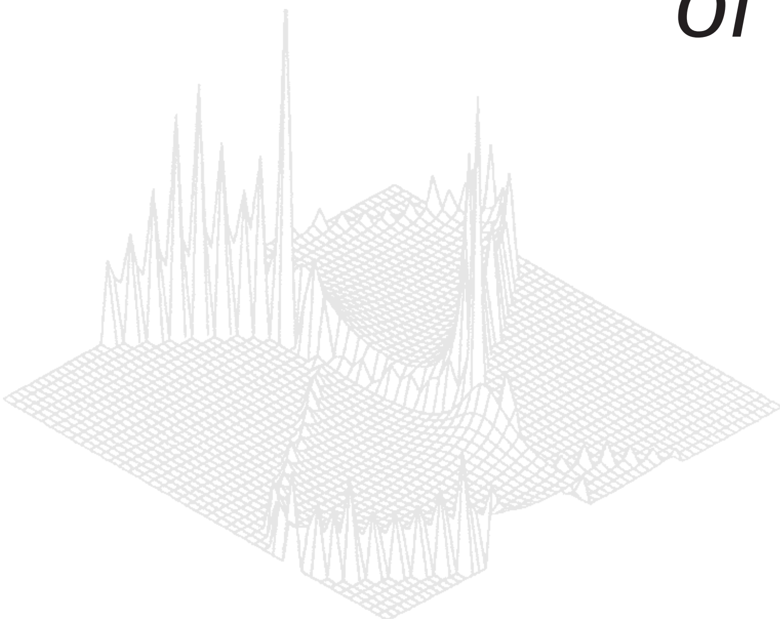
Australian Journal of Physics

Volume 53, 2000



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Aust. J. Phys., 2000, **53**, 477–487

Quantum Measurement and Stochastic Processes in Mesoscopic Conductors*

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Abstract

In quantum measurement theory it is necessary to show how a quantum source conditions a classical stochastic record of measured results. We discuss mesoscopic conductance using quantum stochastic calculus to elucidate the quantum nature of the measurement taking place in these systems. To illustrate the method we derive the current fluctuations in a two terminal mesoscopic circuit with two tunnel barriers containing a single quasi bound state on the well. The method enables us to focus on either the incoming/outgoing Fermi fields in the leads, or on the irreversible dynamics of the well state itself. We show an equivalence between the approach of Büttiker and the Fermi quantum stochastic calculus for mesoscopic systems.

1. Introduction

The theory of conductance in mesoscopic electronics was developed some years ago by Büttiker (1996), following upon earlier ideas of Landauer. The conductance of a mesoscopic system is given in terms of the scattering within and between quantum channels and involves the transmission and reflection coefficients as well as the thermal occupation of reservoirs feeding or draining those channels. The theory makes direct contact with measured currents through averages of quadratic functions of Fermi field operators in the channels. The computation of the scattering matrices depends on the nature of the systems connected to the reservoirs, which could be a simple tunnel barrier or an array of coherently coupled quantum dots. In the Büttiker approach, once the scattering matrices are calculated, we do not need to refer to the dynamics of any local systems to which the input and output channels are coupled. In many ways this theory is a Fermion analogue of the quantum description of optical fields interacting with an optical cavity under the Markov approximation. Such systems are described by the input/output theory of Collett and Gardiner (see e.g. Gardiner 1992). The properties of the fields outside the cavity are determined by a scattering matrix connecting the input and output fields to the cavity and the dynamics inside the cavity. The input/output theory for optical fields has been shown to be an example of the quantum stochastic calculus for boson fields (Barchielli 1986).

The Büttiker approach is particularly useful in determining measured properties of the mesoscopic system, such as conductance. However, recent interest in coherently coupled quantum dots for quantum computation (Kane 1998; Loss and DiVincenzo 1998) has

* Refereed paper based on a contribution to the Ninth Gordon Godfrey Workshop on Condensed Matter in Zero, One and Two Dimensions held at the University of New South Wales, Sydney, in November 1999.

 10.1071/PH00003 0004-9506/00/040477$05.00

focussed attention on the dynamics of localised systems, such as quasibound states on quantum dots, rather than the transport through input and output channels. In the input/output theory of quantum optics, the dynamics of the local system is described through a Markov master equation, and there is a consistency between a description entirely in terms of the input and output modes and the irreversible dynamics of the local system to which they couple. In this paper we establish the connection between a description in terms of input and output channels and the irreversible dynamics of a localised quasibound state on a single quantum dot. The analysis is easily extended to more complex local systems. This enables us to make a connection between two very successful theories, quantum optics and quantum mesoscopics, for the treatment of quantum stochastic processes. We expect that analogies between quantum optics and mesoscopic electronics will prove useful as the latter explores the physics of strong coherent coupling between local systems (e.g. quantum dots, see Blick *et al.* 1998), quantum limited measurements (e.g. using single electronics, see Shirman *et al.* 1997; Gurvitz 1997; Shirman and Schoen 1998), and proposals for quantum computation (see Kane 1998; Loss and DiVincenzo 1998).

Our treatment will be based on a particularly simple system; a single quantum dot coupled to two quantum channels (see Fig. 1), as this system is the electronic analogue of a single Fabry–Perot cavity in quantum optics. The conductance properties of this system can easily be obtained by the method of Büttiker (1992, 1996). Recently a similar result was obtained using a Markov master equation description of the quasibound state of the dot (Sun and Milburn 1999). In this paper we give an equivalent description in terms of the quantum stochastic calculus for Fermi fields.

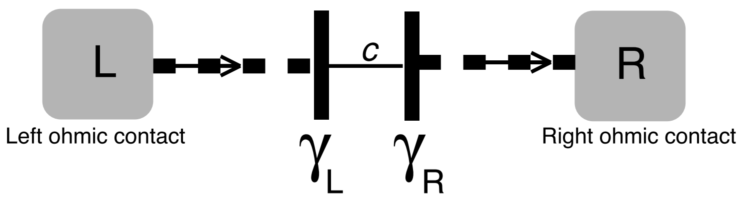

Fig. 1. Schematic representation of tunneling through a single quantum dot.

2. Quantum Stochastic Calculus for Fermions

The free Hamiltonian for a fermion channel is

$$H = \hbar \sum_k \omega_k a_k^\dagger a_k, \tag{1}$$

where a_k is a Fermi annihilation operator satisfying the anti-commutation relations

$$a_k a_l + a_l a_k = 0 \tag{2}$$

$$a_k a_l^\dagger + a_l^\dagger a_k = \delta_{kl} \tag{3}$$

We now define the Fermi field operator

$$a(t) = \sum a_k e^{-i(\omega_k - \omega_0)t} \tag{4}$$

which represents free field dynamics with respect to a frame rotating at frequency ω_0. This frequency will later be taken to characterise the energy of the quasi-bound state to which the Fermi field is coupled. The field operator satisfies the continuum anti-commutation relations

$$a(t)a^\dagger(t') + a^\dagger(t')a(t) = \delta(t - t') \tag{5}$$

Our objective is to define a quantum stochastic process to accurately characterise the Fermi statistics of these fields. To that end we define the integrated operators,

$$A(t) = \int_0^t dt' a(t') \tag{6}$$

and the corresponding Ito increment

$$dA(t) = A(t + dt) - A(t) \tag{7}$$

We now need to specify the state of the free fields. We will take these to be thermal equilibrium states of a noninteracting Fermi system at temperature *T*. It is then easy to show, under appropriate assumptions, that

$$\langle dA^2\rangle = \langle (dA^\dagger)^2\rangle = 0 \tag{8}$$

$$\langle dA^\dagger dA\rangle = f(\omega_0)dt \tag{9}$$

$$\langle dA dA^\dagger\rangle = (1 - f(\omega_0))dt \tag{10}$$

where the equilibrium occupation probability $f(E)$ is evaluated at the reference energy $E_0 = \hbar\omega_0$. Later this will be the probability that a free field Fermi state, resonant with quasi-bound state, is occupied. The important point to note here is that these quantities, while quadratic in the field increments, are only first order in the time increment. This is a quantum analogue of the classical Wiener stochastic process (Gardiner 1991). Stochastic integrals of averaged field operators are found using a generalisation of the Ito calculus for classical stochastic processes (Gardiner 1983). In particular, we have

$$\int_0^{t_1} dt \int_0^{t_2} dt' \langle dA^\dagger(t) dA(t')\rangle X(t)Y(t') = \int_0^{min(t_1,t_2)} dt f(\omega_0)X(t)Y(t) \tag{11}$$

$$\int_0^{t_1} dt \int_0^{t_2} dt' \langle dA(t) dA^\dagger(t')\rangle X(t)Y(t') = \int_0^{min(t_1,t_2)} dt(1-f(\omega_0))X(t)Y(t) \tag{12}$$

where $X(t)$, $Y(t)$ may be operator valued functions. We also note that in the Ito calculus the chain rule is modified according to (Barchielli 1986)

$$d(A(t)B(t)) = dA(t)B(t) + A(t)dB(t) + dA(t)dB(t) \tag{13}$$

The final term must be included to ensure a correct expansion to linear order in the time increment.

We need now to specify how the free field is coupled to a local electronic degree of freedom described by the Fermi annihilation and creation operators $c, c^\dagger$. This degree of freedom could for example be a quasi-bound state of a single quantum dot or donor atom. This interaction will be taken to be linear in the system operators. The usual tunnelling interaction is then specified by the Hamiltonian,

$$H_I = \sum_k g_k a_k^\dagger c + g_k^* c^\dagger a_k \tag{14}$$

Such a coupling is the usual way to describe tunnel coupling between a reservoir and a localised degree of freedom (Mahan 1990). We now anticipate the Markov approximation by assuming that around the reference energy, the coupling constants, g_k are very slowly varying functions of k and replace them by a constant, $\sqrt{\gamma}$ (for further discussion see Gardiner 1992). In the interaction picture we then find that

$$H_I = \sqrt{\gamma}(ca^\dagger(t) + a(t)c^\dagger) \tag{15}$$

The time evolution operator over the time increment dt is given by

$$U(dt) = \exp\{-i\sqrt{\gamma}(cdA^\dagger(t) + dA(t)c^\dagger)\} \tag{16}$$

This enables us to define the 'output' field stochastic process as

$$dA_{\text{out}}(t) = U^\dagger(dt)dA(t)U(dt) \tag{17}$$

This equation suggests that we regard $dA(t)$ as the 'input' field stochastic process $dA(t) = dA_{\text{in}}(t)$. It is then easy to see that

$$dA_{\text{out}}(t) = dA_{\text{in}}(t) + i\sqrt{\gamma}\, c(t)dt. \tag{18}$$

This expression may be written directly in terms of the reservoir field operators as

$$a_{\text{out}}(t) = a_{\text{in}}(t) + i\sqrt{\gamma}c(t) \tag{19}$$

This expression can be used to establish a connection between input and output fields once the dynamics of the local system operators is given. In the frequency (energy) domain the resulting expression is equivalent to the scattering matrix in the method of Büttiker.

The dynamics of the local system is specified by a quantum stochastic differential equation, for example

$$dc(t) = U^\dagger(dt)c(t)U(dt) - c(t) \tag{20}$$

$$= -\frac{\gamma}{2}c(t) + i\sqrt{\gamma}dA(t). \tag{21}$$

The quantum stochastic differential equation for the system number operator is given by

$$dn_c(t) = c^\dagger(t)dc(t) + dc^\dagger(t)c(t) + dc^\dagger(t)dc(t) \tag{24}$$

$$= -\gamma(n_c(t) - f(\omega_0))dt + dN(t) \tag{25}$$

where the noise operator is

$$dN(t) = i\sqrt{\gamma}(c(t)^\dagger dA(t) - dA^\dagger(t)c(t)) \tag{26}$$

Equation (25) correctly reflects the Fermi statistics of the local system. If the Ito correction term $dc^\dagger(t)dc(t)$ had been neglected, this would not have occurred. When this term is evaluated, the dependence of the noise operators on system variables in equation (22) is crucial. As a result the average occupation of the local system in the steady state is given correctly by

$$\langle n_c \rangle = f(\omega_0) \tag{27}$$

as would be expected for a Fermi particle.

We can also obtain the master equation for the local system by

$$d\rho(t) = \mathrm{Trace}_{\mathcal{R}}\left[U(dt)W(t)U^\dagger(dt) - W(t)\right] \tag{28}$$

where $W(t)$ is the state of the total system (localised system and the external fields), and 'Trace' refers to a trace over external field variables. Using the noise moments we find

$$\begin{aligned}\frac{d\rho(t)}{dt} &= -i[H,\rho(t)] + \gamma f(\omega_0)(2c^\dagger\rho c - cc^\dagger\rho(t) - \rho(t)cc^\dagger) \\ &\quad + \gamma(1 - f(\omega_0))(2c\rho c^\dagger - c^\dagger c\rho(t) - \rho(t)c^\dagger c).\end{aligned} \tag{29}$$

It is an easy matter to verify that the mean occupation of the dot $\langle n_c(t)\rangle$ obeys the same equation that results from taking moments of both sides of equation (25). The master equation represents the Schrödinger picture dynamics, while the quantum stochastic differential equation represents the Heisenberg picture.

3. Quantum Stochastic Dynamics of a Single Quantum Dot

The system we discuss in this paper is a standard mesoscopic configuration (Datta 1995) in which ohmic contacts couple to propagating channels to either side of a quantum dot (see Fig. 1). We suppose that there is a single quasi-bound state between two tunnel barriers. Spin will be ignored. It can easily be included as another state in the dot. We will also ignore Coulomb blockade, which for a single bound state simply leads to a shift in the energy of the state. We also take the so-called 'zero-temperature' limit, and assume that the bound state energy is below the effective Fermi energy in the source (L in Fig. 1) and above the effective Fermi energy in the drain (R in Fig. 1). The ohmic contact at the left of the dot is assumed to be a perfect emitter, while the ohmic contact at the right of the dot is a perfect absorber. This ensures that both reservoirs will remain close to thermal equilibrium at all times, provided they are connected to an external EMF. These assumptions will be important when we consider the measured quantities in this system.

We need now to specify how the free fields in the left and right channels are coupled to a local electronic degree of freedom described by the Fermi annihilation and creation operators $c, c^\dagger$. This degree of freedom could for example be a quasi-bound state of a single quantum dot or donor atom. This interaction will be taken to be linear in the system operators. The usual tunnelling interaction, anticipating the Markov approximation as in the previous section, is then specified by the Hamiltonian

$$H_I = \sqrt{\gamma_L}[ca_L^\dagger(t) + c^\dagger a_L(t)] + \sqrt{\gamma_R}[ca_R^\dagger(t) + c^\dagger a_R(t)] \tag{30}$$

where γ_L, γ_R refers to the tunnelling rate across the left and right barrier respectively, while $a_L(t)$, $a_R(t)$ are the Fermi fields in the left channel and right channel respectively. The corresponding unitary evolution operator for a time increment dt is

$$U(dt) = \exp\left(-i\sqrt{\gamma_L}(cdA_L^\dagger + c^\dagger dA_L) - i\sqrt{\gamma_R}(cdA_R^\dagger + c^\dagger dA_R)\right) \tag{31}$$

where $dA_L(t)$,$dA_R(t)$ are the quantum stochastic processes in the left and right channels respectively. The input/output relations are then found to be

$$dA_{L,\text{out}}(t) = dA_{L,\text{in}}(t) - i\sqrt{\gamma_L}c(t)dt \tag{32}$$

$$dA_{R,\text{out}}(t) = dA_{R,\text{in}}(t) + i\sqrt{\gamma_R}c(t)dt \tag{33}$$

with the following averages for the noise

$$\langle dA_L(t)^\dagger dA_L(t)\rangle = dt \tag{34}$$

$$\langle dA_R(t)dA_R^\dagger(t)\rangle = dt \tag{35}$$

All other averages are zero. Alternatively we may write the input–output relations as

$$a_{L,\text{out}}(t) = da_{L,\text{in}}(t) - i\sqrt{\gamma_L}c(t) \tag{36}$$

$$a_{R,\text{out}}(t) = da_{R,\text{in}}(t) + i\sqrt{\gamma_R}c(t) \tag{37}$$

The quantum stochastic differential equation for the destruction operator in the dot is

$$dc(t) = -\frac{(\gamma_L + \gamma_R)}{2}c(t)dt + i\sqrt{\gamma_L}dA_L(t) + i\sqrt{\gamma_R}dA_R(t) \tag{38}$$

The quantum stochastic differential equation for the number operator on the dot is

$$dn_c(t) = \gamma_L[1 - n_c(t)]dt - \gamma_R n_c(t)dt + dN_L(t) + dN_R(t) \tag{39}$$

If we take moments of both sides of equation (39) we find

$$\frac{d\bar{n}}{dt} = \gamma_L(1 - \bar{n}) \tag{40}$$

The first term corresponds to injection from the source onto the dot. This term is zero if the dot is already occupied and $\bar{n}(t) = 1$. The second term corresponds to emission from the dot through the right barrier into the drain. The steady state occupation number on the dot is

$$\bar{n}_\infty = \frac{\gamma_L}{\gamma_L + \gamma_R} \tag{41}$$

The master equation for the dot is found to be

$$\begin{aligned} \frac{d\rho}{dt} = \mathcal{L}\rho = & \frac{\gamma_L}{2}\left(2c^\dagger\rho c - cc^\dagger\rho - \rho cc^\dagger\right) \\ & + \frac{\gamma_R}{2}\left(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c\right) \end{aligned} \tag{42}$$

This equation was previously derived by more direct methods by Sun and Milburn (1999).

4. What is Measured?

It is at this point that we need to make contact with measurable quantities. In the case of electron transport, the measurable quantities reduce to current $I(t)$ and voltage $V(t)$. The measurement results are a time series of currents and voltages which exhibit both systematic and stochastic components. Thus $I(t)$ and voltage $V(t)$ are classical conditional stochastic processes, conditioned by the underlying quantum dynamics of the quasi-bound state on the dot. The reservoirs in the ohmic contacts play a key role in defining the measured quantities and ensuring that they are ultimately classical stochastic processes. Transport through the dot results in charge fluctuations in either the left or the right channels. These fluctuations decay extremely rapidly, ensuring that the channels remain in thermal equilibrium with the respective ohmic contacts. For this to be possible charge must be able to flow into and out of the channels from an external circuit. We assume that a constant potential difference is maintained between the two reservoirs either side of the dot. While the entire system is clearly not in thermal equilibrium, we assume that the left and right channels are themselves close to thermal equilibrium and can each be specified by a separate chemical potential μ_L and μ_R, and these are held constant by a external voltage source V.

If a single electron tunnels out of the dot into the right channel between time t and $t+dt$, its energy is momentarily above the Fermi energy. This electron scatters very strongly from the electrons in that channel and propagates into the right ohmic contact where it is perfectly absorbed. The net effect is a small current pulse $dI_L(t)$ in the external circuit. This is completely analogous to perfect photodetection: a photon emitted from a cavity will be detected with certainty by a detector which is a perfect absorber. Likewise, when an electron in the right channel tunnels onto the dot, there is a rapid relaxation of this unfilled state back to thermal equilibrium as an electron is emitted from the right ohmic contact into the depleted channel. This again results in a current pulse in the circuit connected to the ohmic contacts. The energy gained when one electron is emitted from the left reservoir is, by definition, the chemical potential of that reservoir μ_L, while the energy lost when one electron is absorbed into the right reservoir is μ_R. The net energy transferred between reservoirs is $\mu_L - \mu_R$. This energy is supplied by the external EMF, V, and thus $\mu_L - \mu_R = eV$. It should not be supposed that the electron injected from the left contact and emitted into the right contact have the same energy as the energy of an electron on the dot. In fact any electron energy at all will suffice to restore thermal equilibrium in the left and right leads. If an electron is emitted into the left channel between times t and $t + dt$, the (unnormalised) state of that channel is $\rho_{L,O}(t + dt) = a^{\dagger}_{L,O}\rho(t)a_{L,O}dt$. The probability of this event occurring is simply the normalisation constant and is $p_e(t) = [a^{\dagger}_{L,O}\rho(t)a_{L,O}]dt = \langle a_{L,O}(t)a^{\dagger}_{L,O}(t)\rangle$. That is to say, the probability of emission of electrons into the left channel is determined by the *anti-normally ordered* number flux operator in the left channel. A similar argument indicates that the probability of absorption of an electron in the right ohmic contact is given by the mean of the *normally ordered* number flux operator in the right most channel, $p_a(t) = \langle a^{\dagger}_{R,O}(t)a_{R,O}(t)\rangle$. This is precisely analogous to perfect photodetection from an optical source (Walls and Milburn 1994). Both the emission into the left channel and absorption from the right channel are conditional point processes, conditioned on the quantum state of the quasi-bound state on the dot.

On average of course the same current flows in both reservoirs; however, as the current is stochastic it is made up of contributions from pulses in each lead, which do not necessarily occur at the same time. Indeed they are necessarily separated in time by a degree depending on the lifetime of the quasi-bound state in the dot. In mesoscopic devices, however, current is measured locally in each lead, and thus we can consider either the current in the left lead $I_L(t)$ or the current in the right lead $I_R(t)$ and correlations between them. The current that flows in the right lead is simply given by the probability per unit time that an electron in that channel is absorbed by the perfect absorber that is the right ohmic contact. Thus we have

$$E(I_R(t)) = e\langle a^{\dagger}_{R,O}(t)a_{R,O}(t)\rangle \tag{43}$$

The average current that flows in the left lead is given by the average probability per unit time that an electron is emitted by the perfect emitter that is the left ohmic contact:

$$E(I_L(t)) = e\langle a_{L,O}(t)a^{\dagger}_{L,O}(t)\rangle \tag{44}$$

Note that the average on the left-hand side is an average of a classical stochastic process, while the average on the right is of a quantum stochastic process. We may now substitute the relationship between the output fields, the input fields and the operator for the local state, equation (37) and (38). The average currents are then found to be

$$E(I_L(t)) = e\gamma_L(1 - \langle n_c(t)\rangle) \tag{45}$$

$$E(I_R(t)) = e\gamma_R\langle n_c(t)\rangle \tag{46}$$

In the stationary state both currents are equal and given by

$$I_{L,\infty} = I_{R,\infty} = \frac{e\gamma_L\gamma_R}{\gamma} \tag{47}$$

where $\gamma = \gamma_L + \gamma_R$.

The stationary two time correlation matrix is given by

$$G_{\alpha,\beta}(\tau) = E(I_\alpha(t+\tau), I_\beta(t))_{t\to\infty} \tag{48}$$

and $E(X,Y) = E(XY) - E(X)E(Y)$. The quantity $E(I_\alpha(t+\tau)I_\beta(t))$ is determined by the appropriately ordered two time correlation function for the quantum fields in the channels. As both emission and absorption are point processes, we find

$$\begin{aligned} E(I_L(t+\tau)I_L(t) &= e^2\langle a_{L,O}(t)a^\dagger_{L,O}(t)\rangle\delta(\tau) \\ &\quad + e^2\langle a_{L,O}(t)a_{L,O}(t+\tau)a^\dagger_{L,O}(t+\tau)a^\dagger_{L,O}(t)\rangle_{\tau>0} \end{aligned} \tag{49}$$

$$\begin{aligned} E(I_R(t+\tau)I_R(t)) &= e^2\langle a_{R,O}(t)^\dagger a_{R,O}(t)\rangle\delta(\tau) \\ &\quad + e^2\langle a^\dagger_{R,O}(t)a^\dagger_{R,O}(t+\tau)a_{R,O}(t+\tau)a_{R,O}(t)\rangle_{\tau>0} \end{aligned} \tag{50}$$

$$E(I_R(t+\tau)I_L(t)) = \langle a^\dagger_{R,O}(t+\tau)a_R(t+\tau)a_{L,O}(t)a^\dagger_{L,O}(t)\rangle \tag{51}$$

$$E(I_L(t+\tau)I_R(t)) = \langle a_{L,O}(t+\tau)a^\dagger_{L,O}(t+\tau)a^\dagger_{R,O}(t)a_{R,O}(t)\rangle \tag{52}$$

Using equations (37) and (38), the steady state ($t \longrightarrow \infty$) field correlation functions may be expressed solely in terms of the correlation functions for the quasibound state as

$$\langle a_{L,O}(t)a_{L,O}(t+\tau)a^\dagger_{L,O}(t+\tau)a^\dagger_{L,O}(t)\rangle_{\tau>0} = \gamma_L^2 \mathrm{Tr}\left(cc^\dagger e^{\mathcal{L}\tau}c^\dagger\rho_\infty c\right) \tag{53}$$

$$\langle a^\dagger_{R,O}(t)a^\dagger_{R,O}(t+\tau)a_{R,O}(t+\tau)a_{R,O}(t)\rangle_{\tau>0} = \gamma_R^2 \mathrm{Tr}\left(c^\dagger c e^{\mathcal{L}t\tau}c\rho_\infty c^\dagger\right) \tag{54}$$

$$\langle a^\dagger_{R,O}(t+\tau)a_R(t+\tau)a_{L,O}(t)a^\dagger_{L,O}(t)\rangle = \gamma_R\gamma_L \mathrm{Tr}\left(c^\dagger c e^{\mathcal{L}\tau}c^\dagger\rho_\infty c\right) \tag{55}$$

$$\langle a_{L,O}(t+\tau)a^\dagger_{L,O}(t+\tau)a^\dagger_{R,O}(t)a_{R,O}(t)\rangle = \gamma_L\gamma_R \mathrm{Tr}\left(cc^\dagger e^{\mathcal{L}\tau}c\rho_\infty c^\dagger\right) \tag{56}$$

where ρ_∞ is the steady state solution to the master equation (43) for the quasi-bound state on the dot.

Upon solving the master equation we may evaluate each correlation function to give

$$\begin{aligned} E(I_L(t+\tau), I_L(t)) &= E(I_R(t+\tau), I_R(t)) \\ &= e^2\frac{\gamma_L\gamma_R}{\gamma}\delta(\tau) - e^2\frac{\gamma_R^2\gamma_L^2}{\gamma^2}e^{-\gamma\tau} \end{aligned} \tag{57}$$

$$E(I_R(t+\tau), I_L(t)) = e^2\frac{\gamma_L\gamma_R^3}{\gamma^2}e^{-\gamma\tau} \tag{58}$$

$$E(I_L(t+\tau), I_R(t)) = e^2\frac{\gamma_R\gamma_L^3}{\gamma^2}e^{-\gamma\tau} \tag{59}$$

where $\gamma = \gamma_L + \gamma_R$. The power spectrum of the noise in either the left or the right lead is then given by the Fourier transform of the current correlation function in either lead

$$S_R(\omega) = S_L(\omega) = ei_\infty \left(1 - \frac{2\gamma_R\gamma_L}{\gamma^2 + \omega^2}\right) \tag{60}$$

Note that at zero frequency and symmetric rates ($\gamma_R = \gamma_L$) the current noise is suppressed by a factor of 0.5 over shot noise. This is the same as that obtained by Büttiker (1992), and is equivalent to that obtained Sun and Milburn (1999). Note however that in that paper it was assumed, following earlier work (Chen and Ting 1991), that the measured current is a superposition of the two Poisson processes of emission and absorption. This however is not the case for mesoscopic measurements. As has been stressed by Büttiker, the current is measured in one lead at a time and thus the correct expression for the current noise is that given above.

5. Discussion and Conclusion

In this paper we have shown an equivalence between the approach of Büttiker and the approach of quantum stochastic calculus to the current through mesoscopic systems. To illustrate the equivalence we have discussed the current fluctuations in two terminal mesoscopic circuits with two tunnel barriers containing a single quasi-bound state on the well. The method enables us to focus on either the incoming and outgoing Fermi fields in the leads, or on the irreversible dynamics of the well state itself. We have of course made the Markov approximation in order to obtain the quantum Langevin equations for the system. The Markov assumption is equivalent to the assumption of the Breit–Wigner (Lorentzian) assumption for the transmission coefficients though a double barrier structure, and is discussed in some detail by Sun and Milburn (1999).

We believe there are two advantages in our approach. Firstly, it is useful to be able to refer to the quantum irreversible dynamics of the quasi-bound states on local systems defined by barriers, as well as the input and output Fermi fields. This is particularly important in coherently coupled quasibound states. This is essential for recent condensed matter schemes for quantum computation, where the focus is not so much on the properties of the external, classical currents but rather on the dynamics of the local systems themselves. Secondly, our method parallels a similar approach to strongly coupled field modes in quantum optics, thus suggesting useful directions for future work. We mention one such direction. In quantum optics the method of quantum trajectories (Plenio and Knight 1998) enables a description to be given of quantum limited measurement (Carmichael 1993*a*), quantum control (feedback and feed forward) (Wiseman and Milburn 1993), and cascaded local systems (irreversible but directional coupling) (Carmichael 1993*b*; Gardiner 1993). As mesoscopic technology advances these topics will become increasingly important. The presentation in this paper shows how the quantum stochastic methods of quantum optics may be taken over to Fermi systems.

Acknowledgments

This work was supported by the SP group, Cavendish Laboratory, Cambridge University. I would like to thank Crispen Barnes, Chris Ford and Matthew Collett for useful discussions.

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Manuscript received 4 January, accepted 14 March 2000