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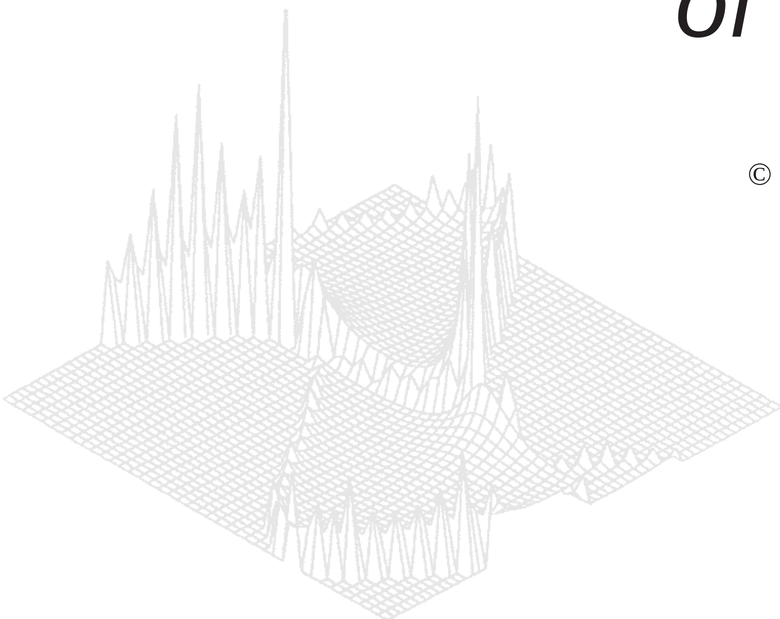

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Scheme for Direct Measurement of the Two-mode Wigner Function in Cavity QED and Ion Traps

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Abstract

A scheme is proposed for the reconstruction of two-mode entangled states in cavity QED and ion traps. For a two-mode field we show that the Wigner function can be obtained by measuring the probability of a two-level atom being in ground states after resonant interaction with two classical fields and dispersive interaction with the two-mode cavity field displaced by resonant sources. For the two-dimensional motion of a trapped ion the Wigner function is obtained by measuring the probability of the ion in its ground electronic state after displacing the ion motion and then resonantly exciting the ion.

1. Introduction

In recent years there has been much interest in the reconstruction of quantum states. Using the homodyne detection and tomographic reconstruction methods, the density matrix of a single-mode running field has been experimentally measured (Schiller *et al.* 1996). Recently, an experimental reconstruction of the motional quantum state of a trapped ion has also been reported (Leibfried *et al.* 1996). However, the scheme involves a complex data analysis.

A scheme for the direct observation of the Wigner characteristic function of a cavity field was proposed by Wilkens and Meystre (1991). In a recent paper, Kim *et al.* (1998) have made a similar proposal applicable to both the cavity field and ion motion. Bardroff *et al.* (1999) have proposed a simple and rapid scheme to measure the Wigner characteristic function of the motional state of a trapped ion. Lutterbach and Davidovich (1997) have presented a scheme for direct measurement of the Wigner function, again in both cavity QED and ion traps. For a cavity field, the Wigner function is obtained by detecting the probabilities of a two-level atom being populated in its excited and ground states after its resonant interactions with two classical fields and dispersive interaction with the cavity field displaced by a microwave source. For a trapped ion, the Wigner function is measured by detecting the probabilities of the ion being populated in its ground and excited electronic states after interaction with two displacement beams and then two carrier beams.

Most of the previous state reconstruction schemes have concentrated on the single-mode case. Raymer *et al.* (1996) proposed a scheme for the reconstruction of a two-mode running field by using balanced homodyne detection. In a more

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recent paper, Kim and Agarwal (1999) have suggested a scheme to reconstruct a two-mode entangled cavity field states via the interaction of a V-type three-level atom with the field displaced by resonant classical sources. More recently still, Solano *et al.* (1999) have proposed a scheme for the measurement of the Wigner function of two trapped ions with centre-of-mass and relative motion modes along their alignment direction. In the present paper we propose a scheme for the measurement of the Wigner function for a two-mode cavity field and the two-dimensional (2D) motion of a trapped ion.

The paper is organised as follows. In Section 2 we show how we can measure the Wigner function for a two-mode field in a cavity and two spatially separated single-mode cavities respectively. We then discuss the reconstruction of an entangled state for the 2D motion of a trapped ion in Section 3. Our conclusions are presented in Section 4.

2. Two-mode Wigner Function in Cavity QED

We first consider a cavity filled with two modes. We assume the initial density operator of the cavity field is $\hat{\rho}_f$. Two microwave sources are connected to the cavity, inducing displacements for the respective cavity modes. This leads to

$$\hat{\rho}_f(\alpha,\beta) = \hat{D}_a(\alpha)\hat{D}_b(\beta)\hat{\rho}_f\hat{D}_b^+(\beta)\hat{D}_a^+(\alpha)\,, \tag{1}$$

where $\hat{D}(\alpha)$ and $\hat{D}(\beta)$ are the displacement operators

$$\hat{D}_a(\alpha) = e^{\alpha\hat{a}^+ - \alpha^*\hat{a}}\,, \tag{2}$$

$$\hat{D}_b(\beta) = e^{\beta\hat{b}^+ - \beta^*\hat{b}}\,, \tag{3}$$

and where $\hat{a}^+$ ($\hat{b}^+$) and $\hat{a}$ ($\hat{b}$) are the creation and annihilation operators for the cavity mode a (b). We then send a two-level atom through the cavity. The atom is initially prepared in the superposition of its excited state $|e\rangle$ and ground states $|g\rangle$,

$$|\phi\rangle = \sqrt{\tfrac{1}{2}}(|e\rangle + |g\rangle)\,, \tag{4}$$

by a classical field. We assume that the atomic transition frequency is somewhat detuned from the frequency of mode a and thus the atom dispersively interacts with mode a. We assume that the difference between the two mode frequencies is large enough so that the atom is far off-resonant with mode b and the coupling between the atom with mode b can be neglected. The effective Hamiltonian for such a system is given by (Holland *et al.* 1991)

$$\hat{H}_e = 2\frac{g_1^2}{\Delta_1}\hat{a}^+\hat{a}\hat{S}_z\,, \tag{5}$$

where $\hat{S}_z$ is the atomic inversion operator, g_1 is the coupling strength between the atom and mode a, and Δ_1 is the detuning between the atomic transition frequency ω_0 and the frequency ω_a of mode a, i.e. $\Delta_1 = \omega_0 - \omega_a$. After an interaction time τ_1 the density operator of the system is

$$\begin{aligned}&\tfrac{1}{2}(e^{-i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}}|e\rangle + e^{i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}}|g\rangle)\hat{\rho}_f(\alpha,\beta)\\ &\quad\times(\langle e|e^{i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}} + \langle g|e^{-i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}})\,.\end{aligned} \tag{6}$$

We then tune the atomic transition frequency so that it is somewhat detuned from the frequency of mode b, and far off-resonant with the frequency of mode a. This can be achieved by the Stark effect induced by a static field applied between the cavity mirrors (Hagley *et al.* 1997). After a dispersive interaction time τ_2 of the atom with mode b, the density operator for the whole system is

$$\begin{aligned}&\tfrac{1}{2}(e^{-i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}-i(g_2^2/\Delta_2)\tau_2\hat{b}^+\hat{b}}|e\rangle + e^{i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}+i(g_2^2/\Delta_2)\tau_2\hat{b}^+\hat{b}}|g\rangle)\hat{\rho}_f(\alpha,\beta)\\ &\quad\times(\langle e|e^{i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}+i(g_2^2/\Delta_2)\tau_2\hat{b}^+\hat{b}} + \langle g|e^{-i(g_1^2/\Delta_1)\tau_1\hat{a}^+\hat{a}-i(g_2^2/\Delta_2)\tau_2\hat{b}^+\hat{b}}),\end{aligned} \tag{7}$$

where g_2 and Δ_2 are the coupling strength and detuning between the atom and mode b. We choose the atomic velocity and the moment when we switch on the Stark field carefully so that

$$\frac{g_1^2}{\Delta_1}\tau_1 = \frac{g_2^2}{\Delta_2}\tau_2 = \frac{\pi}{2}\,.$$

Then we have

$$\begin{aligned}&\tfrac{1}{2}[e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}|e\rangle + e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}|g\rangle]\hat{\rho}_f(\alpha,\beta)\\ &\quad\times[\langle e|e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})} + \langle g|e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]\,.\end{aligned} \tag{8}$$

We then let the atom cross another resonant classical field, undergoing the transition

$$|e\rangle \longrightarrow \sqrt{\tfrac{1}{2}}(|e\rangle + |g\rangle)\,, \tag{9}$$

$$|g\rangle \longrightarrow \sqrt{\tfrac{1}{2}}(|g\rangle - |e\rangle)\,. \tag{10}$$

Then we have

$$\begin{aligned}&\tfrac{1}{4}\{[e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})} - e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]|e\rangle + [e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})} + e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]|g\rangle\}\hat{\rho}_f(\alpha,\beta)\\ &\quad\times\{\langle e|[e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})} - e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}] + \langle g|[e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})} + e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]\}\,.\end{aligned} \tag{11}$$

We now measure the state of the atom. The probability of finding the atom in the state $|g\rangle$ is

$$P_g(\alpha,\beta) = \tfrac{1}{2} + \tfrac{1}{2}\mathrm{Tr}[e^{i\pi(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\hat{\rho}_f(\alpha,\beta)]\,. \tag{12}$$

The Wigner function for a two-mode field is expressed as (Solano *et al.* 1999)

$$\begin{aligned}W(\alpha,\beta) &= \frac{4}{\pi^2}\mathrm{Tr}[e^{i\pi(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\hat{D}_a(-\alpha)\hat{D}_b(-\beta)\hat{\rho}_f\hat{D}_b^+(-\beta)\hat{D}_a^+(-\alpha)]\\ &= \frac{4}{\pi^2}\mathrm{Tr}[e^{i\pi(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\hat{\rho}_f(-\alpha,-\beta)]\,.\end{aligned} \tag{13}$$

Thus we obtain

$$W(-\alpha,-\beta)=\frac{8}{\pi^2}[P_g(\alpha,\beta)-\tfrac{1}{2}]\,. \tag{14}$$

Therefore the measurement of the probability at the point (α,β) directly yields the two-mode Wigner function at the point $(-\alpha,-\beta)$.

We note the method can also be used to reconstruct the entangled state for two spatially separated single-mode cavities. Again, we first displace the two cavity modes by α and β. Then we send a two-level atom, prepared in the superposition state of equation (4), through the cavities. We again assume the atom is initially somewhat detuned from the first cavity mode and thus a dispersive interaction occurs. During the passage through the second cavity the atom is tuned to dispersively interact with the cavity mode. When the atom exits the second cavity the density operator of the whole system is again given by equation (7). We carefully choose the atomic velocity so that $(g_1^2/\Delta_1)\tau_1=\pi/2$. Furthermore, we adjust the strength of the Stark field to obtain an appropriate detuning Δ_2 and thus $(g_2^2/\Delta_2)\tau_2=\pi/2$ (for the case where the two cavities are identical this procedure is not needed). After the atom crosses another resonant classical field, undergoing a $\pi/2$ pulse, the density operator of the whole system is again given by equation (11). Thus the probability of finding the atom in the ground state is again directly related to the two-mode Wigner function by equation (14).

We now the discuss the effect of a detection efficiency of less than 1. Assume that an atom is not detected after interacting with the cavity field. Then the cavity field is described by the reduced density operator

$$\tfrac{1}{2}e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\hat{\rho}_f(\alpha,\beta)e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}+\tfrac{1}{2}e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\hat{\rho}_f(\alpha,\beta)e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\,. \tag{15}$$

After the interaction of the next atom with the cavity field the system combined by the field and this atom is

$$\begin{aligned}
&\tfrac{1}{8}\{[e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}-e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]|e\rangle\\
&+[e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}+e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]|g\rangle\}e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\\
&\times\hat{\rho}_f(\alpha,\beta)e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\{\langle e|[e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}-e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]\\
&+\langle g|[e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}+e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]\}\\
&+\tfrac{1}{8}\{[e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}-e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]|e\rangle\\
&+[e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}+e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]|g\rangle\}e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\\
&\times\hat{\rho}_f(\alpha,\beta)e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}\{\langle e|[e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}-e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]\\
&+\langle g|[e^{i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}+e^{-i\frac{\pi}{2}(\hat{a}^+\hat{a}+\hat{b}^+\hat{b})}]\}\,.
\end{aligned} \tag{16}$$

The probability of detecting this atom in state $|g\rangle$ is again given by equation (12). Thus the scheme is insensitive to the detection effect.

3. Two-mode Wigner Function in Ion Traps

We now show how we can measure the Wigner function for the 2D motion of a trapped ion. We first consider the case where a two-level ion is trapped in a 2D isotropic harmonic potential. The ion is driven by two laser beams, tuned to the ion transition frequency, propagating along the X and Y directions respectively. The Hamiltonian for such a system is

$$\hat{H} = \nu(\hat{a}^+\hat{a} + \hat{b}^+\hat{b}) + \omega_0\hat{S}_z + [\lambda E^+(\hat{x},\hat{y},t)\hat{S}^+ + \text{h.c.}]\,, \tag{17}$$

where $\hat{a}^+$ ($\hat{b}^+$) and $\hat{a}$ ($\hat{b}$) are the creation and annihilation operators for the vibrational mode along the X (Y) axis, $\hat{S}^+$, $\hat{S}^-$, $\hat{S}_z$ are the raising, lowering, and inversion operators for the two-level ion, ω_0 and λ are the transition frequency and coupling constant characterising the transition for the two-level ion, and ν is the vibrational frequency. In equation (17) $E^+(\hat{x},\hat{y},t)$ is the positive part of the classical driving fields:

$$E^+(\hat{x},\hat{y},t) = E_x e^{-i(\omega_0 t - k_0\hat{x} + \phi_x)} + E_y e^{-i(\omega_0 t - k_0\hat{y} + \phi_y)}\,, \tag{18}$$

where E_l and ϕ_l ($l = x, y$) are the amplitudes and phases of the driving fields, respectively, and k_0 is the wave-vector. The position operators $\hat{x}$ and $\hat{y}$ can be expressed by $\hat{x} = \sqrt{1/(2\nu M)}(\hat{a} + \hat{a}^+)$ and $\hat{y} = \sqrt{1/(2\nu M)}(\hat{b} + \hat{b}^+)$, with M being the mass of the trapped ion.

In the resolved sideband limit the vibrational frequency ν is much larger than Rabi frequencies. Then the interactions of the ion with lasers can be treated using the nonlinear Jaynes–Cummings model (Vogel and de Matos Filho 1995). In this case the Hamiltonian for such a system, in the interaction picture, is given by

$$\hat{H}_i = e^{-\eta^2/2}\sum_{j=0}^{\infty}\frac{(i\eta)^{2j}}{(j!)^2}\left[\Omega_x e^{-i\phi_x}\hat{a}^{+j}\hat{a}^j + \Omega_y e^{-i\phi_y}\hat{b}^{+j}\hat{b}^j\right]\hat{S}^+ + \text{h.c.}\,, \tag{19}$$

where $\Omega_l = \lambda E_l$ are the Rabi frequencies of the respective lasers, and the Lamb–Dicke parameter η is defined by $\eta = k_0/\sqrt{2\nu M}$.

We consider the behaviour of the ion in the Lamb–Dicke regime, $\eta \ll 1$. In this limit we can expand the Hamiltonian $\hat{H}_i$ of equation (19) up to second order in η. Furthermore, small Lamb–Dicke parameters lead to $e^{-\eta^2/2} \simeq 1$. Then the Hamiltonian can be simplified to

$$\hat{H}_i = [\Omega_x e^{-i\phi_x}(1 - \eta^2\hat{a}^+\hat{a}) + \Omega_y e^{-i\phi_y}(1 - \eta^2\hat{b}^+\hat{b})]\hat{S}^+ + \text{h.c.} \tag{20}$$

We choose the strengths and amplitudes of the laser beams appropriately so that $\Omega_x = \Omega_y$ and $\phi_x = \pi$, $\phi_y = 0$. Then we obtain

$$H_i = g(\hat{a}^+\hat{a} - \hat{b}^+\hat{b})\hat{S}^+ + \text{h.c.}\,, \tag{21}$$

where $g = \Omega_x\eta^2 = \Omega_y\eta^2$.

We assume that the initial density operator of the 2D motion is $\hat{\rho}_m$. We first displace the motional modes a and b by α and β by application of two displacement beams to each vibrational mode (Monroe et al. 1996), leading to

$$\hat{\rho}_m(\alpha,\beta) = \hat{D}_a(\alpha)\hat{D}_b(\beta)\hat{\rho}_m\hat{D}_b^+(\beta)\hat{D}_a^+(\alpha)\,. \tag{22}$$

The ion is initially in the ground electronic state $|g\rangle$. Then after an interaction time τ of the ion with the lasers the density operator for the whole system is

$$[|g\rangle\cos(\hat{O}\tau) - i|e\rangle\sin(\hat{O}\tau)]\hat{\rho}_m(\alpha,\beta)[\cos(\hat{O}\tau)\langle g| + i\sin(\hat{O}\tau)\langle e|]\,, \tag{23}$$

where $\hat{O}$ is the Hermitian operator

$$\hat{O} = g(a^+a - b^+b)\,. \tag{24}$$

We now detect the internal state of the ion. The probability of measuring the ion in the ground state $|g\rangle$ is

$$P_g(\alpha,\beta) = \tfrac{1}{2} + \tfrac{1}{2}\text{Tr}\{\cos(2\hat{O}\tau)\hat{\rho}_m(\alpha,\beta)\}\,. \tag{25}$$

We choose the Rabi frequency or duration of the lasers appropriately so that $g\tau = \pi/2$. Then the probability of the ion in its ground electronic state is also related to the Wigner function of the 2D motion in the form of equation (14).

In order to detect the electronic state, we employ an electronic V scheme (de Matos Filho and Vogel 1996; Poyatos *et al.* 1996), where the upper levels $|e\rangle$ and $|r\rangle$ couple to the common ground level $|g\rangle$. The transition $|e\rangle \to |g\rangle$ is dipole forbidden, while $|r\rangle \to |g\rangle$ is dipole allowed. After the interaction of the ion with the above-mentioned two carrier beams, a laser on resonance with the transition $|r\rangle \to |g\rangle$ is used to detect the fluorescence. The presence of fluorescence is correlated with the ion being in the electronic state $|g\rangle$, while the absence of fluorescence is correlated with the ion in the state $|e\rangle$.

We now consider the case where the ion is trapped in an anisotropic trap with vibrational frequencies ν_x and ν_y along the X and Y axes. In this case the procedure can be simplified. After the displacement of the motional modes we drive the ion with a laser tuned to the carrier, having a wave vector k_x along the X axis and a wave vector k_y along the Y axis. We assume that ν_x, ν_y, and $|\nu_x - \nu_y|$ are much larger than other characteristic frequencies of the problem. Then in the Lamb–Dicke limit, the Hamiltonian for such a system, in the interaction picture, is

$$\hat{H}_i' = [\Omega e^{-i\phi}(1 - \eta_x^2\hat{a}^+\hat{a} - \eta_y^2\hat{b}^+\hat{b})]\hat{S}^+ + \text{h.c.}\,, \tag{26}$$

where Ω and ϕ are the Rabi frequency and phase of the laser, and the Lamb–Dicke parameters η_x and η_y are defined by $\eta_x = k_x/\sqrt{2\nu_x M}$ and $\eta_y = k_y/\sqrt{2\nu_y M}$. We choose the propagating direction of the laser appropriately so that $\eta_x = \eta_y = \eta$. The initial electronic state is again assumed to be $|g\rangle$. Then after an interaction time τ the density operator for the whole system is

$$[|g\rangle \cos(\hat{O}'\tau) - ie^{-i\phi}|e\rangle \sin(\hat{O}'\tau)]\hat{\rho}_m(\alpha,\beta) \times [\cos(\hat{O}'\tau)\langle g| + i\sin(\hat{O}'\tau)\langle e|e^{i\phi}]\,, \tag{27}$$

where $\hat{O}'$ is defined by

$$\hat{O}' = \Omega(1 - \eta^2\hat{a}^+\hat{a} - \eta^2\hat{b}^+\hat{b})\,. \tag{28}$$

In this case the probability of finding the ion in the ground electronic state $|g\rangle$ is

$$P_g(\alpha,\beta) = \tfrac{1}{2} + \tfrac{1}{2}\mathrm{Tr}\{\cos(2\hat{O}'\tau)\hat{\rho}_m(\alpha,\beta)\}\,. \tag{29}$$

We choose the Rabi frequency or duration of the laser appropriately so that $\Omega\eta^2\tau = \pi/2$. Then we have

$$W(-\alpha,-\beta) = \frac{8}{\pi^2\cos(\pi/\eta^2)}\,[P_g(\alpha,\beta) - \tfrac{1}{2}]\,. \tag{30}$$

In order for the scheme to be valid the condition $\eta^2 \neq 1/(m+\frac{1}{2})$ should be satisfied, where m is an integer. If a trap happens to have such vibrational frequencies such that $\eta^2 = 1/(m+\frac{1}{2})$, then we use a pair of lasers to excite the ion in a Raman manner (Meekhof *et al.* 1996). In this case we can adjust the Lamb–Dicke parameter by setting the directions of the wave vectors of the two lasers appropriately.

4. Conclusion

In summary, we have proposed a scheme for the direct measurement of the Wigner function for an entangled state of a two-mode cavity field and the 2D motion of a trapped ion. In comparison with the scheme of Kim and Agarwal (1999), the present scheme has the following advantages. First, the previous scheme employs V-configuration three-level atoms with two excited states $|a\rangle$ and $|b\rangle$ coupled to the common ground state $|g\rangle$ and requires the transition $|a\rangle \to |g\rangle$ to be resonant with mode a and the transition $|b\rangle \to |g\rangle$ to be resonant with mode b, which might be problematic experimentally. This drawback is avoided in the present scheme. Second, the present scheme is insensitive to the detection efficiency. Third, in order to reconstruct the entangled state for two separated single-mode cavities, the previous scheme additionally requires the measurement of two probabilities of the atom being in the ground state as the atom interacts only with the first or second cavities, respectively. This is unnecessary in the present scheme. Finally, the present scheme is easily generalised to reconstruct the 2D motional state of a trapped ion.

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