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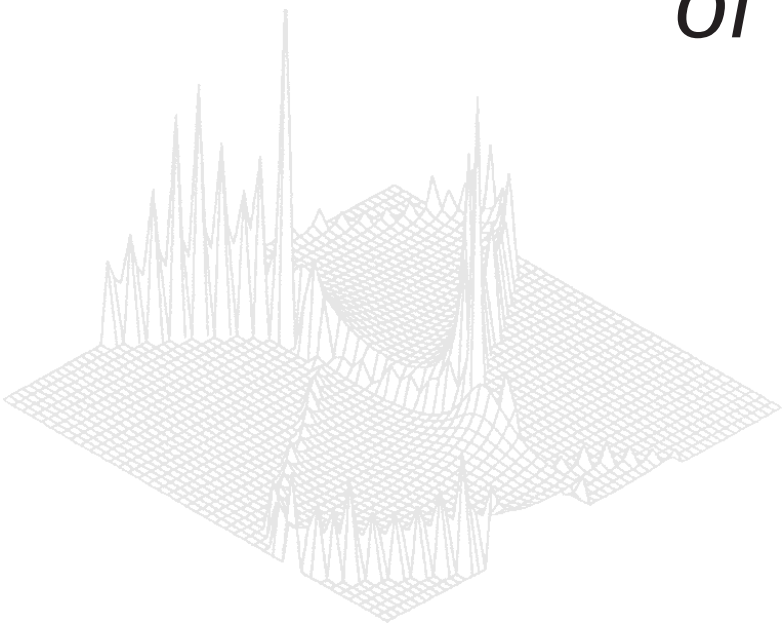

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Characterising the Metal–Insulator Transition in Two Dimensions*

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Abstract

We investigate the metal–insulator transition in 2D electron systems assuming a percolation mechanism connecting through a network of metallic domains. The size of the domains is determined by the level of disorder and the strength of the electron correlations. The domains are linked through quantum tunneling. We determine the dependence of the resistivity on electron density and temperature by calculating the tunnelling transmission through the potential barriers between the domains. The results are in good agreement with recent experimental measurements.

The interplay between electron correlations and disorder plays a central role in the metal–insulator transition in 2D electron systems (Simonian *et al.* 1997). For 2D systems the random potential fluctuations associated with disorder will always localise the electrons if the interactions between the electrons are neglected. In the opposite limit of very strong electron correlations and weak disorder the ground state will be the Wigner solid. Thus, the nature of the localised states is determined by a competition between the disorder and electron correlations.

The random potential fluctuations destroy any first order transition, but may create a percolative second-order transition in the transport coefficients. It has been proposed near the transition that these potential fluctuations create domains (regions of slightly lower and slightly higher density) (Efros 1989; Davies *et al.* 1999; Shklovskii and Efros 1984). As the electron density is decreased this can lead to a classical metal–insulator transition (Nixon and Davies 1990). Experimental evidence for localised domains has recently been found in gated GaAs heterostructures using near field spectroscopy with sub-wavelength resolution (Eytan *et al.* 1999). Negatively charged exciton luminescence is used to image the spatial distribution of the electrons in a 2D layer. In the range of gate voltages where the conductivity drops, the electrons have been shown to be localised inside the potential fluctuations of the remote ionised donors. The spectral signature of regions filled with localised electrons was very different from regions containing conducting electrons.

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Within the domain model it is assumed that the random potential associated with the impurities $V_{\rm imp}(\mathbf{r})$ varies smoothly. When the linear dimensions of the domains l_V exceed the correlation lengths, the local energy density is given by $E(\mu - V_{\rm imp}(\mathbf{r}))$ for the chemical potential μ. Recently, such a model was employed by Shimshoni *et al.* (1998) to study transport properties near the phase transition for quantum Hall insulators and superconductors.

We apply the model by Shimshoni *et al.* (1998) to determine the transport properties of the 2D metal–insulator transition. A $V_{\rm imp}(\mathbf{r})$ which varies smoothly on a scale long compared with correlation lengths leads to spatial inhomogeneities in the local electron density. The transition is density driven so, when the system is very close to the transition boundary, the variations in the carrier density can lead to the formation of domains of coexisting metal and insulating phases, the metallic domains forming in regions of higher electron density $V_{\rm imp}(\mathbf{r}) < \mu$, and the insulating domains taking up the remaining regions.

For 2D systems, the percolation threshold occurs when the total areas of the two types of domains are equal, that is the critical metal area fraction p is $p = p_c = \frac{1}{2}$. For this unique value both the metal and the insulating domains contain at least one connected path across the sample. Right at the transition all neighbouring domains will touch just at one point. Since $V_{\rm imp}(\mathbf{r})$ increases from the contact point as we move into either of the two adjacent metal domains and decreases as we move into either of the insulating domains, it must form a saddle point centred on the contact point.

As the carrier density moves away from the transition, the minority phase retreats from the centre of the junction and is replaced by the majority phase. The area of the majority phase at the junction exceeds the area of the minority phase by $\delta A = \frac{1}{2}\log(l_V/d)\, d^2$, where d is the average distance separating adjacent minority phase domains. For N_{sp} junctions in a system of total area A, the total excess area fraction of the majority phase is

$$p = p_c + \gamma d^2, \tag{1}$$

with $\gamma = (N_{sp}/2A)\log(l_V/d)$. Writing the density within the metal and insulating domains as $n_m(p)$ and $n_I(p)$, the total density for excess area fraction p is

$$\begin{aligned} n(p) &= p n_m(p) + (1-p) n_I(p) \\ &\simeq n_c + (p - p_c)\lambda\,. \end{aligned} \tag{2}$$

Here $\lambda = (n_m^c - n_I^c) + (n_m' + n_I')$ depends on the impurity density in the particular sample. Also $n_m^c - n_I^c$ is the density discontinuity between the metal and insulating phases at the transition boundary, and the derivatives $n_m' + n_I' = \mathrm{d}n_m/\mathrm{d}p + \mathrm{d}n_m/\mathrm{d}p$ are evaluated at p_c. Equations (1) and (2) relate the carrier density $n(p)$ to the junction width d.

To determine the resistivity we express the transmission across the quantum junctions in terms of $n(p)$. The junction barrier height is $V_{\rm barr} = \frac{1}{2}V''(d/2)^2$, so the tunnelling transmission $\mathcal{T} = \mathcal{T}_0 \exp(-2S(d))$ through the barrier at zero temperature is

$$\mathcal{T} = \mathcal{T}_0 \exp\left(-\frac{S''}{\gamma}(p - p_c)\right)\,, \tag{3}$$

where $S(d) \simeq \mathrm{S}(0) + \frac{1}{2}S''d^2$ is the action across the barrier and $\mathcal{T}_0$ is the transmission for $d = 0$. It is easy to show $S'' \equiv \mathrm{d}^2S/\mathrm{d}d^2 = (\pi/2\hbar)\sqrt{mV''}$. At high temperatures transport across the junction is by thermally activated hopping and the transmission

$$\mathcal{T} \sim \int_{V_{\rm barr}}^{\infty} \exp(-E/k_BT)\mathrm{d}E$$

is

$$\mathcal{T} = \mathcal{T}_0 \exp\left(-\frac{\frac{1}{8}\, V''}{k_B T}\ \frac{p - p_c}{\gamma}\right) . \tag{4}$$

Combining equations (3) and (4) and using equation (2), the probability for transmission across the junction in terms of $n - n_c$ is

$$\mathcal{T} = \mathcal{T}_0 \exp\left(-\frac{n - n_c}{\lambda[\alpha + \beta T]}\right) , \tag{5}$$

where we take $\alpha = (\gamma/S'')$ and $\beta = (8k_B/V'')$ as adjustable parameters. Thus, our final expression for the resistivity $\rho = (\hbar/e^2)(1 - \mathcal{T}/\mathcal{T}_0)$ is

$$\rho = \frac{\hbar}{e^2}\left[\exp\left(\frac{n - n_c}{\lambda[\alpha + \beta T]}\right) - 1\right] . \tag{6}$$

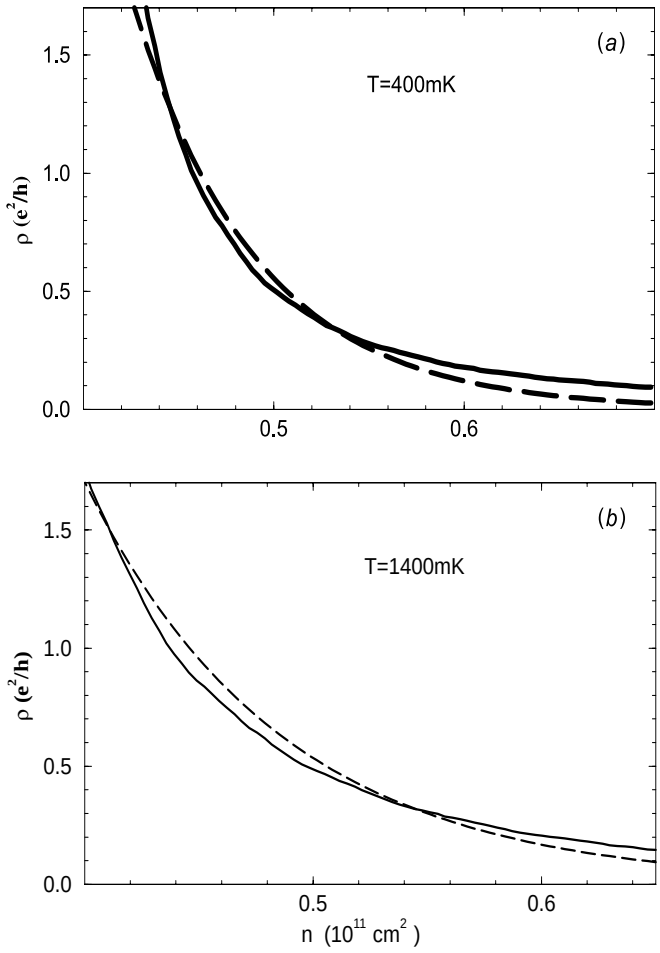

Fig. 1. Best fits to the resistivity data from Simmons and Hamilton (2000): (*a*) Comparison with experimental data for $T = 400$ mK. (*b*) Comparison for $T = 1400$ mK. Calculated values are the dashed curves and experimental values the solid curves.

Fig. 1 plots the resistivity as a function of carrier density n for two different temperatures, $T = 400$ mK and 1400 mK. We show experimental values taken from Simmons and Hamilton (2000). We obtain a good fit for densities within ± 20 % of n_c and a very close fit for the range ± 10 % of n_c. The best fit values of the parameters are $\lambda\alpha = 0.196$ cm^{-2} and $\lambda\beta = 6.24 \times 10^{-4}$ cm^{-2}K^{-1}. In Fig. 2 we compare our results with the experimental data from Hanein *et al.* (1999) at $T = 57$ mK and 214 mK. The agreement is again good.

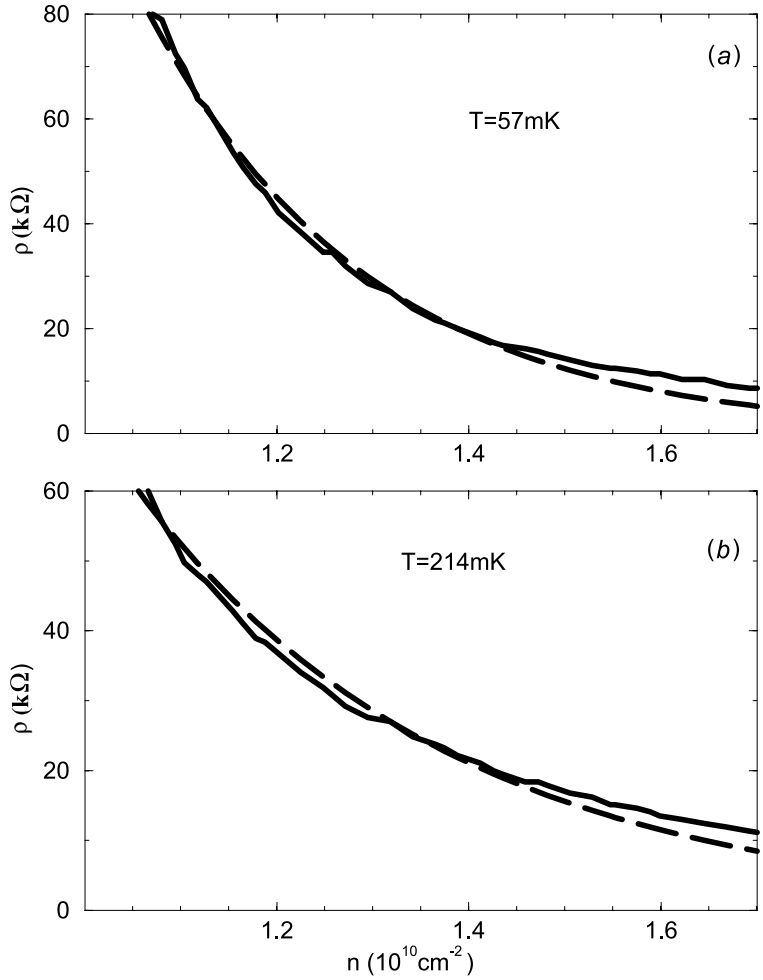

Fig. 2. Best fits to the resistivity data from Hanein *et al.* (1999): (*a*) Comparison with experimental data for $T = 57$ mK. (*b*) Comparison for $T = 214$ mK. Calculated values are the dashed curves and experimental values the solid curves.

In conclusion, a model of formation of domains of metallic and insulating regions near the phase boundary for the metal–insulator transition due to the randomly fluctuating impurity potential leads to calculated resistivities which are in good agreement with two sets of independent measurements of the resisitivity as functions of both carrier density and temperature.

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