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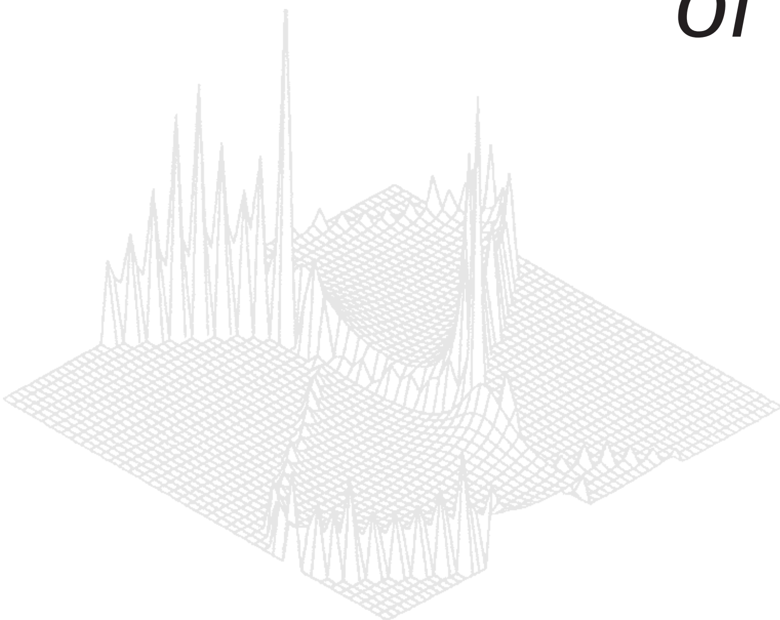
Australian Journal of Physics

Volume 53, 2000



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Aust. J. Phys., 2000, **53**, 499–512

'Anomalous' Resurgence of Shot Noise in Long Conductors*

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Abstract

There has been renewed interest in the physics of the so-called 'crossover' for current fluctuations in mesoscopic conductors, most recently involving the possibility of its appearance in the passage to the macroscopic limit. Shot noise is normally absent from solid-state conductors in the large, and its anomalous resurgence there has been ascribed to a rich interplay of drift, diffusion, and Coulomb screening. We demonstrate that essentially the same rise in shot noise occurs in a much less complex system: the Boltzmann–Drude–Lorentz model of a macroscopic, uniform gas of *strictly non-interacting* carriers. We conclude that the 'anomalous crossover' is a manifestation of simple kinetics. Poissonian carriers, if driven by a *high enough field*, cross the sample faster than any scattering time, thus fulfilling Schottky's condition for ideal shot noise.

1. Introduction

The fine-scale investigation of carrier noise in mesoscopic conductors is now a well-established field within the transport physics of the solid state. It has reached new heights recently, experimentally and theoretically (Blanter and Büttiker 2000). Noise is a unique source of information on the dynamics of microscopic fluctuations. This is notably so at small length scales, already approaching the quantum domain in actual devices.

A particular aspect of mesoscopic noise, in metallic diffusive conductors especially, is the so-called 'crossover' from thermal to shot noise. There are many measurements of it, and almost as many compelling (if frequently quite disparate) theoretical explanations. The term *crossover* refers to the smooth evolution, with increasing voltage, of the current-noise spectral density $S(V;\omega)$ (normally it is sufficient to study its low-frequency limit $\omega \ll \tau^{-1}$, where τ is a characteristic collision time). One sees the onset of a non-dissipative excess component in $S(V;0)$, over and above the dissipative Johnson–Nyquist noise which exhausts the low-field limit. Thus, typically, we have

$$\frac{S(V;0)}{S_0} = 1 + \gamma \left[\frac{eV}{2k_{\rm B}T} \coth\left(\frac{eV}{2k_{\rm B}T} \right) - 1 \right], \tag{1}$$

* Refereed paper based on a contribution to the Ninth Gordon Godfrey Workshop on Condensed Matter in Zero, One and Two Dimensions held at the University of New South Wales, Sydney, in November 1999.

 10.1071/PH00044 0004-9506/00/040499$05.00

where $S_0 = 4Gk_BT$ is the Johnson–Nyquist value and γ the suppression factor; G is the sample conductance and k_BT the thermal energy. The factor γ is a signature of the often subtle correlation effects in the microscopic fluctuations, which are responsible for the form of $S(V; 0)$ as measured (Blanter and Büttiker 2000). At voltages $V \gg k_BT/e$, equation (1) gives $S(V; 0) = 2\gamma eI$, where $I = GV$ is the current. This exhibits suppression of the Schottky formula $S = 2eI$ for classical Poissonian shot noise.

While equation (1) gives an impressive empirical fit to many (though not all) experiments, serious questions arise as to whether any of the prevailing models (Kogan 1996; Blanter and Büttiker 2000) possess the internal consistency expected of a standard microscopic description. We cite Das and Green (2000) and Green and Das (2000*a*) for a description of what may go awry with such theories, all of which rely heavily on hydrodynamic drift–diffusion analogies for mesoscopic transport (Datta 1995; Kogan 1996). Theoretically, the concept of the 'crossover' may be as open to new critiques (Gillespie 2000) as it is to new empirical tests by appropriately designed experiments (Green and Das 1998*a*, 2000*a*, 2000*b*, 2000*c*).

Now, a fresh window on the 'crossover' appears to have been opened by the recent work of Gomila and Reggiani (2000). Foremost is their emphasis on the explicit role of carrier-number fluctuations as generators of the observable shot noise. We note that this perspective was addressed theoretically (with detailed computations) in Green and Das (1998*a*, 2000*a*).

There is a basic difference between fluctuations of carrier number, manifesting at the conductor-lead interfaces and engendering shot noise, and fluctuations of the free energy, manifesting throughout the conductor's volume and generating thermal noise. This understanding is in sharp contrast to the usual phenomenological viewpoint (Kogan 1996) in which no difference is permitted, *even in principle*, between thermally related and carrier-number related processes. In this respect it is useful to bring to mind the textbook distinction between a variation with respect to chemical potential and a variation with respect to particle number. The fact that they are intimately linked by microscopics does not override the fact that they are thermodynamic conjugates, with wholly distinguishable thermodynamic consequences.

Gomila and Reggiani (2000) certainly raise weighty, if not wholly unanticipated, points regarding macroscopic solid-state shot noise (Green and Das 1998*a*; Naveh 1998). Furthermore, these are not easily addressed in strictly low-field, linear, drift-diffusive descriptions (Datta 1995; Kogan 1996; Blanter and Büttiker 2000). Thus, it is well to revisit our own existing non-perturbative Boltzmann theory, allied to a time-of-flight interpretation of shot noise (Green and Das 1998*a*). In Section 2 we briefly recall our formalism, while in Section 3 we give simple Drude-like (but complete and exact) kinetic solutions for shot noise as a function of current, dimensionality, length, and finite radius of the conductor. These show that the 'crossover' anomaly exists with no reference *at all* to long-range Coulomb correlations. In Section 4 we examine the results of Gomila and Reggiani (2000), based on a diffusive Langevin–Poisson scheme, and compare them with those of Green and Das (1998*a*) as recalled in the present paper. Section 5 contains our final remarks.

2. Kinetics of Shot Noise: Theory

(2a) Non-equilibrium Fluctuations

In the context of Gomila and Reggiani (2000) we specialise to a homogeneous metallic wire, subject to a uniform driving field. The mean carrier density is n and the mean total carrier number is $N = \Omega n$ in the sample volume Ω. These are independent of the external field. The kinetic Boltzmann equation for the spatially uniform distribution function $f_{\mathbf{k}}(t)$ is, in the Drude–Lorentz collision approximation,

$$\left(\frac{\partial}{\partial t} + \frac{eE}{\hbar}\frac{\partial}{\partial k_x}\right) f_{\mathbf{k}}(t) = -\frac{1}{\tau}\left(f_{\mathbf{k}}(t) - \frac{\langle f(t)\rangle}{\langle f^{\mathrm{eq}}\rangle} f_{\mathbf{k}}^{\mathrm{eq}}\right). \tag{2}$$

Our electrons are positive for convenience. The field E is in the source–drain x-direction, and τ is the collision time. We denote traces over wave-vector (with a factor of 2 for spin) as $\langle f\rangle \equiv 2\Omega^{-1}\Sigma_k f_k$. The physical constraint on the traces on the right side of equation (2) is, naturally, $\langle f(t)\rangle = \langle f^{eq}\rangle = n$. Finally,

$$f_{\mathbf{k}}^{\mathrm{eq}} = [1 + \exp(\varepsilon_{\mathbf{k}} - \mu)/k_B T)]^{-1}$$

is the usual Fermi–Dirac equilibrium distribution, parametrised by the thermal energy and by μ, the chemical potential. Note that there is no coupling to the Poisson equation, since the system is uniform.

To determine all the relevant microscopic correlation functions in this non-equilibrium system, one must generate the distribution of its *electron–hole pair* fluctuations. That requires variational analysis of equation (2). There are two steps in this, related but treated separately. First we compute the steady-state fluctuation distribution

$$\Delta f_{\mathbf{k}} \equiv k_{\mathrm{B}}T\frac{\delta f_{\mathbf{k}}}{\delta\mu} = \frac{1}{\Omega}\sum_{\mathbf{k}'}\frac{\delta f_{\mathbf{k}}}{\delta f_{\mathbf{k}'}^{\mathrm{eq}}}\,\Delta f_{\mathbf{k}'}^{\mathrm{eq}}, \tag{3}$$

where the equilibrium fluctuation is the mean-square fluctuation of the occupation number, *precisely* as defined in statistical mechanics:

$$\Delta f_{\mathbf{k}}^{\mathrm{eq}} = k_{\mathrm{B}}T\frac{\delta f_{\mathbf{k}}^{\mathrm{eq}}}{\delta\mu} = f_{\mathbf{k}}^{\mathrm{eq}}(1 - f_{\mathbf{k}}^{\mathrm{eq}}). \tag{4}$$

Equation (3) is directly calculable by variation of the one-body Boltzmann transport equation (BTE), and is the exact solution to the linearised BTE (Green and Das 2000*b*, 2000*c*).

For every collision model, there exists a unique one-to-one transformation that maps Δf^{eq} to the functional Δf in the steady state. Together with the fact that Δf exactly satisfies the variational BTE for the electron–hole pair (density–density) correlation function (Kadanoff and Baym 1962), such a mapping establishes equation (3) as the unique mathematical form of the mean-square particle fluctuations out of equilibrium. This explicit and crucial connection, between the equilibrium and non-equilibrium fluctuations, is not model-dependent. It is completely generic to the kinetic description of noise and conspicuously absent from every drift–diffusive description (Kogan 1996).

The next step is to obtain the dynamic response. In our physical picture, a spontaneous thermal energy exchange with the heat reservoir sets up an initial electron–hole pair excitation with average strength $\Delta f_{\mathbf{k}'}$, at some given initial location $\mathbf{r}'$ within volume Ω. The localised spontaneous excitation is not stable. It relaxes back to the steady state according to the time-dependent Green function, derived when equation (2) is perturbed in time, wave-vector space, and (weakly) in real space. This dictates the change in background distribution for state **k** at position **r**, given the initial random excitation at $\mathbf{r}'$. Full technical details are in Green and Das (1998*a*, 2000*b*, 2000*c*).

All of the relaxation dynamics are contained in the *transient* component of the calculable Green function, remaining after the stable long-time adiabatic part is removed from the full response. Let us denote the transient component, in the frequency domain, by $C_{\mathbf{kk}'}(\mathbf{r} - \mathbf{r}'; \omega)$. The dynamic (and non-local) two-body response, namely the electron coupled with its hole, is denoted by

$$\Delta \mathrm{f}^{(2)}_{\mathbf{kk}'}(\mathbf{r} - \mathbf{r}'; \omega) \equiv \mathcal{C}_{\mathbf{kk}'}(\mathbf{r} - \mathbf{r}'; \omega) \Delta f_{\mathbf{k}'}. \tag{5}$$

The full velocity–velocity correlation function is, following the approach of Gantsevich *et al.* (1979),

$$\langle\!\langle \mathbf{v}\mathbf{v}' \Delta \mathrm{f}^{(2)}(\mathbf{r} - \mathbf{r}'; \omega) \rangle\!\rangle'_c \stackrel{\text{def}}{=} \frac{2}{\Omega^2} \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \mathbf{v}_{\mathbf{k}} \mathrm{Re}\{\mathcal{C}_{\mathbf{kk}'}(\mathbf{r} - \mathbf{r}'; \omega)\} \mathbf{v}_{\mathbf{k}'} \Delta f_{\mathbf{k}'}. \tag{6}$$

This completely determines the response of the carrier flux at **r**, induced by a spontaneous thermal fluctuation in the carrier flux at $\mathbf{r}'$. This is *not* a reciprocal process; although uniformity means that the magnitude of the correlator depends on the relative coordinate $\mathbf{r} - \mathbf{r}'$, it matters which of the two positions is upstream. The externally driven system fluctuates asymmetrically in space, just as it fluctuates irreversibly in time.

We now have the basic tool to construct both the thermal noise and the shot noise in the conductor. We discuss the low-frequency case. The thermal spectral density becomes, in a familiar way, the volume integral of all the current–current correlations:

$$S_{\text{therm}}(E) \equiv 4 \int_{\Omega} d\mathbf{r} \int_{\Omega} d\mathbf{r}' \langle\!\langle (ev_x/L)(ev'_x/L)\, \Delta \mathrm{f}^{(2)}(\mathbf{r} - \mathbf{r}'; 0) \rangle\!\rangle'_c; \tag{7}$$

the conductor's length is L. The expression is often portrayed as a real-space symmetrised form (Kogan 1996) which, however, adds little or nothing to the intrinsic physics. We do not elaborate on $S_{\text{therm}}(E)$ except to recall two major properties. The first is the Johnson–Nyquist equilibrium limit

$$S_{\text{therm}}(E \to 0) = 4Gk_{\text{B}}T, \tag{8}$$

where, in our Drude model, the conductance becomes $G = Ne^2\tau/m^*L^2$, for effective mass m^*.

The second property is that equation (7), regardless of driving field, *will never scale other than as* $\Delta f \sim \Delta f^{\text{eq}} \sim T$ *in a degenerate metallic conductor*. This is the strict and inevitable consequence of kinetics, of Fermi-liquid physics, and most of all of asymptotic equilibrium and neutrality in the metallic leads. It has been discussed exhaustively (Das and Green 2000; Green and Das 1998*a*, 2000*a*, 2000*b*, 2000*c*).

(2b) Shot Noise

To set the work of Gomila and Reggiani in context, we must look first at the 'smooth crossover formula' and its inbuilt theoretical deficiency. True shot noise never scales with temperature; for example, it remains well-defined even in the zero-temperature limit. In purporting to make thermal noise integral with true shot noise, the 'smooth-crossover formula' of drift–diffusive theory, equation (1), attempts the kinetically impossible. Every drift–diffusive phenomenology proclaims that equations (1) and (7) are identical (Kogan 1996; Blanter and Büttiker 2000). Yet the rigorous kinetic-theoretical constraint on non-equilibrium thermal noise, namely its abiding proportionality to T, means that equation (7) *cannot possibly describe shot noise* in the presence of strong degeneracy.

It follows that equation (1) is hard to justify. At any rate, the equation's theoretical claim (thermal noise equals shot noise) is unsustainable by a first-principles analysis. We mean an analysis that is conventionally executed, in keeping with the conventional understanding of statistical mechanics and microscopics (Green and Das 2000*a*, 2000*b*). The 'smooth crossover formula' is inconsistent, purely and simply.

That is the background to the shot-noise considerations of Gomila and Reggiani (2000). We do not deny that shot-noise-*like* structure—linear in the current—can emerge from the thermal noise spectrum. Indeed it does, in the semiclassical ballistic limit (Green and Das 1998*b*; Gomila and Reggiani 2000). It is also the case that purely classical models lead to classical shot noise, $2eI$, based on equation (7). Nevertheless, the leading concern is with *metallic* diffusive wires. There, T-independent shot noise cannot be contrived from a strictly thermal basis. Detailed experiments have been proposed to test our claim (Green and Das 2000*a*, 2000*c*).

With the knowledge that true shot noise is fundamentally different from thermal noise, one can build an operationally consistent theory for it. We introduce the idea of the response to variations in the total number of carriers transiting the conductor. In the mean, N is constant in time but fluctuates by $\delta N^{+} = +1$ at any instant that a carrier *first enters at the source*. Similarly it changes by $\delta N^{-} = -1$ as the carrier *finally exits at the drain* (conceptually this amounts to the injection of a hole at the drain). It is not hard to see that this process is described by the non-local correlation

$$\mathcal{C}_{\mathbf{k}\mathbf{k}'}(\mathbf{r}-\mathbf{r}';0)\frac{\delta f_{\mathbf{k}'}}{\delta N}\delta N^{\pm} = \mathcal{C}_{\mathbf{k}\mathbf{k}'}(\mathbf{r}-\mathbf{r}';0)\frac{\Delta f_{\mathbf{k}'}}{\Delta N}\delta N^{\pm}, \tag{9}$$

where $\Delta N = \Omega\langle\Delta f\rangle$. When this correlation records a particle entry, then $\mathbf{r}'$ lies in the cross-sectional region at the source and $\mathbf{r}$ at the drain. When it records a particle exit, then $\mathbf{r}'$ belongs to the drain area and $\mathbf{r}$ to the source. Fig. 1 illustrates the principle. The essence of shot noise is that it is a time-of-flight process, involving many sporadic transits of carriers across a predefined geometry. On this view shot noise has nothing to do with correlations distributed throughout the volume of a conductor. This is totally unlike the guiding assumption for equation (1) (Kogan 1996).

The definition of the measurable shot noise, strictly across the source–drain gap, follows naturally. It simply sums the stochastically independent terms produced, on average, by each of the N active contributors. Each contribution is of equal weight since the carriers are all equivalent (assuming temporal randomness of entry/exit). Thus we have

$$S_{\mathrm{shot}}(E) \stackrel{\mathrm{def}}{=} S_{\mathrm{s;d}}(E)\delta N^{+} + S_{\mathrm{d;s}}(E)\delta N^{-} = S_{\mathrm{s;d}}(E) - S_{\mathrm{d;s}}(E), \tag{10a}$$

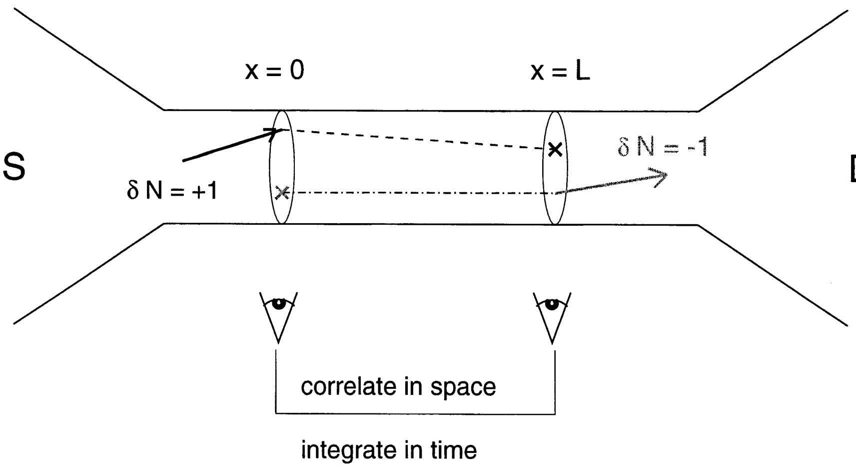


Fig. 1. An operational definition of shot noise. A carrier enters/exits the sample at a random time, augmenting/depleting the carrier population by one. The remotely generated disturbance is also observed at the location of the complementary interface. This correlation of the stimulus and its response is equivalent to a time-of-flight measurement, pairing the carrier states at each observation point. Shot noise is the sum of these stochastically distributed events.

in which the two directional correlations are

$$S_{\mathrm{s;d}}(E) = 2N \int d\mathbf{r}\delta(x-L) \int d\mathbf{r}'\delta(x') \frac{1}{\Delta N} \langle\!\langle (ev_x)(ev'_x)\,\Delta \mathrm{f}^{(2)}(\mathbf{r}-\mathbf{r}';0)\rangle\!\rangle'_c \quad (10b)$$

for injection at the source ($x = 0$) and, for removal at the drain ($x = L$),

$$S_{\mathrm{d;s}}(E) = 2N \int d\mathbf{r}\delta(x-L) \int d\mathbf{r}'\delta(x') \frac{1}{\Delta N} \langle\!\langle (ev_x)(ev'_x)\,\Delta \mathrm{f}^{(2)}(\mathbf{r}'-\mathbf{r};0)\rangle\!\rangle'_c. \quad (10c)$$

Note especially that:

- the space coordinates in the argument of $\Delta \mathrm{f}^{(2)}_{\mathbf{kk}'}(\mathbf{r}'-\mathbf{r};0)$ are reversed in $S_{\mathrm{d;s}}(E)$, and
- $S_{\mathrm{shot}}(E = 0)$ vanishes, since the equilibrium kinetic equation is self-adjoint (time reversible) and entails the identity $S_{\mathrm{s;d}}(0) = S_{\mathrm{d;s}}(0)$.

One easily verifies that equation (10) is explicitly independent of temperature and goes to $2eI$ for current I in the semi-classical ballistic limit (Green and Das 1998*a*). Nevertheless, the intimate microscopic link with the (conjugate) thermal effects remains. It is manifest in the role of the flux auto-correlation $\langle\langle v_x v'_x \Delta \mathrm{f}^{(2)}(\mathbf{r}-\mathbf{r}';0)\rangle\rangle'_c$.

Ultimately, a real measurement of current fluctuations detects them in the access leads for the sample. Hence we expect to detect the sum of thermal and shot-noise contributions, if these effects are statistically independent. Non-equilibrium thermal noise will itself carry a hot-electron excess; it is non-dissipative rather than Johnson–Nyquist in origin (Green and Das 2000*b*, 2000*c*), just as shot noise is non-dissipative. In the macroscopic limit hot-electron noise goes quadratically with E, at least in simple cases. One should ask whether this term could overwhelm an emerging shot-noise signal. This is unlikely, as can be seen from a rough estimate based on the Drude model. The bulk thermal-noise excess goes as

$$S_{\mathrm{exs}}(E) = S_0 \frac{\Delta N}{N} \frac{m^* \mu_{\mathrm{e}}^2 E^2}{k_{\mathrm{B}}T} \leq 4Gm^* \mu_{\mathrm{e}}^2 E^2, \quad (11)$$

since the ratio $\Delta N/N$ is always less than one in a degenerate conductor. (Here $\mu_e = e\tau/m^*$ is the mobility.) By finding the upper bound to $2eI = 2e(GEL) \geqslant S_{exs}(E)$ for typical material parameters, one concludes that pure shot noise—if there were no other mechanism to suppress it—would dominate at least up to fields ~10^6 Vcm^{-1} for a sample 1 mm long. At much shorter (mesoscopic) lengths this simple estimate does not hold; specific modelling is needed.

3. Kinetics of Shot Noise: Application

We can now review our results from Green and Das (1998*a*), built on the form of $S_{shot}(E)$. Despite the relatively crude form of the inelastic collision term in the Drude model, one might expect it to be more relevant at *high fields* than, say, linear treatments that emphasise coherent (or at least elastic) scattering. A driving potential of a few volts is quite enough to place a conductor, some millimetres in length, beyond the validity of drift–diffusive theory (Green and Das 2000*b*).

The calculation is straightforward. The functions $f_{\mathbf{k}}$ and $\Delta f_{\mathbf{k}}$ are first obtained for a given field $E = V/L$. Then the two-point transient response $C_{\mathbf{kk}'}(\mathbf{r} - \mathbf{r}'; \omega)$ is derived from the linearised BTE. Finally, all are combined to yield equation (10).

In Fig. 2 we plot the sum of thermal and shot noises in a one-dimensional (1D) wire calculated within the Boltzmann–Drude model of transport, equation (2). This is in the strongly degenerate carrier regime, at a thermal energy chosen as $k_B T = 0.1\varepsilon_F$, with Fermi

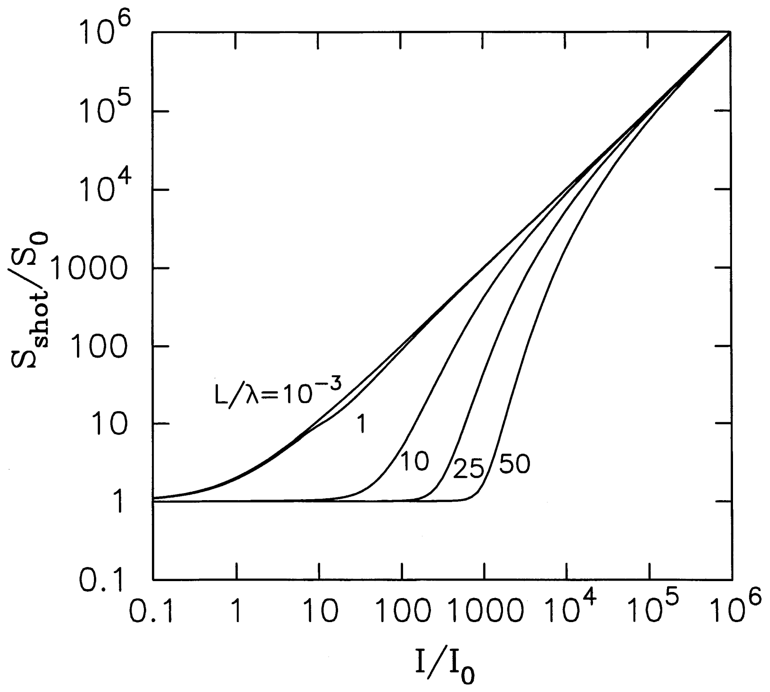


Fig. 2. Sum of thermal and shot noise in a one-dimensional degenerate wire with $k_B T/\varepsilon_F = 0.10$. Noise is scaled to the Johnson–Nyquist value $S_0 = 4Gk_B T$, and the current to $I_0 = 2Gk_B T/e$. Figs 3–5 are structured similarly. The curves are indexed, in descending order, by the ratio of sample length to Fermi mean free path: $L/\lambda = 0.001$, 1, 10, 25 and 50. Note the strong attenuation of low-field shot noise for longer samples. At high enough currents, the shot noise always recovers.

energy ε_F. The spectral density is normalised to the Johnson–Nyquist value S_0 and displayed as a function of current in units of $I_0 = 2Gk_BT/e$. In this and subsequent figures, each curve corresponds to one of five values of the device length as a ratio with the mean free path $\lambda = \tau v_F$ in terms of the Fermi velocity. The curves are always monotonic, tracking down as the ratios rise in the sequence $L/\lambda = 0.001$, 1, 10, 25 and 50. Typically, λ is of the order of 50 to 100 nm.

The shortest wire exhibits full shot noise. This is the semi-classical ballistic limit: the absolute upper bound for our model. In the case of the second shortest wire $L = \lambda$, the curve falls a little below the ideal value $1 + 2eI/S_0$ in the topmost curve. We note a slight shoulder at $I \approx 2G\varepsilon_F/e$. The shot noise remains quasi-ballistic because Pauli blocking in the 1D free electron gas efficiently inhibits any scattering when carriers cannot gain enough energy to leave the Fermi sea. This makes the Fermi distribution fairly robust to moderate external fields.

In longer wires, the shot noise at moderate currents is attenuated more and more. Inelastic-scattering suppression is exponential in the Drude model, taking the low-current form

$$\frac{S_{\rm shot}(E)}{2eI} \to \left(1 + \frac{L}{\lambda} + \frac{L^2}{2\lambda^2}\right) e^{-L/\lambda}, \tag{12}$$

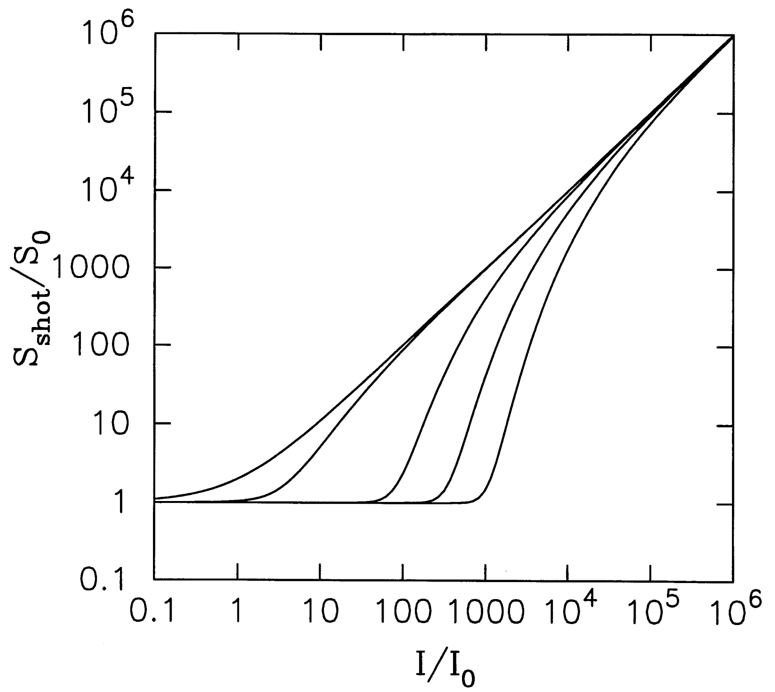


Fig. 3. As for Fig. 2, but for classical carriers at the same density and temperature, and with the same set of physical conductor lengths L. The second curve is much reduced in the low-current region where, in Fig. 2, Pauli blocking inhibits inelastic scattering and sustains the low-current shot noise. Note the congruence of the high-current part of the curves with those of Fig. 2. This shows that true high-field shot noise loses its sensitivity to statistics when the energy scale for transport substantially exceeds the Fermi energy.

which dies very quickly as the length increases. This accounts for the macroscopic extinction of shot noise.

Fig. 3 displays the shot noise of classical 1D carriers, with the same n and T as the degenerate system of Fig. 2. The mean free path is now τv_{th} where $v_{\mathrm{th}} = (2k_B T/m^*)^{\frac{1}{2}}$. We have retained the same physical wire lengths L as in Fig. 2 so that, for Fig. 3 specifically, our chosen ratios of length to mean free path are scaled up by $v_{\mathrm{F}}/v_{\mathrm{th}}$. It is significant that the classical curves fall mostly on top of the degenerate ones. An obvious exception is the second plot, where we saw that degeneracy shields the shot noise from attenuation at lower I. Otherwise the high-current behaviour of the 1D shot noise is unchanged, and thus independent of the carrier statistics. From this it is evident that degeneracy plays no role in the resurgence of high-field shot noise.

To round off our discussion we compare the 1D case with the three-dimensional (3D) case. This is of interest (i) because it is closer to actual experimental devices, and (ii) because it allows us to study the effect of finite, even narrow, wires. Similar results apply in two dimensions (Green and Das 1998*a*).

Fig. 4 shows spectra for the same sequence of lengths, with a wire radius of $R = 100\ \lambda$; very wide. There are minor differences to Fig. 2. For instance, in the second curve Pauli blocking is somewhat less effective in preventing attenuation of low-current shot noise.

Rather more interesting are Figs 5 *a* and 5 *b*. The first is for a wire radius of $R = 0.3\ \lambda$ and the second for $R = 0.05\ \lambda$. (For $\lambda \sim 100$ nm, such thicknesses should be achievable by sophisticated nano-lithography.) There is a major loss of shot-noise spectral strength relative to Fig. 4. Recovery towards full noise does not occur before considerably higher current levels are reached. The freedom to explore shot noise through a new variable, the

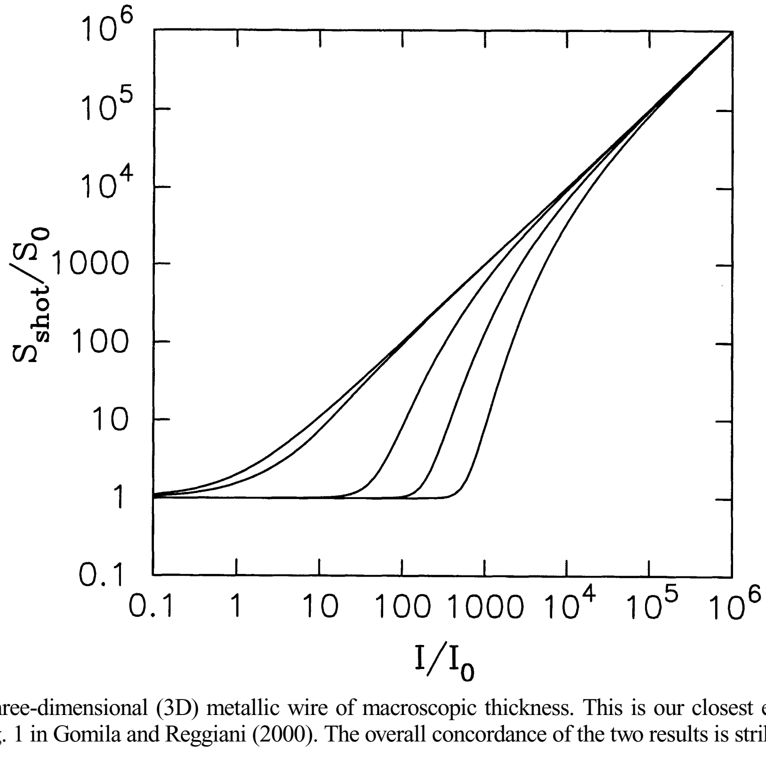
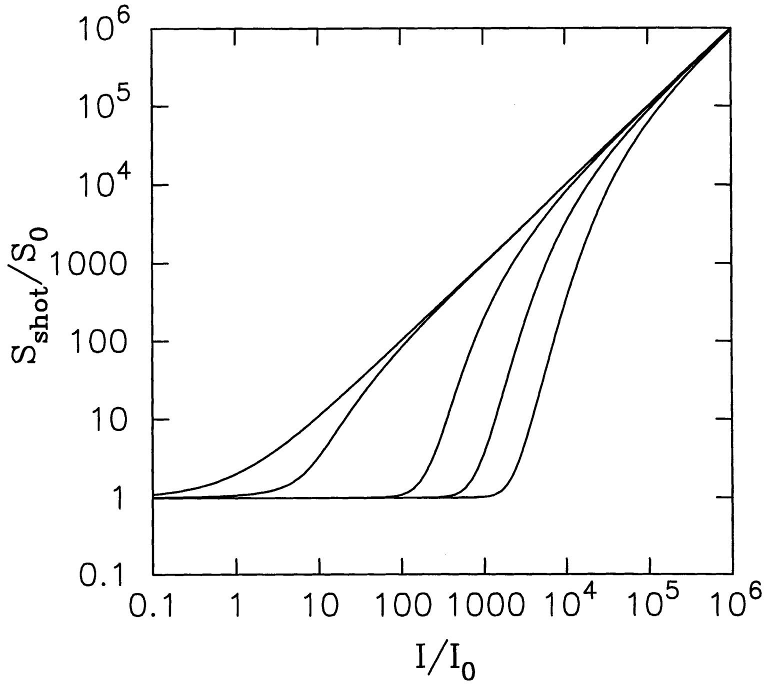


Fig. 4. Three-dimensional (3D) metallic wire of macroscopic thickness. This is our closest example to that of Fig. 1 in Gomila and Reggiani (2000). The overall concordance of the two results is striking.



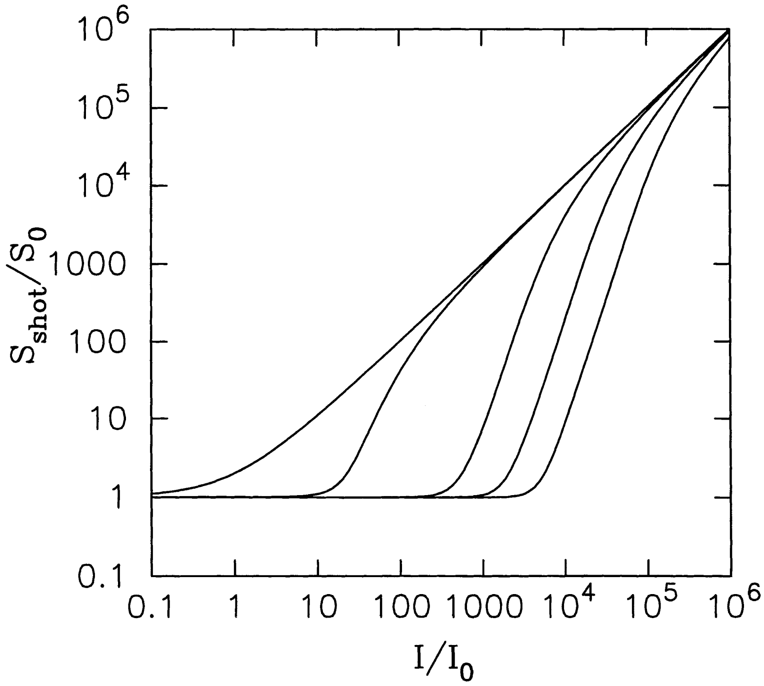


Fig. 5. (*a*) (top) A narrow 3D metallic wire, with radius $R = 0.3\ \lambda$. Note the large loss in shot-noise level, and its recovery at higher currents than for Fig. 4, the large-radius example. (*b*) (bottom) A very narrow wire, $R = 0.05\ \lambda$. The attenuation is severe, with recovery only at the highest values of current.

thickness, suggests a novel range of experiments to complement those proposed for shot-noise resurgence as a function of length (Gomila and Reggiani 2000). Such experiments will in any case require exploration of intrinsically *high-field*, *high-current* behaviour. This is a region dominated by inelastic collisions, and one that has been almost totally neglected in metallic mesoscopic systems.

Space prevents an extended presentation of our work. We invite readers to examine the complete exposition of our general kinetic approach to noise in Green and Das (2000*b*, 2000*c*), as well as the specifics of shot noise in Green and Das (1998*a*).

Before discussing the work of Gomila and Reggiani (2000), we remark on the striking graphical similarity between their calculation of $S_{\mathrm{shot}}(E)$ and the present one. On the other hand, Gomila and Reggiani's theory for the anomalous rise of macroscopic shot noise apparently depends, in large degree, on the role of long-range Coulomb correlations. In our results, Coulomb effects are completely absent in the sample. Nevertheless, the behaviour of the shot noise is practically the same in our inelastic free-electron model.

4. The Gomila–Reggiani Theory

The recent paper of Gomila and Reggiani (2000) presents a much-needed invitation to reassess the microscopic basis of shot noise (unavoidably this brings in the questionable status of the 'smooth crossover'). Certainly Gomila and Reggiani make their argument on the overriding idea of shot noise as a number-fluctuation phenomenon, much in the way of our own philosophy, as already discussed elsewhere (Green and Das 1998*a*, 2000*a*). However, there are some differences between the respective approaches. For example, Gomila and Reggiani propose a drift–diffusive model. This entails additional, intuitive assumptions meant to simplify the underlying transport problem. By contrast, we work directly with the semi-classical kinetic equation. The main points of difference are:

- **Diffusive method**. The drift–diffusive equation of motion embodies a model for current fluctuations in which the diffusion constant D (in microscopic terms, a current–current correlator) appears as a simple scaling parameter for the evolution of the current fluctuations themselves. This means that the solution preconditions its own scaling factor D. If one stays rigidly within the linear-response limit, the Einstein relation can be invoked to constrain the results (Datta 1995; Kogan 1996). This relation is no longer valid for high-field shot noise (Green and Das 2000*b*). Once pushed out of the weak-field limit, drift–diffusive theories face a complex problem of self-consistency in estimating D. The problem is highly non-linear and ill-controlled. In the standard Boltzmann approach, there is no *a priori* distinction between 'drift' and 'diffusion'. The exact non-equilibrium kinetic equation will always be linear in the basic electron–hole pair fluctuations.
- **Langevin sources**. Gomila and Reggiani follow the popular stratagem of generating *single-particle* fluctuations only, in a drift–diffusive setting. This is done by adding *ad hoc* stochastic source terms (Langevin's Ansatz) to the equation of motion (Kogan 1996). The low-order correlators within the phenomenology must be set by hand to meet the presumed constraint of Einstein's relation. The status of this low-field stochastic Ansatz is unclear in high-field situations; it is certainly no clearer for carrier populations with strong internal interactions. Such ambiguities arise simply because the imposition of extraneous, stochastic current sources has no physical or logical basis in the microscopics of an *internally* correlated system (van Kampen 1981). Reliable kinetic descriptions of noise can be set up with no appeal at all to Langevin sources of

the kind adopted for the Gomila–Reggiani model. This is as true of non-degenerate noise (Korman and Mayergoyz 1996) as it is for the semi-classical picture of electron–hole polarisation fluctuations in metallic systems (Green and Das 2000*b*).

- ***T*-dependence**. Equations (12) and (16) of Gomila and Reggiani (2000) both give an overall scaling of their S_{shot} with Johnson–Nyquist noise S_0, and hence with temperature. The shot noise of equation (16) in particular will then be manifestly T-dependent, unlike true shot noise, unless there is a counterbalancing thermal denominator in the non-linear term giving the shot-noise contribution. Such a factor will cancel the thermal dependence introduced through S_0. While this is likely to be so in the classical high-T limit, where high-field excess noise has little dependence on T (Green and Das 1998*a*, 2000*b*), it is not clearly so in the degenerate regime of their model. In that limit, the explicit temperature behaviour of the relevant parameters L_D and I_R is not given. Unless that behaviour is known, one cannot say whether the theory of Gomila and Reggiani recovers *true temperature-independent shot noise* in bulk metallic wires. A direct check of equations (12) and (16) yields no countervailing factor to undo the T-dependence entering through S_0. Hence S_{shot} must scale with T in the Gomila–Reggiani model at strong carrier degeneracy.

5. Summary

We have reviewed some prior results for shot noise in degenerate conductors. We conclude that the anomalous recovery of robust shot noise at bulk scales (where it is normally extinguished by inelastic scattering) depends on pushing the system to large enough currents. In such a strongly non-equilibrium limit, the average transit time of a carrier is well given by eN/I. When this becomes less than the typical scattering time, carriers are ballistic and the high-field shot noise reaches its ideal Schottky value of $2eI$.

A simple but *strictly* kinetic model, informed by a time-of-flight understanding of shot noise, gives a consistent microscopic picture of such noise. The model emphasises plain inelastic scattering in a strongly driven conductor, rather than more subtle and higher-order field effects. On the basis of its clear results, it suggests that Coulomb-fluctuation corrections need not be fundamental to the physics of shot noise in long conductors. Furthermore, recovery of the noise at high currents is not sensitive to quantum statistics. This is because the energy scale for transport will eventually outstrip even a large Fermi energy.

We have demonstrated the resurgence of true *temperature-independent* shot noise at high currents, even in long thin 3D samples. For a set level of the current, shot noise should certainly be much stronger in samples that are relatively wider as well as shorter. With this prediction, namely the inhibition of high-field shot noise in a constricted geometry, we advance an altogether different and major opportunity for new experiments.

We share one common idea with the approach to shot noise proposed by Gomila and Reggiani. It is the importance of (necessarily discrete) number fluctuations as generators of shot noise. This departs from thermal noise, whose character derives from distributed and continuous random changes in the carriers' free energy. In a real sense, we are brought right back to basic thermodynamics. That is because thermal and shot-noise fluctuations are echoes of the thermodynamic conjugacy of the equilibrium variations $\delta\mu/k_{\mathrm{B}}T$ and $\delta\ln N$.

Aside from the crucial difference over the significance of Coulomb corrections in the resurgence of bulk shot noise, there are major differences of method between Green and

Das (1998*a*) and Gomila and Reggiani (2000). The main one is our systematic adherence to strict Boltzmannian kinetics and Fermi-liquid theory (Green and Das 2000*b*). Decidedly, this sets our investigations apart from every one of the drift–diffusive (or so-called Boltzmann–Langevin) works that proliferate in the noise literature. That includes the work by Gomila and Reggiani.

Boltzmann–Langevin phenomenology relies (a) on fictitious stochastic sources—said to generate the individual, one-body current fluctuations—and (b) on essentially classical diffusion to evolve such single-particle objects. Neither (a) nor, worse, (b) makes any identifiable connection with the *electron–hole pair symmetry* that is essential to the actual make-up of the microscopic correlations. It is remarkable that this electron–hole asymmetry of diffusively driven transport models (Büttiker 1986) persists all the way up to the macroscopic scale of the device leads (Fenton 1994). Because this symmetric behaviour is even asymptotic, drift–diffusive descriptions cannot be rid of it.

Each of the assumptions above is equally baseless when it comes to charged Fermi liquids at the level of microscopic many-body physics (Green and Das 2000*b*). The price of their phenomenological simplicity is, precisely, an unphysical electron–hole asymmetry. The consequence of that is the failure of drift–diffusive fluctuations to recover the correct electronic compressibility. Hence, they also fail to meet a most basic condition, namely that metals will not sustain inhomogeneous electric fields beyond the Thomas–Fermi screening length (Green and Das 2000*a*).

Langevin stochastics and pseudo-classical diffusion fail to conform to orthodox quantum kinetics for noise in charge transport, despite frequent claims to the contrary (Kogan 1996). Such artifices are deeply foreign to the real nature of a degenerate, polarisable electron plasma. There, electron–hole pair dynamics, the conservation laws, and the sum rules are utterly central to the physics (Pines and Nozières 1966).

The demonstrable violation by drift–diffusive theories of the sum rules, and neglect of the conservation laws embodied in those rules, provides stark evidence of non-conformity (Green and Das 2000*a*, 2000*b*). It is a state of affairs which seems all the more surprising in that electron-gas theory is extremely well developed, not to say mature. Its essential features have been largely understood—and widely disseminated—for forty years at least. Nevertheless, its lessons (or even its relevance) appear not to have been fully absorbed.

In passing we have called attention, once again, to the problematic status of the ‘smooth crossover’ proclaimed by drift–diffusive phenomenology. We suggest that the time is ripe for a thorough microscopic reassessment of this largely intuitive construct, and most of all for a renewed search for decisive experimental tests of it.

New information on the ‘smooth crossover’ can be expected in two experimental contexts. The first is in the low-field noise signal from quantum-confined devices (Green and Das 2000*a*, 2000*c*). The second is the high-field regime as indicated by us (Green and Das 1998*a*, 2000*b*, 2000*c*), by Naveh (1998) to some extent, and, latterly, by Gomila and Reggiani (2000). In every situation it is important to have a unifying microscopic description, equipped to cover *all* of the many facets of real shot noise. This would be the only way to make the most of any fresh experimental knowledge. It seems most unlikely that piecemeal, *ad hoc* phenomenologies will serve much longer.

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Manuscript received 12 May, accepted 31 July 2000