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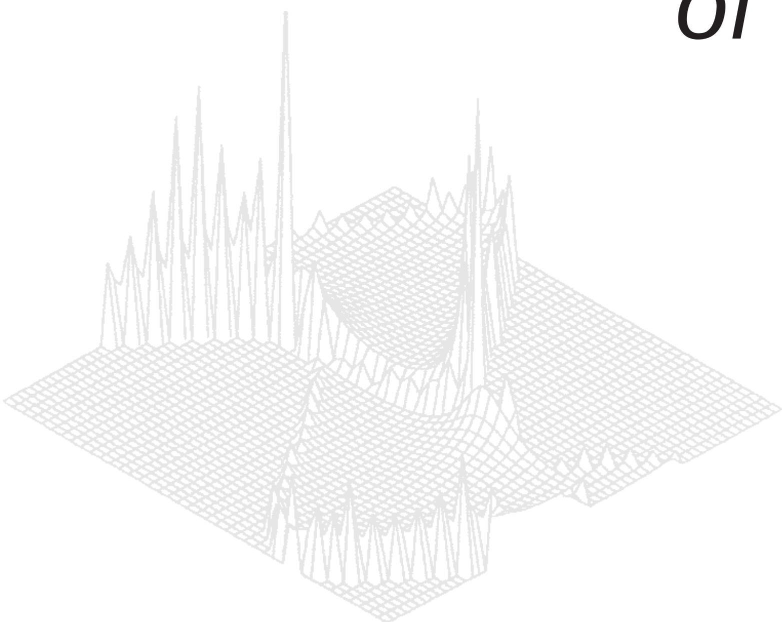

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Generation of Multicavity Entangled States with a Single Three-Level Atom

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Abstract

A simple scheme is proposed for the generation of maximally entangled states for several separated cavities, in which each cavity is in a one-photon state or in the vacuum state. In the scheme a ladder-type three-level atom is sent through the cavities and additional classical fields. The whole system finally evolves into a state, which is given by the product of the highly entangled field state with an atomic state.

In recent years much attention has been paid to entanglement, which is one of the most striking features of quantum mechanics. The correlation between two systems can be used to test local hidden variable theories against quantum mechanics (Bell 1967). Maximally entangled states of three or more systems, referred to as Greenberger–Horne–Zeilinger (GHZ) states (Greenberger *et al.* 1989, 1990) allow a stronger test of local hidden variable theories without using Bell's inequalities. Besides the investigation of fundamental aspects of quantum mechanics, entangled states are useful in fields involving quantum information, such as quantum cryptography (Ekert 1991), quantum computation (Deutsch and Jozsa 1992), and quantum teleportation (Bennett *et al.* 1993).

A scheme, based on the resonant atom–field interaction, has been proposed for preparing two two-level atoms in a maximally entangled state (Cirac and Zoller 1994; Kudryavtsev and Knight 1993; Phoenix and Barnett 1993). Recently, such a scheme has been experimentally realised (Hagley *et al.* 1997). It has also been shown that a GHZ state of three atoms can be generated using the resonant atom–field interaction if the field is initially prepared in a superposition of a three-photon state and the vacuum state (Cirac and Zoller 1994). Other cavity QED methods (Gerry 1996*a*, 1996*b*; Zheng 1998, 1999) have also been proposed for the preparation of multi-atom GHZ states. In a very recent paper, Sackett *et al.* (2000) have reported experimental entanglement of four trapped ions using a new technique proposed. On the other hand, three-photon GHZ entanglement has also been observed (Bouwmeester *et al.* 1999; Pan *et al.* 2000).

Proposals have been suggested to entangle spatially separated cavities. As an intermediate step of teleportation, Davidovich *et al.* (1994) have shown how to produce two-cavity entangled states, in which a single photon resides in either cavity. Using a combination of quantum switches Davidovich *et al.* (1993) have proposed a scheme for the generation of the entangled coherent states (Sanders 1992*a*, 1992*b*) for two cavities. Kim and Lee (2000) have suggested a nonlocal test for entangled states of two spatially separated cavities. On the other hand, Gerry (1996*a*, 1992*b*) has suggested a proposal for the mesoscopic realisation of a three-cavity GHZ state with dispersive cavity QED. Bergou and Hillery (1997)

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have presented a scheme to generate another kind of entangled state of N cavities, in which either each cavity is in a one-photon state or in the vacuum state. The scheme requires that N appropriate atoms be sent through the cavities and additional classical fields. Furthermore, it uses both resonant and dispersive atom–field interactions. In this paper we present a much simpler scheme to generate such states. Our scheme needs only one atom and does not require a dispersive atom–field interaction.

We first show how to generate a two-cavity entangled state. We consider a ladder-type three-level atom, whose states are denoted by $|i\rangle$, $|g\rangle$ and $|e\rangle$. (The energy of the state $|e\rangle$ is highest.) The atom is initially prepared in the superposition state $\sqrt{\frac{1}{2}}(|i\rangle + |e\rangle)$. We then send the atom through two identical cavities initially in the vacuum states $|0\rangle_1$ and $|0\rangle_2$ and two classical fields between them. We assume the cavity modes are resonant with the atomic transition $|g\rangle \to |e\rangle$, and the two classical fields are resonant with the transitions $|g\rangle \to |e\rangle$ and $|i\rangle \to |g\rangle$, respectively. Then, in the interaction picture, the atom–cavity interaction is described by the Jaynes–Cummings Hamiltonian

$$H_{I,j} = g(a_j^+|g\rangle\langle e| + a_j|e\rangle\langle g|), \tag{1}$$

where a_j^+ and a_j are the creation and annihilation operators for the cavity modes $j = 1, 2$, respectively, and g is the atom–cavity coupling strength.

When the atom exits the first cavity the state of the whole system is given by

$$\sqrt{\tfrac{1}{2}}[|i\rangle|0\rangle_1 + \cos(gt)|e\rangle|0\rangle_1 - i\sin(gt)|g\rangle|1\rangle_1]|0\rangle_2, \tag{2}$$

where t is the interaction time. We choose the atomic velocity carefully so that $gt = \pi/2$ is fulfilled. Then we have

$$\sqrt{\tfrac{1}{2}}[|i\rangle|0\rangle_1 - i|g\rangle|1\rangle_1]|0\rangle_2. \tag{3}$$

We adjust the classical fields appropriately so that the atom undergoes the transitions $|g\rangle \to |e\rangle$ and $|i\rangle \to |g\rangle$ during its passage through them. Then we have

$$\sqrt{\tfrac{1}{2}}[|g\rangle|0\rangle_1 - i|e\rangle|1\rangle_1]|0\rangle_2. \tag{4}$$

After the atom exits the second cavity the state of the system is

$$\sqrt{\tfrac{1}{2}}[|0\rangle|0\rangle_2 - |1\rangle_1|1\rangle_2]|g\rangle. \tag{5}$$

In this way the two cavities have been prepared in the maximally entangled state, with the atom left in the state $|g\rangle$.

Now we show how we can entangle three cavities. In order to do so we set a classical field inducing the transition $|g\rangle \to |e\rangle$ between the first cavity and second cavity, and two classical fields inducing the transitions $|g\rangle \to |e\rangle$ and $|i\rangle \to |g\rangle$ between the second cavity and third cavity. After the atom passes through the first cavity and the classical field between the first cavity and second cavity, the system is in the state

$$\sqrt{\tfrac{1}{2}}[|i\rangle|0\rangle_1 - i|e\rangle|1\rangle_1]|0\rangle_2|0\rangle_3. \tag{6}$$

After the atom exits the second cavity the system evolves into

$$\sqrt{\tfrac{1}{2}}[|i\rangle|0\rangle_1|0\rangle_2 - |g\rangle|1\rangle_1|1\rangle_2]|0\rangle_3. \tag{7}$$

Before the atom enters the third cavity the system is in the state

$$\sqrt{\tfrac{1}{2}}[|g\rangle|0\rangle_1|0\rangle_2 - |e\rangle|1\rangle_1|1\rangle_2]|0\rangle_3. \tag{8}$$

After the atom exits the third cavity the system evolves into

$$\sqrt{\tfrac{1}{2}}[|0\rangle_1|0\rangle_2|0\rangle_3 + i|1\rangle_1|1\rangle_2|1\rangle_3]|g\rangle. \tag{9}$$

Thus, the three cavities are prepared in a GHZ state with the atom left in the state $|g\rangle$.

We note the scheme can also be used to prepare three-atom GHZ states. In order to do so we only require two cavities. The first atom, initially prepared in the superposition state $\sqrt{\frac{1}{2}}(|i\rangle_1 + |e\rangle_1)$, is sent through the first cavity, a classical field tuned to the transition $|g\rangle \to |e\rangle$, the second cavity, and two classical fields tuned to the transitions $|g\rangle \to |e\rangle$ and $|i\rangle \to |g\rangle$. In this way the system combined by the first atom and two cavities is prepared in the state

$$\sqrt{\tfrac{1}{2}}[|g\rangle_1|0\rangle_1|0\rangle_2 - |e\rangle_1|1\rangle_1|1\rangle_2]. \tag{10}$$

Then the second atom is sent through the first cavity, while the third atom is sent through the second cavity, respectively. We assume that the two atoms are initially in the states $|g\rangle_2$ and $|g\rangle_3$. In this way the state of each cavity is replicated onto the respective atom. This results in

$$\sqrt{\tfrac{1}{2}}[|g\rangle_1|g\rangle_2|g\rangle_3 + |e\rangle_1|e\rangle_2|e\rangle_3], \tag{11}$$

with the two cavities left in the vacuum states.

We note the method can be further generalised to generate maximally entangled states of N spatially separated cavities. For this purpose we set a classical field tuned to $|g\rangle \to |e\rangle$ between the $(n-1)$th cavity and nth $(1 < n < N)$ cavity, and two classical fields tuned to $|g\rangle \to |e\rangle$ and $|i\rangle \to |g\rangle$ between the $(N-1)$th cavity and Nth cavity. In this case the whole system finally evolves into

$$\sqrt{\tfrac{1}{2}}[|0\rangle_1|0\rangle_2 \cdots |0\rangle_N + (-i)^N|1\rangle_1|1\rangle_2 \cdots |1\rangle_N]|g\rangle. \tag{12}$$

In order to test quantum nonlocality of the N cavities we use N atoms. The kth $(1 \le k \le N)$ atom, initially in the state $|g\rangle_k$, is sent through the kth cavity. This leads to the maximally entangled state of the N atoms

$$\sqrt{\tfrac{1}{2}}[|g\rangle_1|g\rangle_2 \cdots |g\rangle_N + (-1)^N|e\rangle_1|e\rangle_2 \cdots |e\rangle_N], \tag{13}$$

with each cavity left in the vacuum state. In this way the state of each cavity is replicated onto the respective atom. The Pauli operator along any direction for each atom can be measured using the combination of a classical field and two ionisation detectors, which count the atoms in $|g\rangle$ and $|e\rangle$, respectively (Freyberger 1995).

It is necessary to present a brief discussion on experimental matters. For the Rydberg atoms with principal quantum numbers 49, 50 and 51, the radiative time is about $T_r = 3 \times 10^{-2}$ s, and the coupling constant is $g = 2\pi \times 24$ kHz (Brune *et al.* 1996). Thus the interaction time of the atom with each cavity field is $\pi/2g \simeq 10^{-5}$ s. Then the time needed to complete the whole procedure can be assumed to be 10^{-4} s, much shorter than T_r.

Cavities with a quality factor $Q = 10^8$ are experimentally achievable (Brune *et al.* 1996). In the present case the cavity field frequency is $\nu = 51.099$ GHz. Thus the photon lifetime is $T_c = Q/2\pi\nu \simeq 3.0 \times 10^{-4}$ s. The lifetime of the state of the form of equation (9) is about 10^{-4} s, of the order of the whole time needed to complete the procedure. Therefore, based on cavity QED techniques presently, or soon to be, available our scheme might be realisable.

In summary, we have proposed a simple scheme to prepare the maximally entangled state of multiple cavities, in which either each cavity has one photon or each cavity is in the vacuum state. In contrast with the previous scheme, which requires N atoms sent through the cavities, and uses both resonant and dispersive atom–field interactions, our scheme employs only one atom and resonant interaction. Thus, our scheme has the advantage that the procedure is much simpler and thus the time needed to complete the procedure is greatly reduced, which is important in view of decoherence. Our scheme provides a new way for the test of quantum nonlocality and quantum information processing.

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