FLIGHT CHARACTERISTICS OF EXPANSIBLE BALLOONS

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Summary

A general study is made of the factors governing the flight of rubber balloons for cosmic ray research, and a brief account is given of the methods that have been used to obtain long exposures at great heights. The results of incidental upper air wind observations in Victoria over a period of 18 months are included.

I. Introduction

Cosmic radiation investigations are being carried out at the Melbourne University by studying nuclear emulsion photographic plates which have been exposed to the radiation at high altitudes. It is desirable that the plates should spend a long time at altitudes greater than 80,000 ft and, moreover, have a high probability of being recovered.

Non-expanding balloons made of thin plastic sheet stabilize at a height fixed by their size and load and hence can give long exposures. However, such balloons are expensive and difficult to launch and all our work has been done with relatively inexpensive rubber balloons. Such a balloon inflated and sealed at the ground will rise at a nearly constant rate until it bursts. The present paper discusses the behaviour of these balloons together with methods of achieving long flights and improving recovery probabilities.

II. Rate of Rise

For values of Reynolds number met in balloon flights, the resistance to a rising balloon is determined by the transfer of momentum to the air (Gibson 1923); then for a spherical balloon of radius r, rising at a speed v, the free lift F in gravitational units is equal to the air resistance.

$$F=\pi k\rho v^2r^2/g, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where the drag coefficient k is a dimensionless quantity whose value depends on the Reynolds number R, πr^2 is the cross-sectional area of the balloon, and ρ is the density of the surrounding air.

If G is the gross lift, then the speed v is proportional to

$$F^{1/2}/k^{1/2}G^{1/3}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

It follows, as Mallock (1907) has shown, that a spherical balloon should rise at a speed proportional to the inverse sixth root of the density of the air surrounding it. Clarke and Korff (1941) have derived the equivalent result that the speed is proportional to the square root of the balloon radius.

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Figure 1 shows some properties of the standard atmosphere of the International Committee for Air Navigation (Aeronautical Research Council 1952). Curve A shows the density relative to ground ρ/ρ_0, and curve B shows $(\rho/\rho_0)^{1/6}$, as functions of height. Integrating curve B (Fig. 1) gives the height reached by a balloon as a function of $v_0 t$, where v_0 is the initial speed, and t the elapsed time, provided k remains constant.

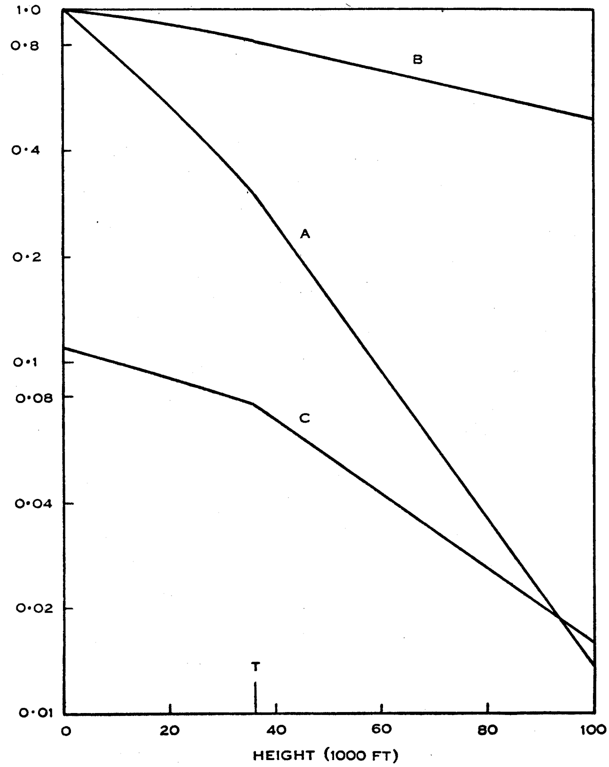

Fig. 1.—Curve A, density relative to ground ρ/ρ_0; curve B, $(\rho/\rho_0)^{1/6}$; curve C, $10^{-5} RF^{-\frac{1}{2}}$ gram$^{\frac{1}{2}}$ (R=Reynolds number, F=free lift) plotted against height for I.C.A.N. standard atmosphere. T is the tropopause.

Above a certain critical value of R, k is fairly constant and has a value near $0{\cdot}1$; at the critical value of R it increases suddenly and for lower values of R has a value k' near $0{\cdot}2$–$0{\cdot}3$. Therefore, if R decreases to the critical value during the flight, the resistance will increase suddenly and the speed will fall in the ratio of the square roots of the values of k.

Now Reynolds number

$$\begin{aligned} R &= rv\rho/\eta \\ &= (g\rho F/\pi k\eta^2)^{\frac{1}{2}}, \qquad \ldots\ldots\ldots\ldots\ldots \quad (3) \end{aligned}$$

where η is the viscosity of the air and for balloons of common size R lies between 10^4 and 10^6 and decreases as the balloon rises. From relation (3), in the stratosphere where η is constant R is proportional to $\rho^{\frac{1}{2}}$.

Figure 2 shows the height *v.* time variation for a flight of a 2000 g balloon with a load of 1000 g and a free lift of 1000 g. The points are heights calculated from observations made with two theodolites and the curve is the theoretical height-time variation for an initial speed of 1100 ft/min. At 53,500 ft there is a sudden change of speed by a factor $0 \cdot 6$; this corresponds to a change of k in the ratio $0 \cdot 36$ which agrees with the values found by Ewald, Pöschl, and Prandtl (1923) in wind tunnel experiments. However, later work (Goldstein 1938) has shown fairly wide variations in k/k', values between $0 \cdot 4$ and $0 \cdot 2$ being found, and also in the critical value of Reynolds number which varies from 10^6 to 10^5. In Figure 1 curve C, $R/F^{\frac{1}{2}}=(g\rho/\pi k\eta^2)^{\frac{1}{2}}$ is plotted for $k=0 \cdot 1$; at 53,500 ft with

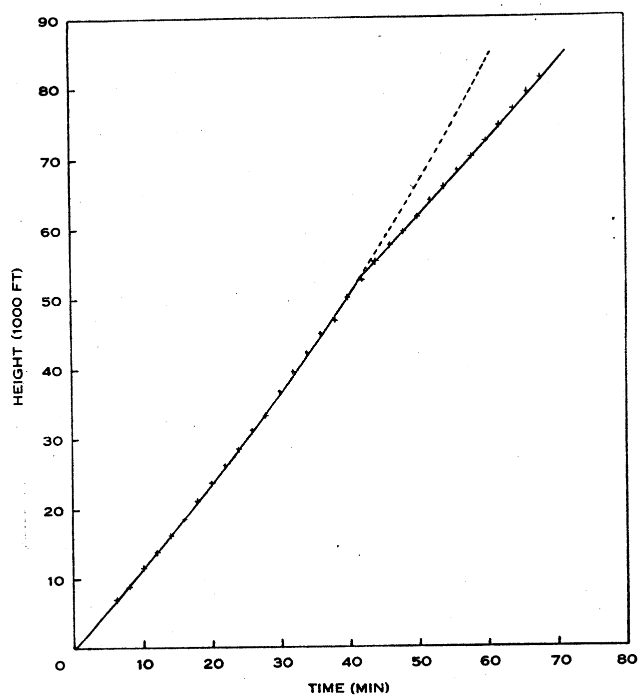

Fig. 2.—Flight of a 2000 g balloon. Solid line shows theoretical height with k changing by $1/0 \cdot 6$ at 53,500 ft ; crosses are observed heights. Dotted line shows expected height if k does not change.

free lift 1000 g the calculated critical value of R is $1 \cdot 5 \times 10^5$. Two other flights that have been analysed in detail have given speed changes by factors of $0 \cdot 7$ and $0 \cdot 9$ at Reynolds numbers of $1 \cdot 9 \times 10^5$ and $1 \cdot 6 \times 10^5$ respectively, which are in the range mentioned above. The result of this investigation has therefore shown that a variation of rate of rise does occur in flights with these large balloons which is in general agreement with theory. For most purposes the standard method of assuming a constant rate of rise is sufficiently accurate.

III. Excess Pressure in Spherical Balloons

Owing to the tension in the rubber, the pressure inside a rubber balloon exceeds the external pressure when the diameter is greater than the flaccid

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diameter. This excess pressure must not be neglected when considering the small free lifts required for nearly stabilized balloons at great heights. If it is assumed that Poisson's ratio for the balloon rubber remains at 0·5 and Young's modulus stays constant throughout the expansion of the balloon, then the excess pressure Δp at radius r is given by

$$\begin{aligned} \Delta p &= 0, && \text{for } n \leqslant 1, \\ \Delta p &= (CM/r_f^3)(n-1)/n^3, && \text{for } n > 1, \end{aligned} \qquad (4)$$

where r_f = flaccid balloon radius,

$n = r/r_f$,

M = mass of balloon rubber,

C = constant depending on the elastic moduli of the rubber and its density.

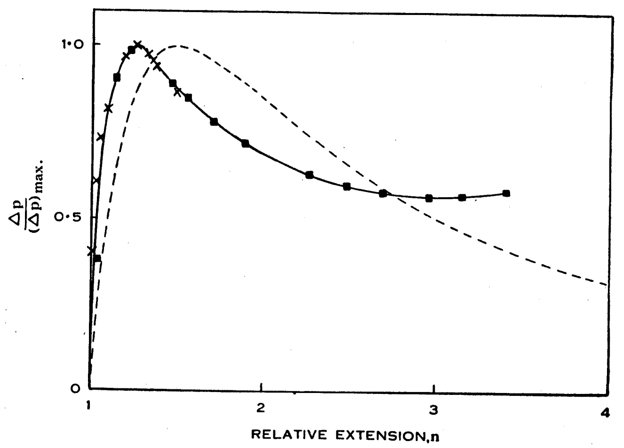

Fig. 3.—Ratio of excess pressure to its maximum value as a function of relative extension n. Squares are experimental values for a 700 g balloon, crosses for a 2000 g balloon. Dotted curve theoretical values from equation 4. $\Delta p_{max.} = 18 \cdot 3$ mm H_2O for 700 g, 7·9 mm H_2O for 2000 g.

Figure 3 compares the variation of excess pressure obtained from the above expression with experimental values for two sizes of " Beritex " balloons made by the Guide Bridge Rubber Company. The excess pressures were measured with a micrometer water manometer. As might be expected from the assumptions used in its derivation, relation (4) does not usefully represent the variation of excess pressure with relative inflation, particularly in that it fails to predict a further rise after the initial maximum.

Väisälä (1937) has previously measured the excess pressure variation inside balloons of less than 70 cm flaccid diameter. He found that the excess pressure Δp is given by

$$\Delta p = 2d_0 P(n)/r_f, \qquad (5a)$$

where d_0 is the thickness of unstretched balloon rubber. $P(n)$ is characteristic of the rubber and is a complicated function empirically determined to be given by

$$P(n)=(\tau/n^3)\ \exp\ [a(n-1)-b/(n-1)], \quad \text{(5b)}$$

τ, a, and b and hence the shape of the excess pressure curve are constant for all balloons made of the same rubber. This agrees with our experimental results since curves of $\Delta p/(\Delta p)_{\text{max.}}$ for the 700 and 2000 g balloons coincide (Fig. 3).

Both relations (4) and (5) give the ratio of the excess pressures at the same relative extension of two balloons made from the same rubber as

$$\frac{\Delta p_1}{\Delta p_2}=\frac{M_1}{M_2}\left(\frac{r_{f2}}{r_{f1}}\right)^3. \quad \text{(6)}$$

For the balloons used in the present experiment this gives

$$\frac{\Delta p_{2000\ \mathrm{g}}}{\Delta p_{700\ \mathrm{g}}}=0\cdot 42.$$

The experimental value was 0·44.

IV. Height Stabilization of Rubber Balloons

In order to keep a balloon at great heights over long periods, it is necessary that its free lift be made very close to zero at the required height. Because of the excess pressure inside a rubber balloon its free lift is not quite independent of height but diminishes slowly. This effect can be used to fix the ceiling height of a balloon by giving it a small initial free lift which can be calculated by reference to Figure 3 and is reduced to zero at the required ceiling height. However, the initial free lift needed to do this is small ($\sim$40 g for a 700 g balloon+400 g load to stabilize at 80,000 ft) so the rate of rise would be very slow and the time necessary to reach the proposed ceiling height might exceed the balloon's life, or the wind might take it out of sight before the ceiling height is reached. Also it is difficult to set the free lift accurately to the required value. A fairly rapid rise to the ceiling is wanted and the free lift must then be removed. We have tried to do this in the following ways :

(i) A commonly used method is to have strings of balloons which burst in succession until the free lift becomes negative ; then the remaining balloons carry the load slowly back to ground. With this method there is always something to sight on and to mark the load after falling, but one cannot know how much rubber from the burst balloons will remain as load. This is serious when few balloons are used.

(ii) The disadvantage of method (i) can be overcome by placing one balloon inside another. Then when the outer one bursts, the inner one is left unencumbered. This restricts the size of the load to what can be forced through the neck of a balloon ; also it was found that there was a danger of the inner balloon bursting at the same time as the outer one.

(iii) Another way to separate two balloons completely is to use a release mechanism, operated by a clock, and then the load can be bulky. In practice, the clock was set so that when the pair of balloons had reached about 70,000 ft the upper one which had provided most of the lift was disconnected and the lower balloon, which carried the plates, rose more slowly until it burst. In methods (ii) and (iii) the free lift of one of the balloons must be set so as to have a low value at the ceiling, and this was found to be difficult to arrange in practice.

(iv) A simple way to remove the excess free lift at a preset height is to release the corresponding amount of gas by means of a valve operated by a string attached across a diameter (Hopper and Wilson 1953). A wide tube is inserted in the neck of the balloon; we have found a diameter of $1\frac{1}{4}$ in. satis-

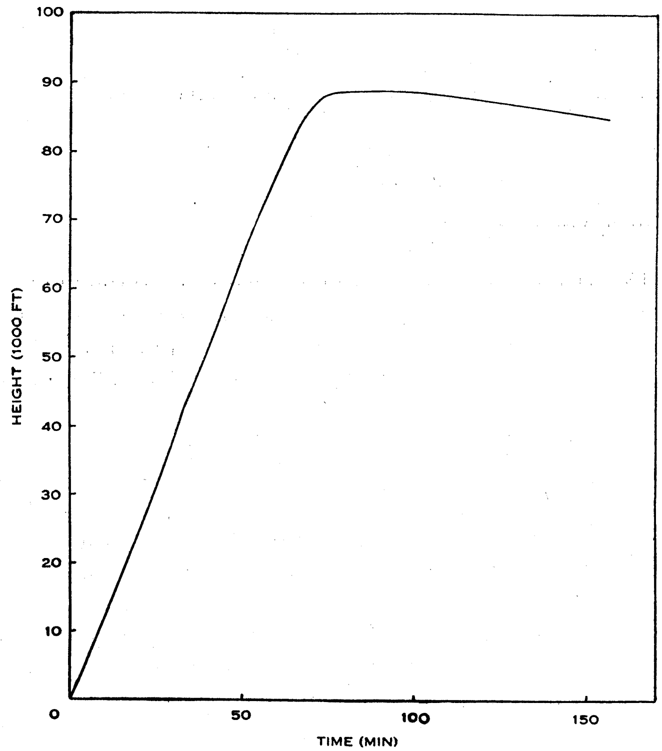

Fig. 4.—Flight of a 2000 g balloon with string-operated valve.

factory. A valve rests on top of the tube and a string from the valve is attached to the opposite end of the diameter of the balloon. When the balloon reaches a height at which the string becomes taut the valve opens and gas is released until the gross lift equals the load. Figure 4 shows a height-time curve for a flight of this kind.

(v) Any levelling-off process prevents the balloon from reaching its maximum (bursting) height. If maximum height as well as a long flight is wanted, the balloon must retain a small positive free lift. This can be achieved by first using the levelling-off process described in (iv) with a length of string such that the balloon levels off at a diameter below its bursting diameter.

When the balloon has levelled off, a clock closes the valve and then drops a small weight. This will cause the balloon to rise again until it reaches its bursting diameter or until the tension in the rubber causes it to level off at a new height.

This new height can be found in the following way. Assuming that the balloon has levelled off at a radius r_1, pressure p_1, and gross lift G_1 equal to the total load, then

$$G_1=\frac{4}{3}\pi r_1^{3}(\rho_1^{\mathrm{A}}-\rho_1^{\mathrm{H}}),$$

where ρ_1^{A} is the density of the air at pressure p_1,
ρ_1^{H} is the density of hydrogen at pressure p_1.

The pressure of hydrogen is in fact $(p_1+\Delta p_1)$ but $\Delta p_1 \ll p_1$ at all heights considered. If the valve is now closed and a small weight dropped, the load is changed to G_2, and the balloon will stabilize at a new pressure level p_2, with radius r_2 such that

$$G_2=\frac{4}{3}\pi r_2^{3}(\rho_2^{\mathrm{A}}-\rho_2^{\mathrm{H}}),$$

where ρ_2^{A} and ρ_2^{H} are densities at the new pressure.

Then

$$\frac{G_1}{G_2}=\frac{r_1^{3}}{r_2^{3}}\cdot\frac{\rho_1^{\mathrm{A}}-\rho_1^{\mathrm{H}}}{\rho_2^{\mathrm{A}}-\rho_2^{\mathrm{H}}}\approx\frac{r_1^{3}\rho_1^{\mathrm{A}}}{r_2^{3}\rho_2^{\mathrm{A}}}=\frac{r_1^{3}T_2p_1}{r_2^{3}T_1p_2}, \quad \ldots\ldots\ldots\ldots (7)$$

where T_1, T_2 are temperatures at the two heights.

Now considering the hydrogen

$$(p_1+\Delta p_1)r_1^{3}/T_1=(p_2+\Delta p_2)r_2^{3}/T_2.$$

Substitute in (7)

$$(p_1+\Delta p_1)T_2/(p_2+\Delta p_2)T_1=G_2p_1T_2/G_1p_2T_1,$$

or

$$\Delta p_2/p_2=G_1(1+\Delta p_1/p_1)/G_2-1.$$

From Figure 3, Δp is approximately constant for $n>2$. Hence

$$p_2=\Delta p_1/[G_1(1+\Delta p_1/p_1)/G_2-1].$$

Loss of gas through expanded pin-holes which sometimes occurs in balloon rubber has so far prevented us from stabilizing a balloon at a new height after dropping a weight.

V. Tracking

We have followed balloons usually by the double theodolite method and occasionally, through the courtesy of the Commonwealth Meteorological Branch, by radar. On good days balloons can still be seen 90 miles or so away, though they would usually be lost in haze at much shorter distances. To follow a complete flight it is desirable that the balloon should not recede more than about 50 miles. Fortunately it has been found that the wind structure in Victoria in clear weather in spring and summer often allows this. Figure 5 shows average winds that we have observed in flights over a period of 18 months. We have plotted westerly and southerly components and standard deviation following Brooks, Durst, and Carruthers (1946). The easterly component above 70,000 ft during spring and summer has also been observed by the Commonwealth

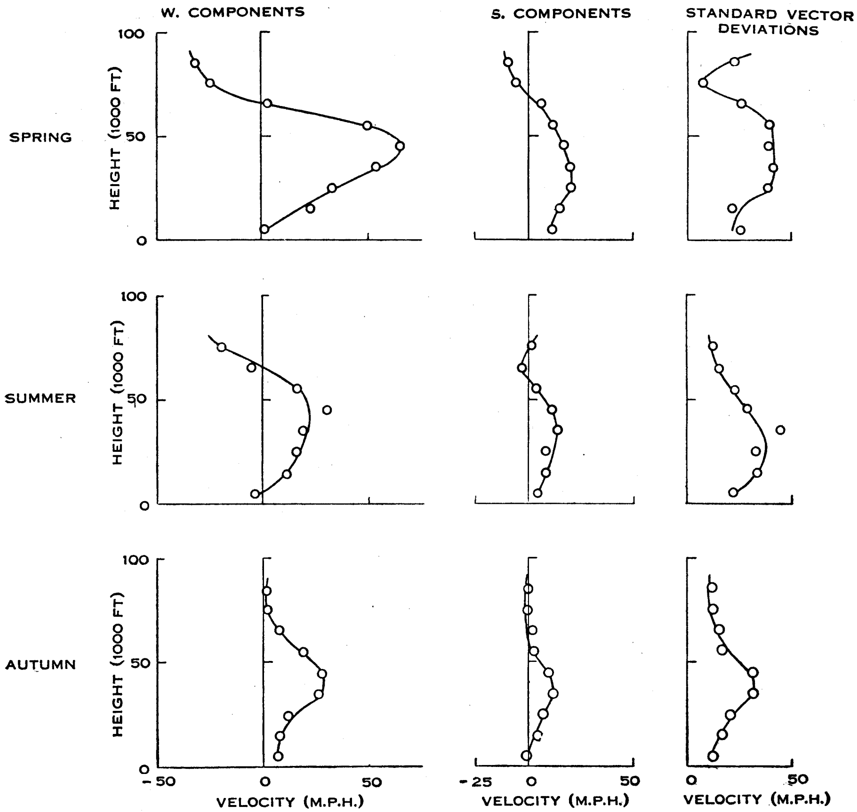

Fig. 5.—Seasonal mean winds and standard vector deviations as functions of height.

Meteorological Branch over South Australia (Trefry, personal communication 1952) and a similar pattern by the New Zealand Meteorological Service over Nandi and Auckland (Porter 1952).

Often from 20,000 to 60,000 ft the wind is between SW. and W. and above 70,000 ft between E. and NE., so the rising balloon travels east and returns. A stabilized balloon may remain so long in the easterly wind at its maximum height that it travels further than it did in the stronger westerly below. Figure 6 shows the path followed by the balloon in the flight shown in Figure 4. The dotted part is deduced from the wind structure disclosed by the observed part of the flight. This return also helps recovery, as eastern Victoria is mountainous and thinly populated. Even so it is best to attach a marker that is easily seen and looks strange enough to attract the passer-by. A sphere made of a light frame covered with aluminium foil was first introduced by Mr. G. R. Trefry of Commonwealth Meteorological Branch and is quite suitable. In sunshine it can be distinguished several miles away and the recovery rate in Victoria since using such markers has been 29 out of 33 balloons, whereas previously it was 25 per cent.

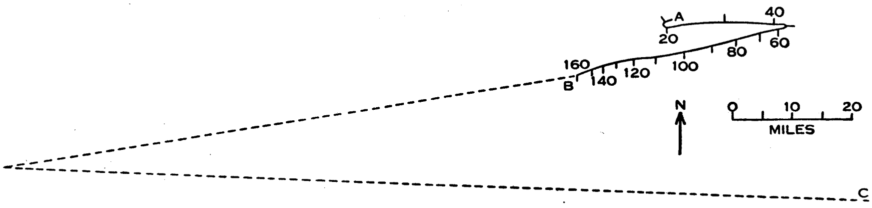

Fig. 6.—Ground projection of track of a balloon, stabilized by a valve, released at *A*, observed to *B*, and found at *C*. Solid curve is observed path, dotted curve path deduced from observed winds. Graduations on curve are times from release in minutes.

VI. Acknowledgments

The work described in this paper summarizes a programme of work carried out by a team engaged in cosmic ray research at the University of Melbourne. Other members of the team who have contributed to preparing balloon flights and taking observations are Dr. R. Parsons, Dr. S. Biswas, Miss F. Brisbout, and recently Mr. I. Macaulay and Mr. Y. K. Lim. Special thanks must be given to Mr. H. A. Waters for technical assistance.

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