SPACE CHARGE WAVE AMPLIFICATION IN A SHOCK FRONT AND THE FINE STRUCTURE OF SOLAR RADIO NOISE

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[*Manuscript received August* 17, 1953]

Summary

The Mott-Smith (1951) interpolation method gives a non-Maxwellian velocity distribution for the particles in a shock front. The dispersion equation corresponding to the non-Maxwellian distribution is derived by Vlasov's (1945) formula. The roots of the dispersion equation indicate frequency bandwidths of space charge wave amplification that decrease with the shock strength. It is suggested, in agreement with Denisse and Rocard (1951), that the storm bursts of narrow bandwidth originating in shock fronts constitute the elementary fine-structure components of solar radio noise bursts.

I. Introduction

Solar radio noise bursts have a fine structure (Blum and Denisse 1950 ; Reber, unpublished data ; Wild 1951) in the frequency bandwidth of amplification, of the order of a few per cent. of the base frequency. Denisse and Rocard (1951) suggested the origin of these elementary storms to be the space charge wave amplification in a shock front. They drew attention to the fact that the electron velocity distribution in the shock front of an ionized gas will depart from the Maxwellian one, developing a secondary hump at a velocity exceeding the mean thermal motion of the electrons. Applying the Landau criterion (Landau 1946 ; Bohm and Gross 1949), they concluded that the secondary hump would be responsible for the excitation of electronic oscillations and indicated its application to the observed fine structure of solar radio noise bursts.

Denisse and Rocard, however, used the Enskog-Chapman approximation (Chapman and Cowling 1939), which, on account of its slow convergence, is applicable only to very weak shocks, as pointed out by Wang Chang (1948). They admitted in their paper that, though there was every reason to believe that the effects discussed by them would augment with the strength of the shock, their analysis did need extension to strong shocks.

In a recent paper, Mott-Smith (1951) indicated a promising method of approach to the velocity distribution in a strong shock. He pointed out that the distribution of molecular velocities in a strong shock wave in a gas would be bimodal in view of the fact " that a considerable number of the Maxwellian molecules of the bounding supersonic and subsonic streams penetrate into the centre of the shock ". He found the velocity distribution function on the

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assumption that it would be a sum of two Maxwellian terms with temperatures and mean velocities corresponding to the subsonic and supersonic streams. He further showed that his distribution function was an approximate stationary solution of the Boltzmann equation for strong shocks.

II. The Dispersion Equation

We have followed in this paper* the Mott-Smith approach, as we believe that it is applicable to strong shocks and also brings out clearly the physical concept of the bimodal velocity distribution.

We assume the gas to be completely ionized with equal numbers of positive ions and electrons. We also neglect the small polarization effect (Denisse and Rocard 1951) and assume the electrons and ions to have a common mass velocity. On account of our neglect of the polarization effect, the concentration functions will refer indifferently to the ion or the electron.† The Mach number, however, will refer to the ions, which mainly determine the shock wave.

We shall use the following notation :

e, charge of electron (in e.s.u.),
m, mass of the electron (or ion),
$\varkappa$, Boltzmann's constant,
u_α, stream velocity before shock,
u_β, stream velocity behind shock,
T_α, stream temperature before shock,
T_β, stream temperature behind shock,
γ, ratio of specific heats,
$a=2\gamma/(\gamma-1)$,
M, Mach number of stream before shock $=u_\alpha\sqrt{\{(a-2)/a\}(m/\varkappa T_\alpha)}$,
n_0, particle concentration before shock,
n_0', particle concentration behind shock,
n, particle concentration in shock centre,
u, particle velocity along the direction of propagation of shock,
$\bar{u}$, common mass velocity of electrons and ions,
$U=u-\bar{u}$,
$U'=\sqrt{(m/2\varkappa T_\alpha)}\,U$,
k=wave number of A.C. perturbation,
ω=angular frequency of A.C. perturbation.

We define as the centre of the shock front the point where the particle concentration is the mean of the concentrations before and behind the shock, that is, where

$$n=\frac{1}{2}(n_0+n_0').$$

* The basic equation (1) used in this paper has been derived in a paper on " The non-Maxwellian distribution in a shock front and the anomaly of the chromospheric temperature " (Sen 1953).

† The suffix e will be used when the concentration refers specifically to the electron.

For simplicity, we shall regard the shock centre as the representative point in the shock front and investigate the conditions at this point. Then the normalized electron velocity distribution function $f(U')$ at the shock centre, referred to axes moving with the stream velocity $\bar{u}$, can be shown to be (Sen 1953)

$$f(U')=\frac{1}{2\sqrt{\pi}}\frac{n_0}{n}\{e^{-(U'-f)^2}+de^{-b(U'+c)^2}\}, \qquad \ldots\ldots\ldots\ldots (1)$$

where b, c, d, and f are constants given by

$$\left.\begin{aligned} b&=\frac{(a-1)^2M^2}{(M^2+a-2)(aM^2-1)},\\ c&=\sqrt{\frac{a(a-2)}{2}}\frac{(M^2-1)(M^2+a-2)}{M(a-1)\{a(M^2+1)-2\}},\\ d&=\frac{(a-1)^2M^3}{(M^2+a-2)^{3/2}(aM^2-1)^{1/2}},\\ f&=\sqrt{\frac{a(a-2)}{2}}\frac{M(M^2-1)}{a(M^2+1)-2}. \end{aligned}\right\} \quad \ldots (2)$$

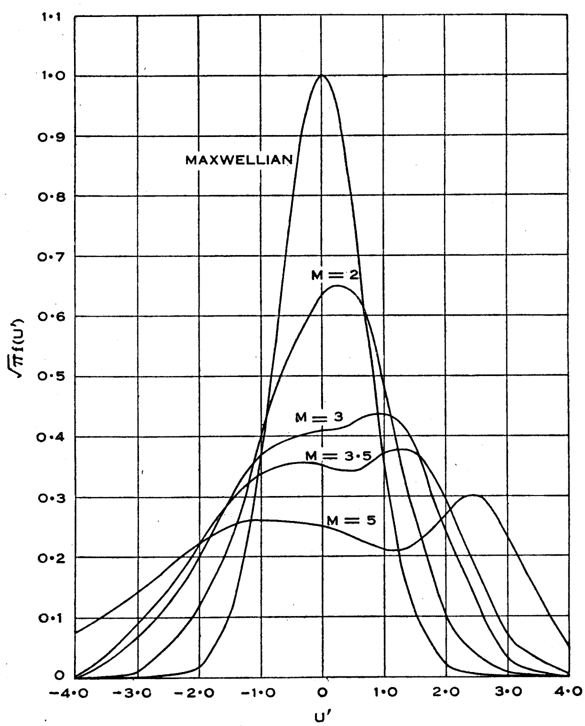

Fig. 1.—Velocity distribution at centre of shock front.

We take $\gamma=5/3$ for the electronic gas. Figure 1 gives the plot of the function $\sqrt{\pi}f(U')$ against U' for different values of the Mach number M. For comparison, we have also drawn in the same figure the corresponding Maxwellian distribution.* It is apparent that, with increasing strength of shock, the velocity distribution curve becomes progressively non-Maxwellian. For $M \simeq 3$, the curve develops a secondary hump that accentuates with M.

The secondary humps in our electron velocity distribution curves in Figure 1 give us reason to expect space charge wave amplification in the shock front. We shall see if this is so from the dispersion equation† corresponding to the distribution function (1). Assuming that all A.C. quantities vary as $Re \exp \mathrm{j}(kx-\omega t)$, Vlasov (1945) derived, in the linear approximation, the following dispersion equation :

$$\frac{4\pi n_e e^2}{m}\int\frac{f(V_0)\mathrm{d}V_0}{(\omega-kV_0)^2}=1, \qquad (3)$$

where $f(V_0)$ is the normalized electron velocity distribution function.

Applying (3) to (1) and assuming a series expansion for small k (i.e. long waves), we derive the following dispersion equation :

$$\left(\frac{3c^2d}{2\sqrt{b}}+\frac{3d}{4b^{3/2}}+\frac{3}{2}f^2+\frac{3}{4}\right)k'^2+\frac{d}{2\sqrt{b}}+\frac{1}{2}-\alpha^2=0, \qquad (4)$$

where

$$k'=k/\omega \text{ and } \alpha=\omega/\omega_0, \qquad (5)$$

ω_0 being the plasma frequency in the medium before the shock, that is,

$$\omega_0{}^2=\frac{4\pi n_0 e^2}{m}. \qquad (6)$$

Table 1 gives the upper limit of α as a function of M, for complex roots‡ k' of the dispersion equation (4). Figure 2 gives the corresponding graph.

TABLE 1

UPPER LIMIT OF α AS A FUNCTION OF M

M	1·0	1·2	2·0	2·5	3·0	3·5	4·0	5·0	7·0	10·0
α	1·00	1·07	1·28	1·36	1·41	1·45	1·48	1·51	1·54	1·56

* $e^{-U'^2}=\underset{M\to 1}{\mathrm{Lt}}\sqrt{\pi}f(U')$.

† We consider only the motions of the electrons and neglect the motions of the ions, on account of the relatively larger mass of the latter.

‡ It is true that the roots of k' in the equation as it stands are purely imaginary. The reason lies in the nature of the approximation and the neglect of collisions. Inclusion of higher powers of k' in (4) or of collision effects will introduce the real part of the propagation constant k.

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Presumably, only values of $\alpha > 1$ will be significant for space charge wave amplification, as frequencies below the plasma frequency will not have much chance of escape from the overdense atmosphere. We see from Figure 2 that the frequency bandwidth of amplification becomes narrower with decreasing strength of shock.

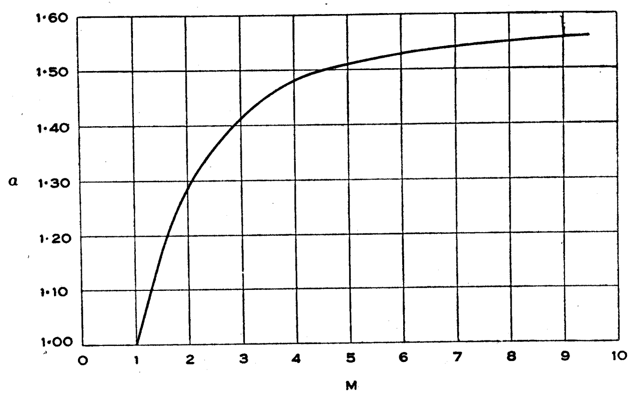

Fig. 2.—Frequency bandwidth of amplification, α, as a function of the shock strength M.

III. Discussion

Frequency bandwidths of a few per cent. of the base frequency have been observed in the fine structure of solar radio noise bursts (Blum and Denisse 1950; Reber, unpublished data; Wild 1951). It is true that the Mott-Smith analysis does not apply to weak shocks.* Nevertheless, in the absence of any compelling reasons to suspect sharp discontinuities, the progressive decrease of the bandwidth of amplification with the shock strength would presumably carry over into the region of weak shocks, and is, moreover, in agreement with the analysis of Denisse and Rocard (1951).

We may suppose that the shock waves originate as weak shocks from the convective cells in the subphotospheric layers that are responsible for the photospheric granules (Schwarzschild 1948). The author (Sen 1953, Appendix) has shown that even in the low chromosphere the shock waves will be frequent enough to preserve the non-Maxwellian distribution as a quasi-steady state against the disruptive effect of collisions.

It is beyond the scope of the present paper to enter into the controversial and difficult question of whether the conditions are as favourable in the solar atmosphere as in discharge tubes for conversion of space charge wave energy into electromagnetic radiation.

The author believes, however, that the discontinuity at the shock front may favour such conversion. In that case, the shock fronts traversing the solar atmosphere would give rise to storm bursts of narrow bandwidth that form the elementary fine-structure components of solar radio noise bursts.

* It gives the wrong slope to the shock thickness as a function of the shock strength. See Wang Chang (1948) and Mott-Smith (1951).

IV. Acknowledgment

The author wishes to acknowledge his indebtedness to Miss Loris B. Perry for computing Table 1 and drawing the figures.

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