HARMONICS IN THE SPECTRA OF SOLAR RADIO DISTURBANCES

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Summary

The paper describes observations with a 40–240 Mc/s radio spectroscope leading to the discovery and preliminary investigation of harmonics in the spectra of sporadic bursts from the Sun. It is found that spectral features of bursts are commonly duplicated with a 2 : 1 frequency separation. The ratio is sometimes appreciably lower though apparently never greater. This and other features of the spectra are shown to support the hypothesis that the fundamental frequency corresponds to the natural plasma frequency of the corona in the vicinity of the source. By applying this result to a standard model of the corona, information on the position, velocity, and size of the sources is deduced. The results suggest that the generation of bursts may be associated with longitudinal plasma oscillations excited by fast streams of charged particles.

I. Introduction

Most of the energy that reaches us from the Sun at wavelengths in the radio spectrum between 1 and 20 m does so in short-lived sporadic bursts of seconds' or minutes' duration and in occasional storms which may continue for days. Since its discovery by Dr. J. S. Hey in 1942, the intense variable radiation has been studied extensively on a world-wide scale. But its origin is still largely unknown. We know from the application of the theory of ionized media to optical data that waves of such length must originate entirely in the outer layers of the solar atmosphere. We know from radio observations that the sources of some of the greater bursts move outwards through several hundred thousand kilometres of the solar corona during the few minutes in which they are received. We know also that the radio disturbances have a general association with visible activity, particularly large sunspots and solar flares. The experimental work which led to the present paper was undertaken with the object of looking for further clues on the physical nature of the radio disturbances.

The experiment consists of observing the spectrum of the high intensity radiation, as a function of time, over a wide continuous range of frequency. The range extends from 40 to 240 Mc/s, that is to say between wavelengths of 1·25 and 7·5 m. Some previous spectroscopic observations were made in the range 70–130 Mc/s (Wild 1950*a*, 1950*b*, 1951; Wild and McCready 1950); experience gained in this work emphasized the importance of extending the frequency range and helped to shape the design of the present experimental programme.

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On November 21, 1952, some four months after the start of regular observations, a large outburst of radio noise was recorded which revealed a new phenomenon. The record showed that spectral features in the lower part of the observed range were unmistakably duplicated at about double the frequency. In a preliminary report of this occurrence (Wild, Murray, and Rowe 1953), the authors pointed out that the duplication was almost certainly due to the emission of fundamental and second harmonic frequencies from a common source. In view of subsequent evidence this conclusion now seems definite. Indeed it appears that harmonics are received in a considerable proportion of all sporadic bursts.

In the present paper we give details of observations on the harmonic effect (Section III), suggest an interpretation of the results (Section IV), and show how the data may be used to study the location, speed, and size of the sources responsible for generating sporadic bursts (Section V). The conclusions of the paper are summarized in Section VI. Before considering these topics a brief outline is given of the experimental method.

II. The Experimental Method

(a) *Choice of Site*

The choice of site for this investigation was influenced mainly by the requirement of avoiding interfering signals, especially those from high frequency radio transmitters. The observing station was set up near Dapto, N.S.W., some 50 miles to the south of Sydney. It is effectively screened from Sydney by nearby mountains.

(b) *The 40–240 Mc/s Spectroscope*

A detailed description of the instrument is beyond the scope of the present paper. Here we shall consider only the main features which determine its capabilities and the type of record produced.

The spectroscope is represented, in considerably simplified form, by the block diagram in Figure 1. Spectra are obtained by rapidly tuning through the range with a receiver of small bandwidth. The whole range is covered by three separate receiving units sweeping in succession through ranges of 40–75, 75–140, and 140–240 Mc/s. The three ranges have their outputs connected to common displays.

Each receiving unit consists of a broad-band rhombic aerial connected to a swept-frequency superheterodyne receiver. The receivers are tuned by continuously rotating tuning condensers connected to a common shaft which is driven by a motor at 2 r.p.s. Each range is swept in one-quarter of a revolution, the complete range in three-quarters. Thus a complete spectrum is swept out in $\frac{3}{8}$ sec and two complete spectra are obtained each second.

The receivers are controlled by an electronic sequence switch which allows the output of each receiver to be passed to the display only during its operative periods.

The frequency resolution of the instrument is determined by the response curve of the intermediate-frequency amplifier whose bandwidth between half-power points is 0·5 Mc/s. The output time-constant, which must necessarily be short in view of the rapid sweep, is about 10^{-3} sec. The noise factor of the receivers varies slightly over the range with an average value of about 10.

The aerials are equatorially mounted and motor driven to follow the Sun. Their effective areas are between 5 and 10 m^2.

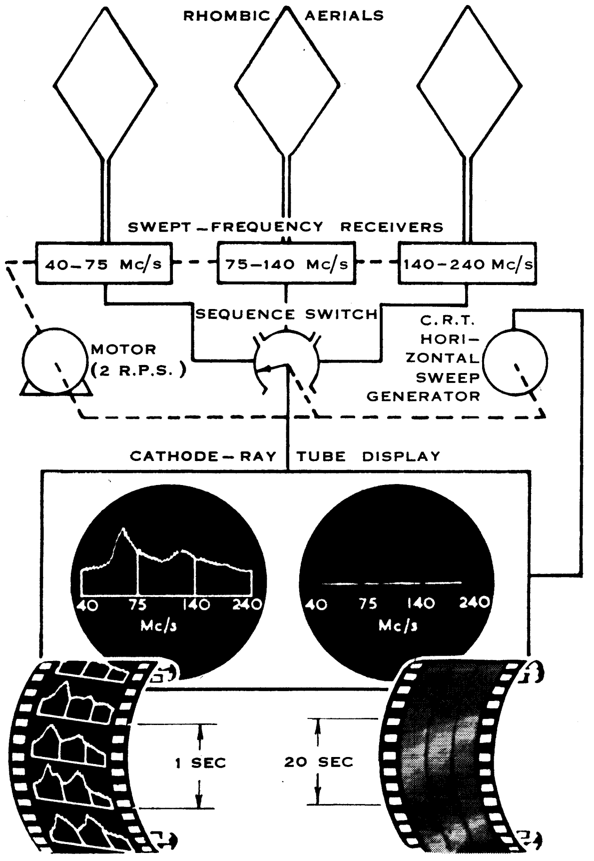

Fig. 1.—Simplified block diagram of the 40–240 Mc/s spectroscope.

(c) *The Display and Recording Unit*

Two types of recording display are used, each having a 12-in. cathode-ray tube and a camera operated with continuously moving 35-mm film.

One display is similar to that used for the previous observations (Wild and McCready 1950) and corresponds to the " A scan " of radar sets ; frequency is displayed horizontally and receiver output vertically. This arrangement is suitable for precise measurements of absolute intensity but suffers from the disadvantages that (i) the rate of film consumption (36 in/min in our case) is excessively high for continuous operation and (ii) the process of reducing records to a useful form is too laborious for handling extensive data.

For continuous operation the second display is used in which frequency is again displayed horizontally but receiver output is registered by modulating the intensity of the spot. Since the response of photographic film is approximately logarithmic it is possible to record intensities of vastly different magnitudes. Using Ilford HP3 film it is found that aerial temperatures in a 1000 : 1 range can be satisfactorily accommodated. With a film consumption of 1 in/min (120 traces per inch), successive traces are recorded with no overlap.

(*d*) *Calibration*

Frequency calibration is provided by injecting into the receiver signals from a standard signal generator at frequency intervals of 5 or 10 Mc/s. The probable error associated with the frequency measurement of a sharply defined feature is determined mainly by the reading accuracy and is estimated to be $\pm\frac{1}{2}$ per cent.

Intensity calibration is provided by switching on temperature-limited noise diodes which are *permanently* connected across the aerial terminals of each receiver. The noise-source impedance is then provided by the aerial itself. This arrangement has two advantages : firstly, the anode-cathode capacity of the diode (which would be difficult to tune out over a broad frequency band with the conventional method of substitution) forms part of the input tuned circuit, thus eliminating calibration errors due to this cause ; and secondly, if calibration is performed with the aerial directed towards the undisturbed Sun, the power injected by the noise generator corresponds to the excess power above cosmic noise, which is the quantity required to be measured. The standard calibration procedure consists of injecting power at seven levels each separated by a 2 : 1 interval. By this means the records are directly calibrated for levels of flux density between 5×10^{-21} and $3\cdot2\times10^{-19}$ $\mathrm{Wm^{-2}\,(c/s)^{-1}}$; for higher levels it is necessary to extrapolate according to the receiver gain law.

Time marks are inserted on the record by photographing a clock beside the display which is illuminated by a flash at 1-min intervals.

The complete calibration procedure is performed at least once per day.

(*e*) *The Reduction of Records*

Spectra which vary with time are conveniently presented in the form of intensity-frequency-time diagrams in which intensity is shown by contours in the frequency-time plane. Such *dynamic spectra* were used to present the results of the previous observations. Experience then gained indicated that a practical limit to the amount of data that could be examined was set by the labour involved in producing the diagrams. However, by using the intensity-modulated display, we obtain the information in a form from which the salient features of the dynamic spectrum can be immediately recognized (see Plate 1 for examples). It is a simple process to trace these features and redraw them on a proper scale of frequency. These " sketches " are adequate for many purposes.

When higher precision is required (e.g. for the detailed study of the harmonic effect) it is more satisfactory to use an A-scan record and deduce the fully calibrated profiles and dynamic spectrum. When only the intensity modulated record is available, it is necessary to analyse the film with a high resolution microphotometer.

III. The Observations

Solar observations were started in August 1952 over the frequency range 40–140 Mc/s, and in November 1952 over the complete range 40–240 Mc/s. The observations described here cover the period August 1952 to August 1953. The equipment was operated for about 1000 hr during the year, preference being given to periods of solar activity.

In addition to several days of continuous activity (" noise storms ") we recorded at other times several hundred bursts of sporadic occurrence. In the present paper we shall be concerned only with the sporadic bursts. In describing their spectra the following terms, previously introduced by Wild and McCready (1950), will be adopted and where necessary extended :

> *Spectral Type II.*—The type of dynamic spectrum exhibited by bursts or " outbursts " of some minutes' duration in which the spectral features drift slowly, though perhaps irregularly, in the direction of decreasing frequency. The typical drift rate is of the order of $\frac{1}{4}$ Mc/s per sec. These bursts often occur at the time of solar flares.
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> *Spectral Type III.*—The type exhibited by sporadic bursts of a few seconds' duration in which the frequency of maximum intensity drifts rapidly in the direction of decreasing frequency at a rate of about 20 Mc/s per sec.

In the current series of observations the great majority of sporadic bursts were found to belong to one of these two spectral classes ; four type II bursts were recorded and several hundred type III, the latter occurring mainly in small groups or compact clusters lasting for about 1 min.

The coexistence of first and second harmonics has been recognized in both classes of spectra—in two of the four observed type II bursts, and in 20 type III bursts. The spectrum of many other type III bursts suggested that harmonics might have been present though their certain recognition was masked by the large bandwidth of the bursts.

(*a*) *Harmonics in Type II Spectra*

The two recorded harmonic outbursts of spectral type II occurred on November 21, 1952 and May 5, 1953. The earlier one is the larger and more complete disturbance but the later one has yielded more exact data because an A-scan record was obtained. Details of the two outbursts are as follows.

(i) *The Outburst of November 21, 1952*

General data.—The outburst lasted from 23 hr 50 min to 24 hr 05 min U.T. and was followed immediately by a noise storm lasting for about 2 hr. The disturbance was accompanied by a large solar flare starting at 23 hr 45 min located above a big spot group near the centre of the Sun's disk. It was also accompanied by a radio fadeout and geomagnetic crochet.

The dynamic spectrum.—The outburst was recorded on the intensity-modulated display and the record reduced photometrically. Part of the record is shown in Plate 1 (*a*) and the dynamic spectrum in Figure 2 (*a*). It consists of

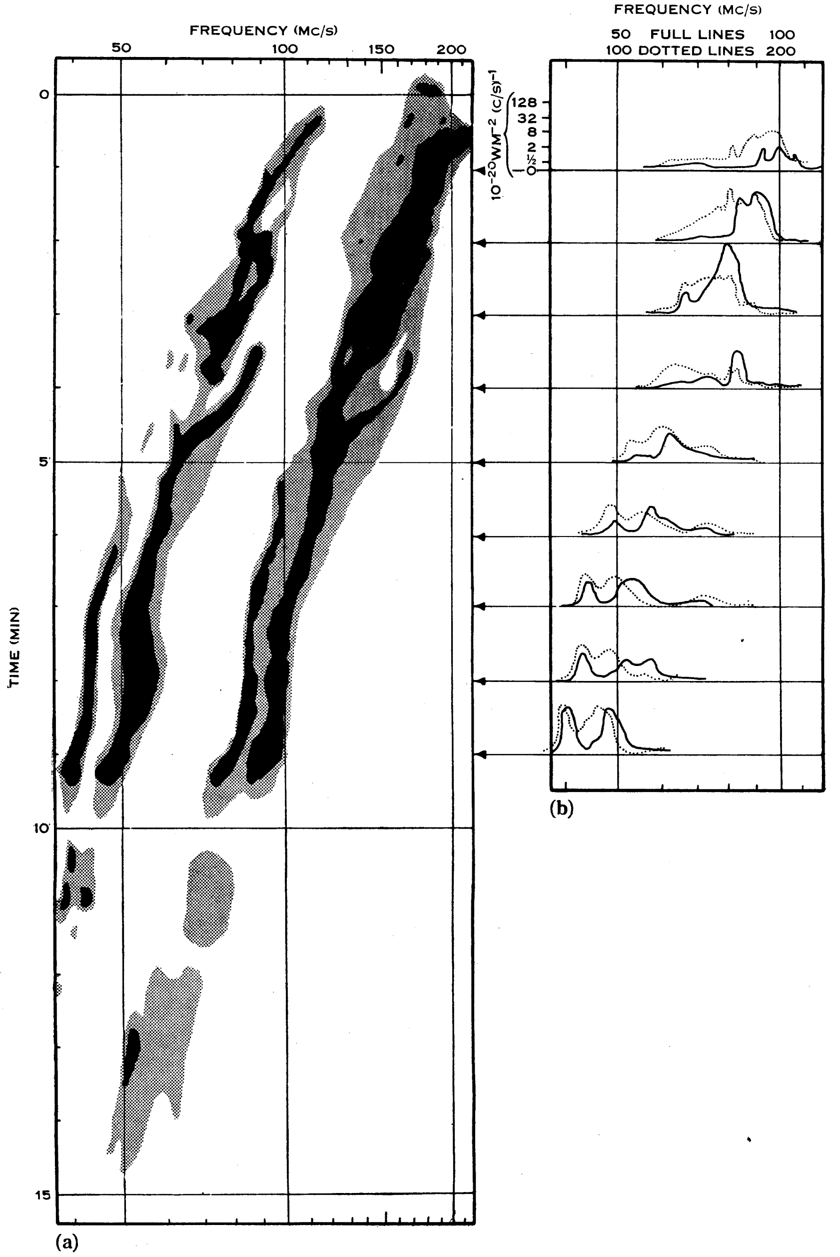


Fig. 2.—The outburst of November 21, 1952, 23 hr 50 min U.T. (*a*) The dynamic spectrum. The intensity contours correspond to levels of approximately 5 and 20 Wm^{-2} (c/s)$^{-1}$. (*b*) Profiles at 1-min intervals. The second harmonic is shown dotted and displaced in frequency by a factor of 2.

two main bands, widely spaced in frequency, which drift at a rate typical of type II spectra. Each band shows a fine structure which for much of the time consists of two well-defined peaks. Two features are of special interest. Firstly, the structure of the two bands is strikingly similar ; peaks appearing in one band seem to be duplicated simultaneously in the other. Secondly, although the frequency of each band drifts over a range of at least 3 : 1, the frequency ratio of corresponding features in the two bands remains approximately constant and lies within a few per cent. of 2. The first of these features was interpreted as signifying that the two main bands were emitted from the same source ; the second that the duplicity was due to radiation from the source at a fundamental frequency and its second harmonic.

The profiles.—Figure 2 (*b*) shows the instantaneous spectral profiles of the two main bands taken at 1-min intervals ; the bands are plotted in superposition such that a frequency f in the fundamental (full line) coincides with a frequency $2f$ in the harmonic (dotted line). When compared in this way the two sets of profiles are by no means identical in shape. One feature seems to be systematic : peaks in the harmonic band always lie slightly to the left of corresponding ones in the fundamental, i.e. the frequency ratio is consistently less than 2. The measured ratios lie between $1 \cdot 96$ and $1 \cdot 99$ for sharp peaks, and as low as $1 \cdot 90$ for the more diffuse peaks. Peak amplitudes in the two bands are of comparable magnitude.

(ii) *The Outburst of May 5, 1953*

General data.—The type II burst lasted from 04 hr 59 min to 05 hr 02 min U.T. It was preceded by a cluster of type III bursts and other activity starting at 04 hr 55 min, and was followed by a general increase in level at the lower frequencies lasting from 05 hr 04 min to 05 hr 08 min. No flare observations are available but the outburst accompanied a partial radio fadeout. Visible activity on the disk was confined almost entirely to western heliographic longitudes, between 30° and the limb.

The dynamic spectrum.—The outburst was recorded on both displays, the A-scan record being used exclusively for deriving the data presented below. The dynamic spectrum is shown in Figure 3 (*a*). Both the duration and rate of frequency drift are smaller than for the previous outburst. The spectrum is simpler and, except near the finish, each harmonic band consists of a single peak.

The profiles (Fig. 3 (*b*)).—Owing to the greater precision of the A-scan record, the profiles have been plotted on a larger scale and at more frequent intervals (15 sec). The profiles of the fundamental peaks are seen to be markedly asymmetrical, the low frequency edge being the steeper of the two. Indeed in the last four profiles, the slope of this low frequency " cut-off " is indistinguishable from that of the receiver's response curve, indicating that the slope on the true spectrum was too steep to be resolved by the instrument. The harmonic band is more symmetrical but on the average the low frequency edge is again the steeper. With the passage of time the ratio of peak frequencies gradually increases from about 1·7 near the start to $2 \cdot 00 \pm 0 \cdot 01$ at the finish and the amplitude of the fundamental relative to the harmonic increases from about 0·5 to 10.

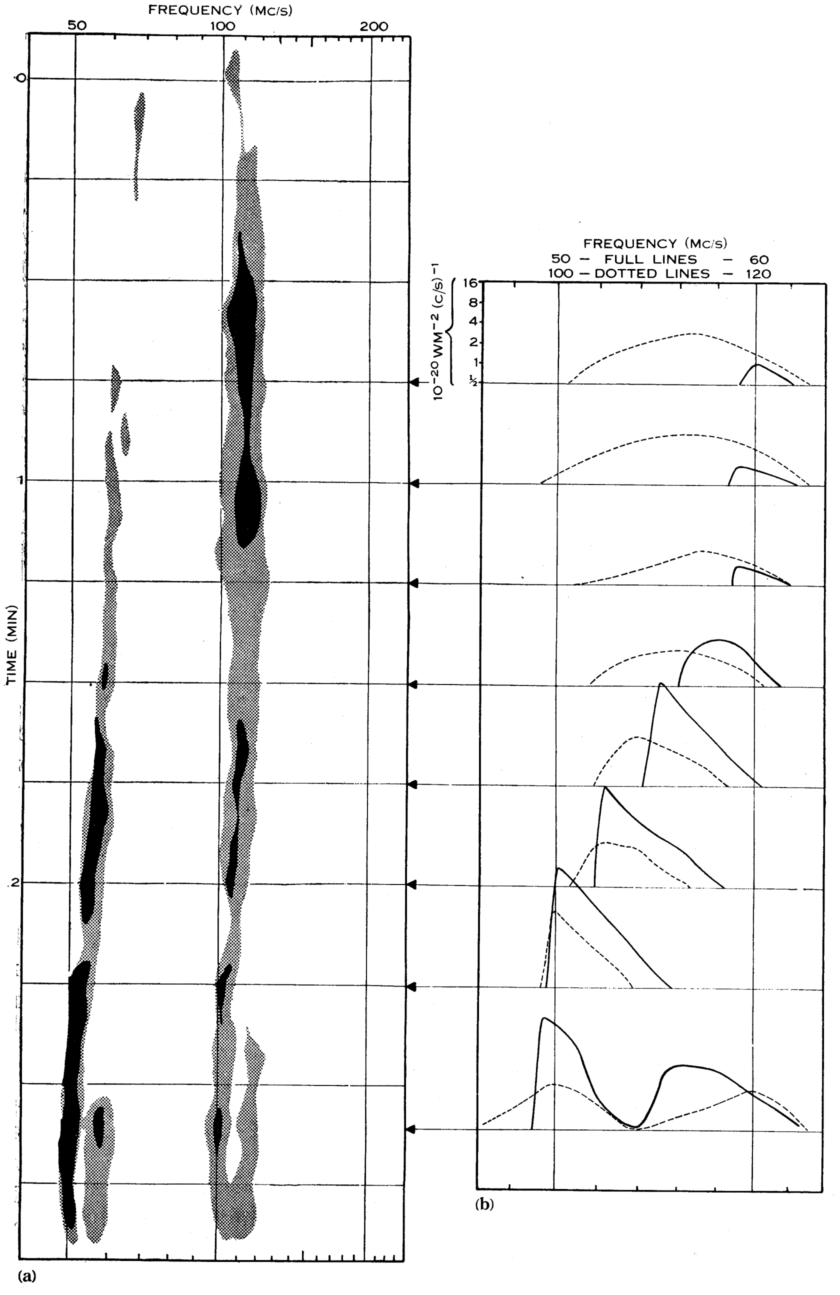


Fig. 3.—The outburst of May 5, 1953, 04 hr 59 min U.T. (*a*) The dynamic spectrum (contour levels as in Fig. 2), (*b*) profiles at 15-sec intervals. Note that scales are different from those in Figure 2.

(*b*) *Harmonics in Type III Spectra*

The fast-drift, short-lived bursts of spectral type III, first observed with the previous equipment (Wild 1950*b*), have been found to constitute the great majority of all bursts observed at times other than noise storms. With the wider frequency range of observation it is now possible to give what seems to be a complete description of their spectral characteristics. The new finding of immediate interest is the recognition of dynamic spectra consisting of two similar formations in which the features of one are duplicated at about double the frequency. Following the evidence given above for type II spectra, there seems little doubt that this effect is again due to the emission of a fundamental frequency and its second harmonic.

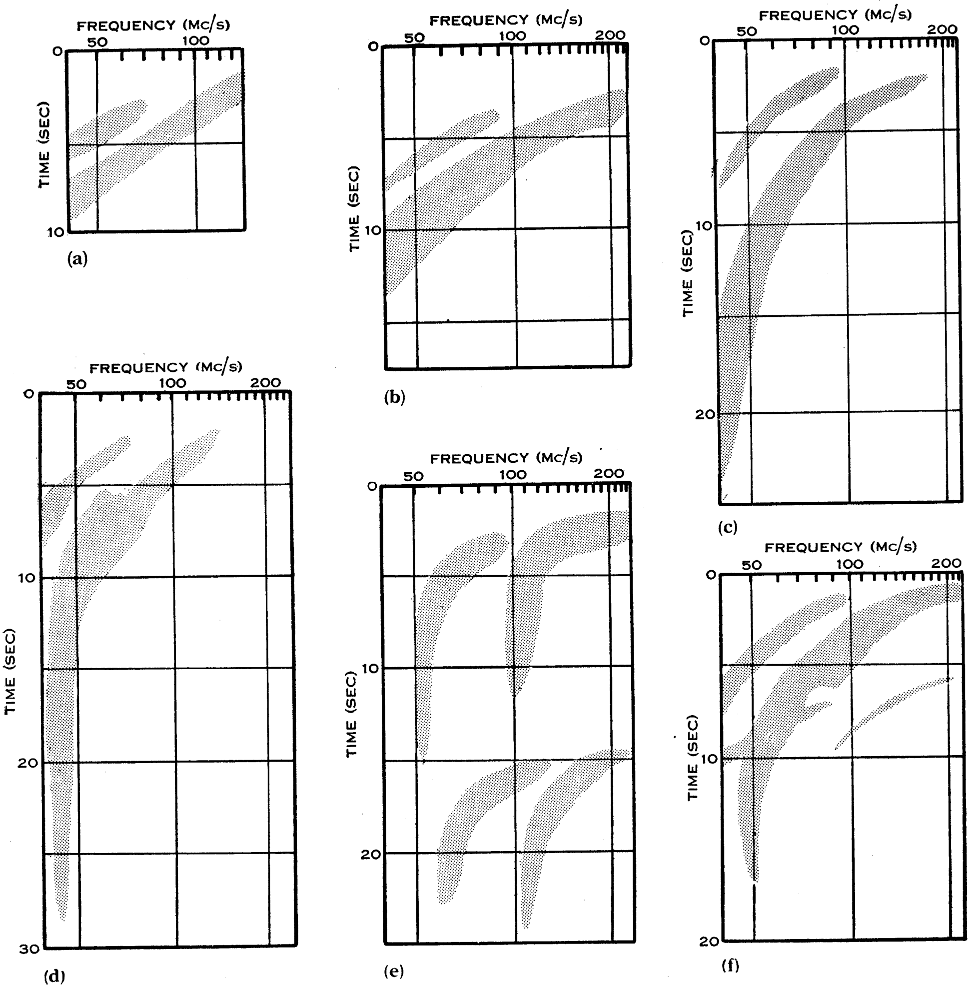


Fig. 4.—Dynamic spectra of harmonic type III bursts. Times (U.T.): (*a*) October 3, 1952, 23 hr 21 min; (*b*) June 7, 1953, 01 hr 08 min; (*c*) June 5, 1953, 01 hr 35 min; (*d*) June 5, 1953, 01 hr 37 min; (*e*) January 14, 1953, 06 hr 07 min; (*f*) June 5, 1953, 01 hr 32 min.

Figure 4 shows examples of harmonics in type III bursts. In cases (*a*), (*b*), and (*c*) both fundamental and harmonic bands are seen to drift off the low frequency edge of the observed range, the rate of frequency drift decreasing continuously with time. In cases (*d*), (*e*), and (*f*), however, the frequency drift decreases so rapidly at the lower frequencies that one or both bands become

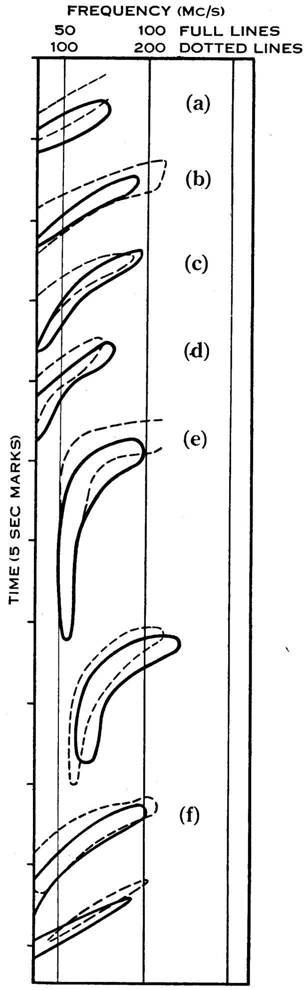


Fig. 5.—Comparison of the fundamental and second harmonic bands of the type III bursts shown in Figure 4.

almost stopped at a fixed frequency : the spectrum finishes in a vertical " tail ". In all cases the fundamental and harmonic parts of the dynamic spectrum are similar in shape although the former tends to be of narrower relative bandwidth ($\Delta f/f$).

In the type II spectra considered above it was shown that the ratio of peak frequencies in the two bands never exceeded 2 but was often slightly lower. A similar effect is demonstrated for type III spectra in Figure 5 in which relevant portions of the spectra in Figure 4 have been replotted with the harmonic band displaced two-to-one in frequency. There is seen to be a consistent displacement of the harmonic band to the left of the fundamental, again indicating a frequency ratio less than 2. In the great majority of cases the ratio lies between 1·85 and 2·00.

Of the several hundred type III bursts observed during the year, the recognition of harmonics was considered certain on 20 occasions. In addition there were many doubtful cases. Certain recognition is often made difficult owing to the combination of wide bandwidth and rapid frequency drift, which may result in the merging of the two bands. In cases where the bursts show tails, however, it is possible to decide without ambiguity whether or not the two bands are present. Of 19 such cases recorded, 12 were found to have the two bands. If it is assumed that the cause of harmonic production is independent of the cause of tail production, this result suggests that the proportion of bursts in which fundamental and harmonic are both of detectable magnitude may exceed 50 per cent.

(c) *Summary of Observational Results*

The main observational results are summarized below.

(1) The dynamic spectrum of sporadic bursts, both type II (slow drift) and type III (fast drift) sometimes show simultaneous duplication of features separated by a frequency ratio of about 2 : 1. The duplication is attributed to the emission of both fundamental and second harmonic frequencies from a common source.

(2) Bursts showing detectable fundamental and harmonic bands in their spectra account for a considerable proportion of all bursts, perhaps 50 per cent. or more. At the higher frequencies, in the vicinity of 200 Mc/s, it is not unreasonable to surmise that the greater part of radiation received in sporadic bursts is due to second-harmonic emission.

(3) The intensity of the second harmonic may be comparable with, or even greater than that of the fundamental. No third and fourth harmonics have been detected ; had they been present with an intensity one-tenth of that of the second harmonic, they would certainly have been detected on several records.

(4) There appears to be a general rule that the ratio of peak frequencies never exceeds 2 but is often slightly lower.

(5) Detailed analysis of one outburst of type II for which sufficiently precise data were available indicated that the low frequency edge of the fundamental band may be extremely sharp and that the relative amplitude of the fundamental increases as the frequency ratio approaches 2.

IV. Interpretation

(a) *General Inferences*

The first three conclusions listed above imply that the sporadic solar radiation is rich in second harmonics. This surely means that the emitting process is one involving oscillations of charge, as distinct from non-periodic accelerations such as those of thermal motions in the absence of a magnetic field. To produce a harmonic the oscillators must be non-linear. Also, since an even harmonic is observed, the emitted waveform must be asymmetrical in the sense that values of the electric vector at two instants of time separated by half a cycle are not equal and opposite. This asymmetry may provide an important restriction on the types of processes admissible.

In some bursts, especially those of type II, it was noted that the width of each band is extremely narrow, perhaps only 2 or 3 per cent. of the mid frequency, between half-intensity points. We infer that in these cases all the oscillatory charges which constitute the source oscillate at roughly the same frequency. In other words there appears to be some proper frequency controlling the oscillations.

We know of two classes of proper frequency for oscillations in an ionized medium : the gyro frequencies of charged particles in a magnetic field, of which the electron gyro frequency is the most relevant in the present problem ; and the plasma frequency.

(i) *The Electron Gyro Frequency*, $f_H = eH/2\pi mc$.—Here e and m denote the electronic charge (e.s.u.) and mass, H the magnetic field, and c the velocity of light. It has been shown by Schwinger (1949) and others that electrons gyrating in a magnetic field generate harmonics when the orbital velocity approaches the velocity of light. The difficulties associated with the escape from the solar atmosphere of radiation at the fundamental of the gyro frequency (Ryle 1948) are well known, though it is possible that the harmonics $2f_H$, $3f_H$, . . . could escape (Roberts 1952). Indeed Roberts suggested that spacings between harmonics offered a possible experimental test for the gyro theory of generation ; he pointed out that adjacent harmonics would be in the ratios 3 : 2, 4 : 3, 5 : 4, etc., but not 2 : 1. The observation of harmonics at a 2 : 1 spacing now provides a negative answer to this test and indicates that a process is required in which fundamental frequencies can escape.

(ii) *The Plasma Frequency*, $f_0 = e\sqrt{(N/\pi m)}$.—Here N is the electron density of the medium. In the presence of a magnetic field, not considered here, the plasma frequency is split into three components one of which is f_0.

Several authors (see for instance Shklovsky 1946 ; Martyn 1947 ; Bohm and Gross 1949 ; Jaeger and Westfold 1949) have suggested oscillations at the plasma frequency as the source of high intensity solar radio noise although no complete theory has yet been given. Difficulties are again encountered regarding escape because the plasma frequency coincides with the critical frequency at which the refractive index reduces to zero. If emission takes place from a localized region in which only the plasma frequency is excited, the radiation can escape only within an infinitesimally narrow cone normal to the surface of zero refractive index. However, there is no such escape restriction on the higher harmonics.

Let us now suppose that the excited region generates not merely the plasma frequency but rather a narrow band of frequencies about the plasma frequency together with their harmonics. Since propagation can take place at frequencies above the critical frequency, the received spectrum would consist of all second and higher harmonic frequencies generated but only the high frequency part of the fundamental band. This is strongly suggestive of the observed harmonic spectra and seems to offer a natural explanation both of the reduction of the harmonic ratio below 2 and the sharp cut-off in the fundamental band. We now examine this interpretation in more detail by considering the spectrum from a point source in a model corona.

(b) *The Escape of Frequencies near the Plasma Frequency from a Model Corona*

The approach to be adopted may be summarized as follows. We consider a localized source in the solar corona situated in a region specified by an electron density N. We suppose that if a probe connected to a radio spectroscope were inserted into this region the resulting spectrum would consist of a narrow-band peak centred on the plasma frequency f_0 and others of similar shape but lower amplitude at the harmonic frequencies $2f_0$, $3f_0$ . . . We refer to this spectrum as the *natural spectrum*. Next we consider the propagation of energy through the solar atmosphere and calculate the fraction of the source brightness capable of reaching a terrestrial observer. The fraction transmitted is a function of frequency which we call the *propagation characteristic*. Finally we deduce the *received spectrum* by multiplying the natural spectrum by the propagation characteristic.

For the electron-density distribution in the corona we shall assume the idealized, spherically symmetrical model given by the Baumbach-Allen formula. The effects of coronal irregularities and magnetic fields will be neglected for the present calculation but will be considered in Section V. We shall be concerned merely with illustrating the type of result obtained and it will be sufficient to consider the case of a fixed point source whose height we choose to be at the 50 Mc/s plasma level ($2\cdot3\times10^5$ km above the photosphere in the Baumbach-Allen corona).

(i) *The Natural Spectrum*.—For present purposes it is convenient to consider a natural spectrum of definite form, though its choice is relatively unimportant. We shall assume the profile of each harmonic to be symmetrical, on a logarithmic frequency scale, about the plasma frequency and to have the shape of the Gaussian error curve. The assumed spectrum is shown in Figure 6 (*a*). The bandwidth has been chosen to be comparable with observed profiles and the amplitude of the second harmonic has been set arbitrarily to one-tenth that of the fundamental.

(ii) *The Propagation Characteristic*.—The propagation and escape of radio waves from point sources within the Baumbach-Allen corona has been treated in detail by Jaeger and Westfold (1950). The problem consists of calculating (1) the paths of rays between source and observer, in general these paths are curved owing to the steady increase with height of refractive index in the corona; and (2) the absorption along each path.

Figure 7 shows the paths of rays of different frequencies escaping from a source at the 50 Mc/s plasma level. The lowest frequency capable of escape is the plasma frequency itself which can be propagated along the radial ray only. Higher frequencies can escape in directions contained within a sharply defined cone about the radial direction ; the higher the frequency the wider the cone.* Thus to a terrestrial observer the propagation characteristic depends on the position of the source on the Sun's disk. Figure 6 (*b*) shows the derived characteristic for various source angles θ (defined in the inset below Figure 6 (*a*)). The effect of absorption is included and accounts merely for the slight drop in values of transmission at frequencies immediately above the cut-off frequency. Apart from this small effect the cut-off is perfectly sharp. As implied by Figure 7, the cut-off frequency coincides with the plasma frequency for a source at the centre of the disk and increases with the source angle.

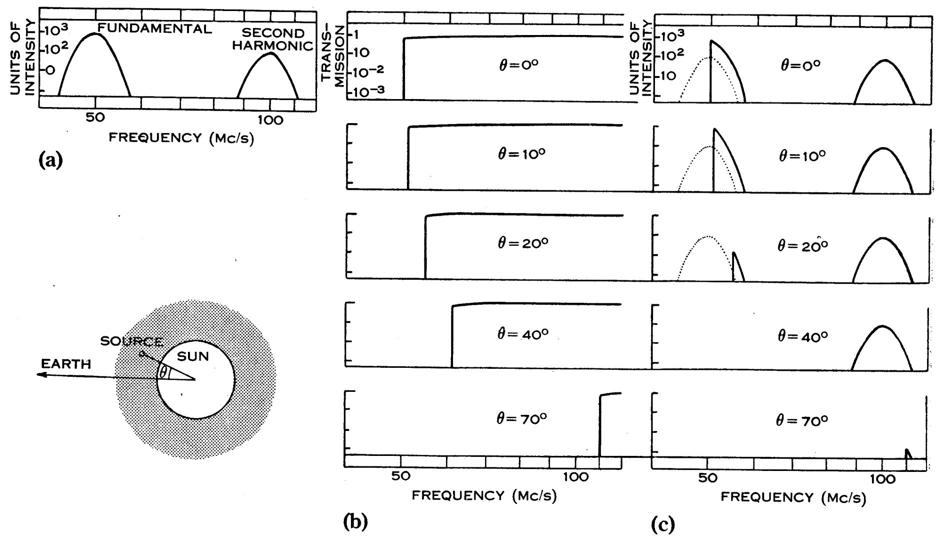


Fig. 6.—The derivation of spectra for an assumed model (see Section IV). (*a*) The assumed natural spectrum for a point source, located at the 50 Mc/s plasma level of the solar corona, emitting harmonics of the plasma frequency ; (*b*) the calculated propagation characteristic (from Jaeger and Westfold) for radiation escaping from the 50 Mc/s level at various angles θ (see inset below (*a*)) ; (*c*) the received spectrum derived from (*a*) and (*b*).

(iii) *The Received Spectrum.*—Combining the data of Figures 6 (*a*) and 6 (*b*) we obtain the received spectra of Figure 6 (*c*). For a source at the centre of the disk (θ=0°), the ratio of peak frequencies of the fundamental and second harmonic is exactly 2, but the low frequency half of the fundamental band is completely cut off. For θ=10° the ratio is reduced to 1·96 and for θ=20° to 1·90. In the latter case the amplitude of the fundamental is greatly reduced, and at θ=40° it is no longer present. For these values of θ the second harmonic

* Each ray path shown in Figure 7 is that which leaves the source tangentially to the spherical strata of the assumed atmosphere. In general the " outermost " ray (i.e. that which emerges from the atmosphere at the greatest angle) is one which leaves the source in a direction having a slight inward component. The difference in the angles of emergence of these two types of limiting ray is negligible except for very oblique emergence.

is unaffected, but for sources near the limb (e.g. $\theta = 70°$) it too becomes modified by the cut-off characteristic.

(iv) *Comparison with Observations*.—The derived spectra of Figure 6 (*c*) illustrate how localized emission at harmonics of frequencies around the plasma frequency can explain the following characteristics observed in type II and type III spectra :

(1) The normal occurrence of harmonic peak-frequency ratios slightly less than 2.

(2) The sharp low frequency cut-off observed in the fundamental band.

(3) The reduction in amplitude of the fundamental peak relative to the harmonic as the peak-frequency ratio diminishes.

There is seen to be a marked resemblance between the derived profiles and the detailed observed profiles shown in Figure 3.

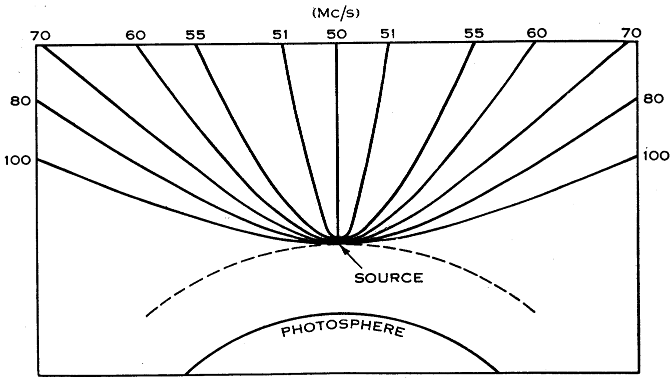


Fig. 7.—The limiting rays of outward emission at various frequencies from a source at the 50 Mc/s plasma level in the Baumbach-Allen corona. The numbers refer to frequencies in Mc/s. Data from Jaeger and Westfold (1950).

(*c*) *Conclusions on the Mechanism of Burst Generation*

The general agreement between the above interpretation and the observations supports the basic proposition that the sources of sporadic bursts radiate their energy mainly at harmonics of frequencies near the plasma frequency of the surrounding medium. Indeed it appears that, of the processes which have been suggested to explain the high intensity of solar noise, those involving plasma oscillations are the only ones capable of accounting for the harmonic phenomenon. Nevertheless, it should be stressed that our knowledge of plasma oscillations is far from complete and the conditions under which electromagnetic radiation can be obtained from them are not properly understood.

Evidence will be given in Section V that the source regions are in rapid motion through the solar corona and it is likely that the observed movements are associated with streams of ionized matter. As Shklovsky (1946), Bohm and Gross (1949), and others have pointed out, ionized streams projected through the corona provide a means of exciting plasma oscillations. Oscillations of this kind are longitudinal and, if the amplitude becomes appreciable in comparison with the wavelength, the charge density exhibits excess " bunching " near the nodal points. This effect may provide the required non-linearity and asymmetry for the generation of the second harmonic.

V. The Position, Movement, and Size of the Sources of Bursts

(a) *The Position and Speed of the Sources of Type II Outbursts*

The conclusions of Section IV (*b*) suggest that in idealized circumstances it should be possible to determine both the source height and the source angle as a function of time directly from the dynamic spectrum of a burst. It is of course necessary to assume a standard electron-density model of the corona, and it is only possible to treat simple cases in which the observed profiles resemble the theoretical ones (Fig. 6 (*c*)). Under these conditions we can determine (1) the plasma frequency (and hence the height in the corona) from the observed peak frequency of the second harmonic, and (2) the critical escape frequency (and hence the source angle) from the observed cut-off frequency of the fundamental.

The relation between the critical escape frequency f_c, the plasma frequency f_0, and the source angle θ is a complex one, but it has been shown by Smerd (unpublished data) that for the Baumbach-Allen corona the approximate formula

$$f_c = f_0 \sec(0 \cdot 87\theta)$$

agrees with the values calculated by Jaeger and Westfold (1950) to within a few per cent. for frequencies between 20 and 100 Mc/s and source angles between 0 and 80°. A tabulation of plasma frequency for various coronal heights has been given by Smerd (1950).

The application of this method of source location is demonstrated in Figure 8 for the outburst shown in Figure 3. Figure 8 (*a*) shows plots of the observed values corresponding to f_0 and f_c ; Figure 8 (*b*) the derived source height and angle as a function of time ; and Figure 8 (*c*) successive positions of the source at ½-min intervals. The calculated path is seen to be mainly across the line of sight. Since the active areas on the disk at the time of the outburst were confined almost entirely to western heliographic longitudes (see Fig. 8 (*c*)) we might infer that the source travelled across the disk from west to east. The mean speed of travel is found to be about 4000 km/sec.

The results given in Figure 8 emphasize the importance of knowing the rate of change of source angle. In previous determinations of the velocity of outburst sources, either the determination was confined to the radial component (spectroscopic method, Wild 1950*a*) or the speed was calculated on the assumption that the motion was strictly radial (directional method, Payne-Scott and Little 1952). Had the speed been derived on the assumption of radial motion in the present case, the less plausible conclusion would have been reached that the source initially travelled slowly inwards and subsequently turned and accelerated outwards.

The outburst of Figure 2 is less suitable for similar analysis partly because of its greater complexity and partly because the profile data are less accurate. Near the finish, however, the peaks are well defined, and the closeness of the

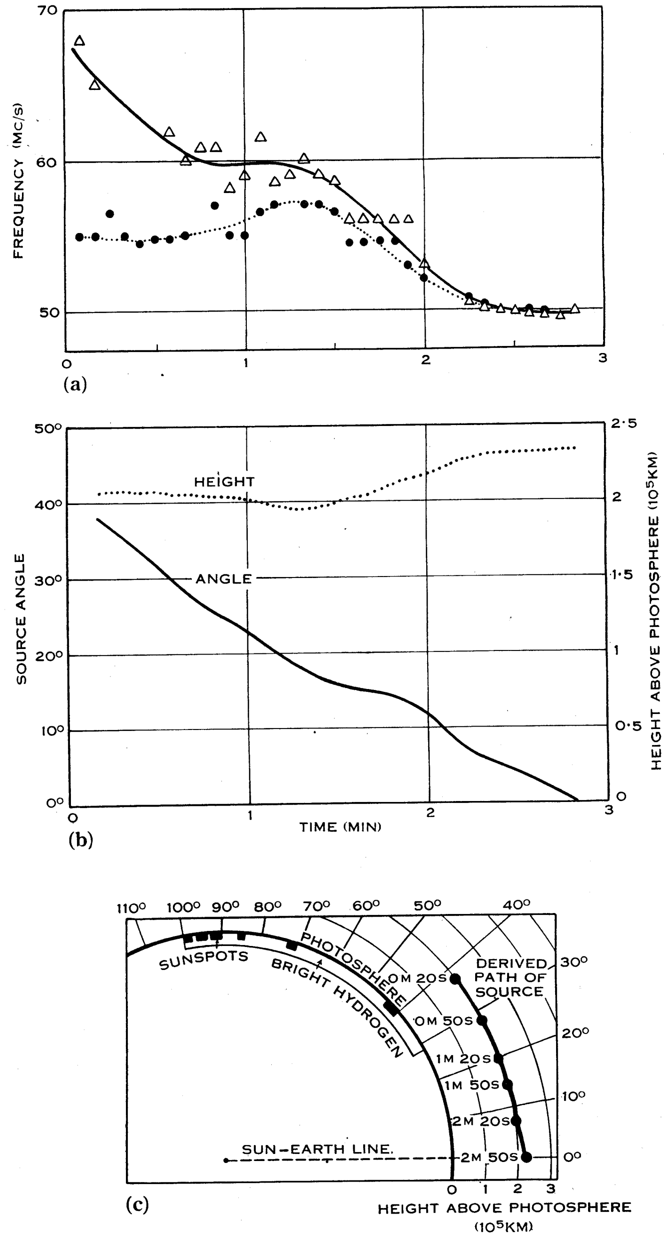


Fig. 8.—Position determinations for the outburst in Figure 3. (*a*) The variation with time of (i) the peak frequency of the fundamental (triangles) and (ii) half the peak frequency of the second harmonic (dots); (*b*) The derived height and "source angle" (θ in Fig. 6); (*c*) Successive positions of the source at ½-min intervals. The heliographic longitudes of sunspots etc. are indicated, and refer to the western hemisphere.

peak-frequency ratio to 2 suggests a central position on the disk. Since the associated flare was also located near the centre, it may be inferred that the source travelled approximately radially outwards in the direction of the Earth. The derived height plot is shown in Figure 9, and represents the most extensive range of travel we have yet observed in an outburst. The plot indicates an approximately constant velocity of 475 km/sec over a 6 : 1 range in heights. The time delay between the fadeout and the start of the outburst suggests that the source could have been ejected simultaneously with the onset of the ultra-violet emission from a region low in the corona, some $2 \cdot 5 \times 10^4$ km above the photosphere.

The emission of second harmonics in outbursts suggests a reason for the unexpectedly large source heights deduced by Payne-Scott and Little (1952) from their directional observations. These authors considered the typical

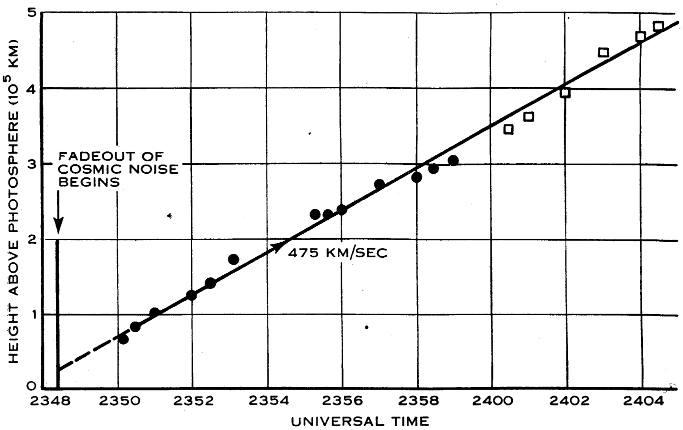


Fig. 9.—Derived height plot for the outburst in Figure 2. Where possible heights are derived from the fundamental frequency band (dots). The range is extended to greater heights by using half the frequency of the second harmonic (squares).

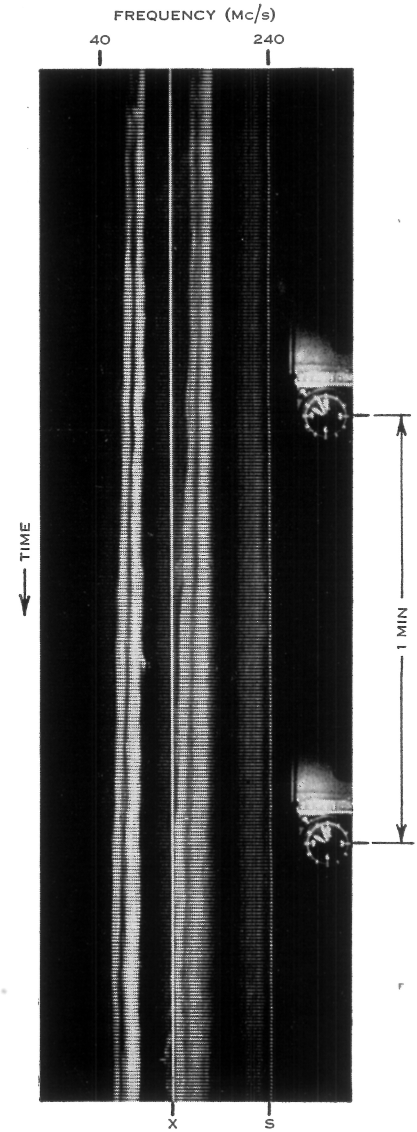
minimum height for 97 Mc/s radiation to be some 2×10^5 km above the photosphere. This, they point out, is about three times as high as the 97 Mc/s plasma level in the Baumbach-Allen corona. Rather than indicating a radical alteration to the electron density distribution, as suggested by these authors, the discrepancy could be simply accounted for in terms of emission at the second harmonic: for the assumed electron-density distribution, the height of the 97/2 Mc/s plasma level is $2 \cdot 4 \times 10^5$ km.

(*b*) *Sources of Error in Velocity Determinations*

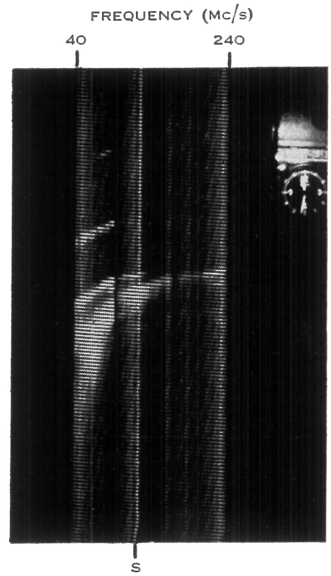
The two main sources of error in the above method of velocity determination are likely to be (i) the use of a particular spherically symmetrical model of the corona, and (ii) the neglect of coronal magnetic fields. We now consider these in turn.

(i) *Coronal Irregularities.*—It is well known that the electron-density distribution of the corona can depart considerably from the spherically sym-

HARMONICS IN THE SPECTRA OF SOLAR RADIO DISTURBANCES



(a)



(b)

Intensity-modulated records showing harmonics in (*a*) part of a type II burst (corresponding to Fig. 2) and (*b*) a type III burst (corresponding to Fig. 4 (*f*)). The constant frequency signals marked *S* are due to local transmitters. The vertical white line *X* is not part of the record.

metrical model we have assumed. In the first place the *mean* electron-density distribution changes with the solar cycle. From his analysis of optical data, van de Hulst (1950) concludes that the mean density at any height decreases by a factor of about 1·8 between maximum and minimum phases. For the equatorial region, his sunspot-maximum model agrees closely with the Allen-Baumbach model, while heights in his sunspot-minimum model are about 5×10^4 km lower in the range of interest here. However, the spacing between levels of different electron density is scarcely affected ; for instance the spacing between the 50 and 100 Mc/s levels changes by only 5 per cent. This indicates that the determination of velocity along radial directions is not significantly affected by changes in the mean distribution with the solar cycle. Effects due to departures from the mean values, manifest in the complex structure of the corona, are likely to be much more important. The optical data, which have recently been reviewed by van de Hulst (1953), seem to indicate that the electron density in coronal rays may be some 10 times that between the rays while the gradients in the two regions are similar.

Let us now re-examine the outburst analysed in Figure 9, bearing in mind the effects of coronal structure. Assuming the true source velocity to be constant, the linearity and smoothness of the height plot indicate that the " shape " of the assumed electron-density law is the correct one along the path of the source. Suppose, however, that the source travelled along a coronal ray whose density was 10 times that given by the assumed model. Source heights are then increased by some 4×10^5 km, and the velocity (still essentially constant) by a factor of 1·6. Taking van de Hulst's sunspot-minimum model to represent the other extreme, it is estimated that the outburst's true outward component of velocity probably lay between 400 and 750 km/sec.

In cases where the motion is largely transverse (e.g. Fig. 8), the derivation of speed depends on the large-scale angular distribution of electron density, and the quantitative results are less certain.

(ii) *Coronal Magnetic Fields*.—The presence of a magnetic field at the source causes two additional proper frequencies to exist either side of the plasma frequency (Westfold 1949). The three frequencies correspond to those of the magneto-ionic theory at which the refractive index vanishes ; each shows a characteristic polarization. At the coronal heights with which we are concerned the three frequencies are probably closely spaced (i.e. $f_H \ll f_0$) and the effects of neglecting magnetic fields could cause no significant errors in the derivation of velocities.

Application of the magneto-ionic theory shows that from any one point only two of the three frequencies can escape from the Sun. It was previously suggested (Wild 1950*a*) that the presence of closely spaced double peaks, like those in Figures 2 and 3, could be due to this effect of magnetic splitting. An alternative explanation is in terms of two sources separated in space. The former explanation now seems more likely because in both instances described here the two peaks are seen to fade simultaneously, thus suggesting a common source. Combined observations of spectrum and polarization should help to decide this question.

(c) *The Size of Outburst Sources*

In the simple interpretation outlined in Section IV it was assumed that burst radiation emanates from a point source. For sources of finite size we should receive the spectrum due to the excitation of a finite band of plasma frequencies. Lack of knowledge of the natural spectrum prohibits the estimation of source dimensions from the width of the observed peaks. We can, however, use the bandwidth data to set an upper limit on at least one of these dimensions.

In the vicinity of 50 Mc/s, bandwidths as small as 1·5 Mc/s between half-power points are not uncommon. With the standard coronal model this means that the source responsible for the peak is contained within a height range of less than 10^4 km. Also, in cases such as Figure 3, it can be inferred from the abruptness of the cut-off that the gradient of the outer edge of the source is such that the intensity may change by a factor of 2 within a height range of less than 500 km.

(d) *The Sources of Type III Bursts*

The main conclusions of Section IV, that the burst-generating sources probably radiate at harmonics of the plasma frequency of the surrounding medium, apply to type III as well as to type II bursts. This provides an experimental foundation to the speculation (Wild 1950*b*) that the fast frequency drift of type III bursts is to be interpreted in a similar fashion to the slow drift of type II bursts. On this basis the type III sources are found to move outwards through the solar corona with initial radial velocity components of between 3×10^4 and 10^5 km/sec, showing steady deceleration along their path. Those with " tails " are apparently brought to rest at heights of a few hundred thousand kilometres.

The interpretation of type III bursts has been discussed briefly by Wild, Roberts, and Murray (1954) and will be considered in more detail in a later paper.

VI. Summary of Conclusions

The observational results of this paper are summarized at the end of Section III. The presence in the spectra of sporadic bursts of intense narrow emission bands accompanied by their second harmonics indicates that the sources consist of oscillatory charges which oscillate, according to a non-linear law, at or near some proper frequency. From considerations of the escape of radiation from resonance levels in the solar atmosphere it is concluded that the plasma frequency is the only known proper frequency capable of accounting for the observations. If the natural spectrum of emission from the plasma level is assumed to show a slight spread in frequency about the plasma frequency, it is possible to give a natural explanation of certain peculiar features of observed spectra. The sources are found to be in rapid motion through the corona and it is suggested that the generation of high intensities is associated with longitudinal plasma oscillations excited by fast streams of ionized matter.

Assuming a standard, spherically symmetrical corona, application of the plasma hypothesis to observed spectra yields information on the position (both height and angular displacement from the centre of the disk), velocity, and size of the source. Velocities of 500 and 4000 km/sec were deduced for two long-duration outbursts, and velocities as great as 10^5 km/sec for the short-lived type III bursts.

VII. Acknowledgments

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