THE EMISSION OF RADIATION FROM MODEL HYDROGEN CHROMOSPHERES. II

By J. T. Jefferies* and R. G. Giovanelli*

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Summary

An improved method is presented for calculating the characteristics of the radiation field of Hα, Lα, Lβ, and the Lyman continuum emitted by model hydrogen atmospheres which are isothermal at one of a number of kinetic temperatures in the range 10^4 to $2 \cdot 5 \times 10^5$ °K. It is found that earlier estimates of the intensities of these lines need revision and improved values have been obtained.

I. Introduction

Excitation in and the quantity of radiation emitted by high temperature hydrogen atmospheres have been discussed by a number of authors. Basically, excitation is due to collisions ; self-absorption of radiation may, however, cause a profound modification of excitation conditions. Since the radiation emitted in or transmitted through such an atmosphere may undergo scattering or absorption, excitation and the transfer of radiation are interrelated in a manner which is further complicated by possible changes in wavelength upon interaction between radiation and atoms.

The problem may be formulated in terms of a set of simultaneous equilibrium equations, one for the population of each atomic level, and a set of second order differential equations, one for the transfer through the atmosphere of radiation of each wavelength. But the solution of these equations has proved so formidable that substantial simplifications have always been unavoidable. The two most important of these have been

(i) a restriction in the number of energy levels and corresponding wavelengths considered, and

(ii) the derivation of approximate rates of emission and absorption by assuming the radiation intensities and then estimating excitation conditions.

Thus, in considering the emission of radiation from model solar chromospheres, Giovanelli (1949) and Jefferies (1953) assumed black body radiation at 5000 °K except in the Lyman lines and continuum, while Thomas (1949) and Matsushima (1952) assumed either no radiation or black body radiation at 6000 °K other than in the Lyman lines.

Nevertheless, assumptions as to radiation intensities, particularly of Hα, can have a significant effect on the computed emissions, not so much of the

* Division of Physics, C.S.I.R.O., University Grounds, Sydney.

Lyman α line but of the other Lyman and subsidiary lines. We present here a method which substantially avoids these assumptions, the Hα intensity being carried as a parameter.

Previous work of the present authors has differed from that of Thomas and of Matsushima in that we have restricted consideration to the states and substates of principal quantum number 1, 2, and 3 and to the ionized state. Thomas considered states of principal quantum number 1–10 and the ionized state, collisions other than with ground state atoms being neglected. Matsushima extended Thomas's results by including collisions with excited atoms.

There is considerable simplification in ignoring the fine structure of the quantum states, and there is now some evidence that this may be justified. Giovanelli (1948) pointed out that transitions between the $2S$ state and the $2P$ state could be ignored unless the appropriate collision cross section was abnormally large. But Purcell (1952) has shown that the effective cross section for collision with positive ions may be as high as $10^7\pi a_0{}^2$, which is sufficient to ensure that the principal transitions involving the $2S$ state will be to and from the $2P$ state. At electron concentrations of the order of 5×10^{11} cm^{-3} or more, this alone would ensure that the $2S$ and $2P$ populations are in the ratio of their statistical weights. Further, while computations ignoring $2S\rightleftarrows 2P$ transitions have shown a strong overpopulation in the $2S$ state in the presence of low radiation densities (i.e. very thin atmospheres), the Lyman radiation intensities in hot atmospheres will normally ensure that the populations of even the $2S$ and $2P$ substates are of the same order. This follows, for example, from explicit solutions of the equilibrium equations given by Giovanelli (1949) and Jefferies (1953). A high $2S\rightleftarrows 2P$ collision rate will promote an even closer approach to populations distributed according to the statistical weights. We therefore propose to ignore substates and consider the simpler problem so formulated. As before, only the three lower quantum states and the ionized state will be considered.

The present work is confined to a range of physical conditions in which excitation by collisions between neutral atoms may be neglected. This will almost certainly apply for temperatures of about 10^4 °K and higher, and may apply for lower temperatures depending on the cross sections (as yet unknown) for excitation from the excited states.

II. Excitation and Collision Rates

The total rate of excitation per unit volume from state j to state l is denoted by $P_{jl}N_j$, where N_j is the population of the j state. P_{jl} may be dissected into a radiative component, A_{jl}, and a collisional component, R_{jl}, which includes the electron concentration N_e as a factor.

The rates of collision excitation between substates have been discussed by Jefferies (1953)—henceforth referred to as paper I. Owing to an error in numerical integration, the collision rates in Table 1 of that paper are too large by a factor of 2. The rates used here are the corrected values.

III. The Equation of Radiative Transfer

With Eddington's approximation, the equation of radiative transfer is

$$\frac{1}{3}\frac{\mathrm{d}^2 J}{\mathrm{d}\tau^2}=J-4\pi B, \qquad (3.1)$$

which may be written, for a coherently scattering atmosphere, as

$$\frac{1}{3}\frac{\mathrm{d}^2 J}{\mathrm{d}\tau^2}=\lambda J-4\pi b, \qquad (3.2)$$

where $1-\lambda$ is the fraction of the absorbed radiation which reappears as scattered radiation in the same line.

The present authors (Giovanelli and Jefferies 1954) have shown that in a four-level atom

$$\lambda_{ab}=\frac{R_{ba}(1-p_{cd}p_{dc})+P_{bc}(p_{ca}+p_{cd}p_{da})+P_{bd}(p_{da}+p_{dc}p_{ca})}{P_{ba}(1-p_{cd}p_{dc})+P_{bc}(p_{ca}+p_{cd}p_{da})+P_{bd}(p_{da}+p_{dc}p_{ca})}, \qquad (3.3)$$

where P_{lj} is the rate of transition $l \rightarrow j$ per atom in the l state, and is the sum of a radiative term A_{lj} and a collision term R_{lj}; and

$$p_{lj}=\frac{P_{lj}}{\sum_k P_{lk}}. \qquad (3.4)$$

Further,

$$4\pi b_{ab}=\rho_{ab}\frac{R_{ab}(1-p_{cd}p_{dc})+P_{ac}(p_{cb}+p_{cd}p_{db})+P_{ad}(p_{db}+p_{dc}p_{cb})}{D}, \qquad (3.5)$$

where D is the denominator of (3.3), and

$$\rho_{ab}=\frac{8\pi h\nu_{ab}^3}{c^2}\,\frac{\omega_a}{\omega_b}, \qquad (3.6)$$

ω_a and ω_b being the statistical weights of the states, for line transitions. For transitions involving the continuum

$$\rho_{ab}=\frac{\pi h^4\nu_{ab}^3}{c^2}\left(\frac{2}{\pi mkT}\right)^{3/2}N_e. \qquad (3.7)$$

For transitions for which the atmosphere is of great optical depth, P_{lj} and P_{jl} may be replaced by R_{lj} and R_{jl}, while for transitions for which the atmosphere is of small optical depth, self-absorption may be neglected; and, if either of these conditions applies to every spectral line other than the one under consideration, interlocking can be neglected.

In an atmosphere in which λ and b do not vary with τ, the solution of (3.2) may be written

$$J=\frac{4\pi b}{\lambda}+\alpha\exp(\sqrt{3\lambda}\tau)+\beta\exp(-\sqrt{3\lambda}\tau), \qquad (3.8)$$

where the integration constants α and β are determined by the boundary conditions. In particular, for a very thick atmosphere with no external illumination, the emergent intensity is given by

$$J=\frac{4\pi b}{\lambda}\,\frac{2\sqrt{\lambda/3}}{1+2\sqrt{\lambda/3}}, \qquad (3.9)$$

while for a very thin atmosphere, $\sqrt{3\lambda}\tau^1 \ll 1$,

$$J = 4\pi b\tau^1 \text{ or } 8\pi b\tau^1, \qquad \text{(3.10)}$$

depending on whether reflection at the base is negligible or complete. The symbol τ^1 represents the total optical depth of the atmosphere.

IV. The Ground State Population

The intensity of any spectral line is closely associated with the populations of the upper and lower states involved, except in the special case of an optically thin atmosphere. Consequently the population of both the ground and ionized states will generally be related to the intensity of the Lyman continuum.

If Lyman line and continuum radiation is negligible, i.e. if the atmosphere is optically very thin, particularly in Lα, almost all ionizations occur directly by electron collision from the ground state. These are balanced by recombinations, so that

$$\frac{N_1}{N_4} = \frac{\Sigma P_{4l}}{R_{14}}, \qquad \text{(4.1)}$$

where N_4 $(=N_e)$ is the ion concentration, the subscript denoting the ionized state, and P_{4l} the rate of recombination per ion to the l-quantum state.

In an atmosphere optically thick in the Lyman lines, any excited atom has a high probability of returning direct or by cascade to the ground state, emitting a Lyman quantum. But, except in the unimportant case near the top of the atmosphere, this quantum will be reabsorbed and the process repeated until an ionization occurs. Consequently almost every collision excitation from the ground state results eventually in an ionization. These, together with direct ionizations from the ground state, are balanced by recombinations direct to the ground state, since any recombination to an excited state also results in a sequence of emissions and absorptions of Lyman quanta until an ionization occurs. Thus

$$\frac{N_1}{N_4} = \frac{P_{41}}{R_{12}+R_{13}+P_{14}}. \qquad \text{(4.2)}$$

This relation is valid provided the rate of superelastic collision from any excited state (effectively the 2-quantum state) is small compared with the rate of ionization from that state. It follows from the values of the excitation rates that this holds for all values of N_e at kinetic temperatures T of $1\cdot5\times10^4$ °K or higher, and for $N_e \leqslant 10^{12}$ cm^{-3} for $T = 10^4$ °K.

Provided the atmosphere is optically thin and not strongly irradiated in the Lyman continuum, P_{14} may be replaced by R_{14} in (**4.2**) so that

$$\frac{N_1}{N_4} = \frac{A_{41}}{\Sigma R_{1l}}, \qquad \text{(4.3)}$$

where, since recombinations are predominantly radiative, P_{41} has been replaced by the rate of radiative recombination A_{41}. In (**4.1**) and (**4.3**) the ratio N_1/N_4 is independent of electron concentration and of thickness of the atmosphere.

If the atmosphere is not optically thin in the Lyman continuum, we require the rate of absorption A_{14}. The rate of emission of Lyman continuous quanta of frequency ν is given by the expression

$$[4\pi b+(1-\lambda)J]\alpha_\nu d\nu/h\nu, \qquad (4.4)$$

where α_ν is the absorption coefficient. With J written as

$$J=4\pi\frac{b}{\lambda}\cdot\varphi(\lambda, \tau), \qquad (4.5)$$

(**4.4**) becomes

$$4\pi b\left[1+\frac{1-\lambda}{\lambda}\varphi(\lambda, \tau)\right]\frac{\alpha_\nu d\nu}{h\nu}. \qquad (4.6)$$

Since the ratio of the rate of emission of such quanta (which equals the rate of ionization in accordance with (**4.2**)) to the rate of absorption is increased by the factor

$$1+\frac{1-\lambda}{\lambda}\varphi(\lambda, \tau)$$

over the corresponding case for $\tau\ll 1$, it follows that

$$\frac{N_1}{N_4}=\frac{A_{41}}{\Sigma R_{1l}\left[1+\frac{1-\lambda}{\lambda}\varphi(\lambda, \tau)\right]}, \qquad (4.7)$$

which varies between the limit $A_{41}/\Sigma R_{1l}$ and $\lambda A_{41}/\Sigma R_{1l}$ according as $\tau^1\ll 1$ $(\varphi=0)$ and $\tau,\tau^1\gg 1$ $(\varphi=1)$.

From (**3.3**) it follows on neglecting small terms that

$$\lambda_{14}=\frac{R_{41}+\alpha(P_{42}+P_{43})}{P_{41}+\alpha(P_{42}+P_{43})}, \qquad (4.8)$$

where $\alpha=(r_{21}+r_{31})/(p_{24}+p_{34})$ and $r_{lj}=R_{lj}/\sum_k P_{lk}$.

In the range of N_e and T of interest here, the transition rates are such that $\alpha\simeq r_{21}/p_{24}$ and R_{41} is negligible, so that

$$\lambda_{14}\simeq\frac{\alpha(P_{42}+P_{43})}{P_{41}+\alpha(P_{42}+P_{43})}, \qquad (4.9)$$

which is approximately proportional to N_e, and so is dependent on τ_{14}. We shall consider only two broad regions of the atmosphere such that, at the head of the Lyman continuum, $\sqrt{3\lambda}\tau_{14}$ is greater than or less than unity. In the lower region

$$\frac{N_1}{N_4}=\frac{A_{41}}{\Sigma R_{1l}}\lambda_{14}, \qquad (4.10)$$

with λ_{14} given by (**4.9**). In the upper region, the intensity of the Lyman continuum will be roughly that coming from a uniform atmosphere with physical conditions the same as those at $\sqrt{3\lambda}\tau_{14}\simeq 1$. It follows from (**3.9**) and (**4.5**) that

$$\varphi(\lambda, \tau)\simeq\frac{2\sqrt{\lambda/3}}{1+2\sqrt{\lambda/3}}$$

in this region, λ being computed for conditions applying at $\sqrt{3\lambda}\tau_{14} \simeq 1$, so that for λ not too large, (**4.7**) reduces to

$$\frac{N_1}{N_4} = \frac{A_{41}}{\Sigma R_{1l}} \frac{\sqrt{3\lambda}}{2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{4.11})$$

where A_{41} and R_{1l} are values at the electron concentration under consideration ($A_{41}/\Sigma R_{1l}$ is actually independent of N_e).

From (**4.9**) and (**4.11**) we may compute the values of N_e at $\sqrt{3\lambda}\tau_{14}=1$ and so estimate the electron concentrations for which (**4.10**) and (**4.11**) apply. Results are given in Table 1. In the intermediate region where $\sqrt{3\lambda}\tau_{14} \simeq 1$, the ratio N_1/N_4 changes rapidly with depth by a factor of the order of 10.

TABLE 1

VALUES OF N_e AND N_1/N_4 AT VARIOUS TEMPERATURES

T (10^4 °K)	N_e at $\sqrt{3\lambda}\tau_{14}=1$ (cm^{-3})	Corresponding Values of N_1/N_4	
1·0	5×10^{10}	$\frac{A_{41}}{\Sigma R_{1l}} \times 1\cdot9\times10^{-13}N_e$	$(10^{12} \geqslant N_e > 5\times10^{10})$
		$\frac{A_{41}}{\Sigma R_{1l}} \times 9\cdot5\times10^{-2}$	$(N_e \leqslant 5\times10^{10})$
1·5	5×10^{11}	$\frac{A_{41}}{\Sigma R_{1l}} \times \frac{2\cdot2\times10^{-13}N_e}{1+1\cdot5\times10^{-12}N_e}$	$(10^{12} \geqslant N_e > 5\times10^{11})$
		$\frac{A_{41}}{\Sigma R_{1l}} \times 2\cdot8\times10^{-1}$	$(N_e \leqslant 5\times10^{11})$
2·5	$>10^{12}$	—	—

The computed values of N_1 are shown in Table 2, for a variety of physical conditions.

TABLE 2

VALUES OF N_1/N_e FOR VARIOUS ATMOSPHERIC MODELS

T (°K)	Thin Atmospheres Eqn. (**4.1**)	Thick Atmospheres Eqn. (**4.3**)	Adopted
$1\cdot0\times10^4$	$5\cdot3\times10$	$8\cdot0\times10^{-1}$	See Table 1
$1\cdot5\times10^4$	$1\cdot8\times10^{-1}$	$9\cdot7\times10^{-3}$	See Table 1
$2\cdot5\times10^4$	$1\cdot7\times10^{-3}$	$2\cdot2\times10^{-4}$	$2\cdot2\times10^{-4}$
$5\cdot0\times10^4$	$3\cdot8\times10^{-5}$	$9\cdot4\times10^{-6}$	$3\cdot8\times10^{-5}$
$1\cdot0\times10^5$	$4\cdot4\times10^{-6}$	$1\cdot4\times10^{-6}$	$4\cdot4\times10^{-6}$
$2\cdot5\times10^5$	$7\cdot8\times10^{-7}$	$3\cdot1\times10^{-7}$	$7\cdot8\times10^{-7}$

For $T=5\times10^4$ °K, the Lα and Lβ optical depths are about unity, which makes computations of radiation intensities difficult. In general, we shall interpolate in this range.

V. The Lyman Continuum Intensity

The emergent intensity at the Lyman series limit follows from the solution of equation (**3.2**) with λ_{14} given by (**4.9**) and, provided the optical thickness in $L\alpha$ and $L\beta$ is large,

$$4\pi b_{14}=\rho_{14}\,\frac{\Sigma R_{1l}}{A_{41}}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{5.1})$$

where ρ_{14} is given by (**3.7**).

Evaluating λ_{14} and $4\pi b_{14}$ for the electron concentration at $\sqrt{3\lambda}\tau_{14}=1$ and substituting in the solution of (**3.2**) we obtain the emergent intensity. For kinetic temperatures greater than $1\cdot 5\times 10^4$ °K, the atmosphere is thin in the Lyman continuum if $N_0<10^{12}$ cm^{-3}, and assuming λ and b to be independent of τ, the emergent radiation is, from (**3.10**),

$$\begin{aligned} J_c &= 4\pi b_{14}\tau_{14} \\ &\simeq \rho_{14}N_e z\alpha(1,\,\nu_0), \end{aligned}$$

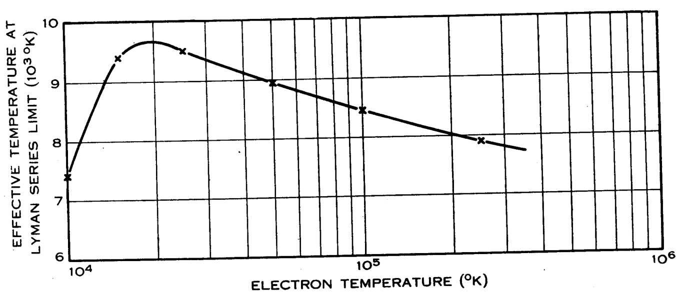

Fig. 1.—Effective temperature at the beginning of the Lyman continuum. $N_0=5\times 10^{11}$ cm^{-3}, $\beta=6\times 10^{-9}$ cm^{-1}.

where z is the scale height for electrons, $\alpha(1,\,\nu_0)$ the Lyman continuum absorption coefficient per $1S$ atom and $\rho_{14}N_e$ has the value appropriate to the base of the atmosphere.

Neglecting reflection at the base, the emergent Lc intensity, expressed in terms of the equivalent black body temperature for hemispherical emission, is shown in Figure 1, for $N_0=5\times 10^{11}$ cm^{-3} and $z=1\cdot 66\times 10^8$ cm. A comparison with the results in paper I shows that, for $T=10^4$ and $1\cdot 5\times 10^4$ °K the emission found here is much greater. This difference arises from neglect of scattering, resulting in too high a ground state population, in the previous work. For the higher temperatures the present results are in agreement with those before.

VI. The Lyman Line Intensities

To evaluate the Lyman line intensities the scattering parameters and source functions are required. From (**3.5**) we find, on neglecting small terms, that, when the atmosphere is opaque to $L\beta$,

$$4\pi b_{12}\simeq \rho_{12}\frac{\Sigma R_{1l}-p_{41}R_{14}}{A_{21}}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{6.1})$$

and

$$\lambda_{12} \simeq \frac{R_{21}+(P_{24}+P_{23}p_{34})p_{41}}{A_{21}}. \quad \ldots\ldots\ldots\ldots\ (6.2)$$

If the atmosphere is optically thin in Lα and Lβ the emergent Lα intensity is independent of λ_{12}, while

$$4\pi b_{12} \simeq \rho_{12}\frac{R_{12}+R_{14}p_{42}+(R_{13}+R_{14}p_{43})p_{32}}{A_{21}}. \quad \ldots\ldots\ (6.3)$$

From equations (**6.1**), (**6.2**), and (**6.3**), $4\pi b_{12}$ and λ_{12} depend on N_e and so on τ_{12}. However, for atmospheres of high optical depth, the emergent Lα radiation is approximately that from a uniform atmosphere of N_e and λ equal to that at $\sqrt{3\lambda}\tau_{12}=1$. For $\sqrt{3\lambda}\tau_{12}^{\frac{1}{2}}<1$, the emission has been computed for an atmosphere of constant b_{12} using the electron concentration at the base of the atmosphere and assuming complete reflection there.

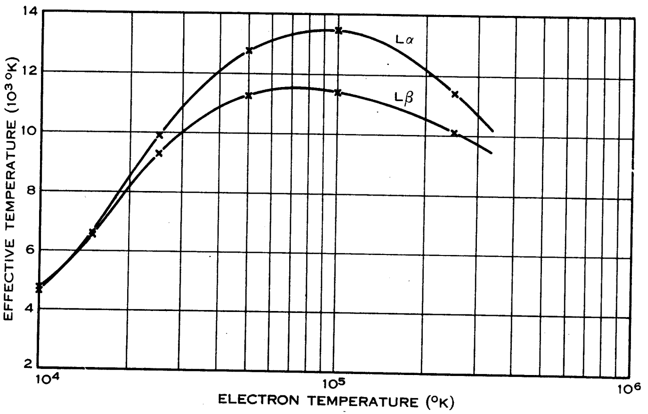

Fig. 2.—Effective temperature at the centre of Lα and Lβ. $N_0=5\times10^{11}$ cm^{-3}, $\beta=6\times10^{-9}$ cm^{-1}.

For Lβ, in cases where $\sqrt{3\lambda}\tau_{12}>1$, neglect of small terms yields

$$4\pi b_{13} \simeq \rho_{13}\frac{\Sigma R_{1l}-p_{41}R_{14}}{A_{31}}, \quad \ldots\ldots\ldots\ldots\ldots\ (6.4)$$

$$\lambda_{13} \simeq \frac{R_{31}+(P_{34}+P_{32}p_{24})p_{41}}{A_{31}}, \quad \ldots\ldots\ldots\ldots\ (6.5)$$

while, if $\sqrt{3\lambda}\tau_{12}^{\frac{1}{2}}\ll 1$,

$$4\pi b_{13} \simeq \rho_{13}\frac{R_{13}+R_{14}p_{43}+(R_{12}+R_{14}p_{42})p_{23}}{A_{31}+A_{32}}. \quad \ldots\ldots\ (6.6)$$

The values of λ_{13} and $4\pi b_{13}$ depend on the rate of absorption of Hα, and so on the Hα intensity. Since the mean level of origin for the escaping Lβ radiation is given by $\sqrt{3\lambda}\tau_{13}=1$, a level where the optical depth is Hα is small, P_{23} has been computed adopting Hα radiation intensities appropriate to the top of the atmosphere (see Section VII).

With boundary conditions that there is no incident radiation on the top of the atmosphere and that perfect reflection occurs at the base, the central intensities of the Lyman lines can now be found, and are shown in Figure 2 in terms of equivalent hemispherical black body temperatures. These values are not directly comparable with those in paper I, which are calculated for an atmosphere of uniform electron concentration $N_e=5\times10^{11}$ cm^{-3} and thickness equal to $1/(6\times10^{-9})$ cm, while here we have adopted $N_e=N_0e^{-\beta z}$ with $\beta=6\times10^{-9}$ cm^{-1} and $N_0=5\times10^{11}$ cm^{-3}. The effect of this difference may be seen from Table 3 where we have given values of N_e such that $\sqrt{3\lambda}\tau=1$ for Lα and Lβ. For $T\geqslant10^5$ °K, $\sqrt{3\lambda}\tau^1<1$.

TABLE 3

ELECTRON CONCENTRATIONS AT THE MEAN LEVEL OF ORIGIN OF THE ESCAPING Lα AND Lβ RADIATION

T (°K)	N_e at $\sqrt{3\lambda}\tau_{12}=1$ (cm^{-3})	N_e at $\sqrt{3\lambda}\tau_{13}=1$ (cm^{-3})
$1\cdot0\times10^4$	$1\cdot4\times10^8$	$2\cdot6\times10^7$
$1\cdot5\times10^4$	$5\cdot2\times10^9$	$1\cdot3\times10^9$
$2\cdot5\times10^4$	$7\cdot1\times10^{10}$	$2\cdot1\times10^{10}$

VII. THE Hα INTENSITY

To find the Hα intensity we proceed slightly differently. The intensity of Hα is given by the equation

$$\frac{1}{3}\frac{d^2J}{d\tau^2}=J-\rho_{23}\frac{N_3}{N_2}, \quad \ldots\ldots (7.1)$$

in which N_3 may be eliminated using the equilibrium equation

$$(P_{31}+P_{32}+P_{34})N_3=P_{13}N_1+P_{23}N_2+P_{43}N_4. \quad \ldots\ldots (7.2)$$

For regions in which $\sqrt{3\lambda}\tau_{13}\gg1$, which, for the lower temperatures, is certainly so where $\sqrt{3\lambda}\tau_{23}\simeq1$, we may put $A_{31}N_3=A_{13}N_1$, when (7.2) reduces to

$$(R_{31}+P_{32}+P_{34})N_3=R_{13}N_1+P_{23}N_2+P_{43}N_4. \quad \ldots\ldots (7.3)$$

The transfer equation (7.1) may then be written

$$\frac{1}{3}\frac{d^2J}{d\tau^2}=J-\rho_{23}\frac{R_{13}N_1+P_{43}N_4+P_{23}N_2}{(R_{31}+P_{34}+P_{32})N_2}, \quad \ldots\ldots (7.4)$$

and so

$$\lambda_{23}\simeq\frac{R_{31}+P_{34}+R_{32}}{A_{32}}. \quad \ldots\ldots (7.5)$$

Further,

$$4\pi b_{23}\simeq\rho_{23}\frac{R_{13}N_1+P_{43}N_4+R_{23}N_2}{A_{32}N_2}, \quad \ldots\ldots (7.6)$$

a result which may also be obtained from (3.5) on omitting very small terms.

If, for Lβ, $\sqrt{3\lambda}\tau^1\leqslant1$, we find

$$\lambda_{23}\simeq\frac{A_{31}}{A_{31}+A_{32}}=0\cdot56, \quad \ldots\ldots (7.7)$$

$$4\pi b_{23}\simeq\rho_{23}\frac{R_{23}N_2+R_{13}N_1+P_{43}N_4}{(A_{31}+A_{32})N_2}. \quad \ldots\ldots (7.8)$$

In paper I, λ_{23} was found to be $\sim 1/3$ in the chromosphere and to be almost independent of N_e and T, whereas here, for $\sqrt{3\lambda}\tau_{13} \gg 1$ it is found to be much smaller ($\sim 10^{-2}$) and to vary with the physical conditions. This difference is due to the explicit neglect, in paper I, of Hα scattering following the transition $2S \rightarrow 3P \rightarrow 1S$. For $\sqrt{3\lambda}\tau^1_{13} \ll 1$, λ_{23} is of the same order as that found previously, as expected.

The value of N_2 needed in (**7.6**) and (**7.8**) is found from the relationship, valid if the Lα optical depth is not too small,

$$\frac{N_2}{N_1} \simeq \frac{A_{12}}{A_{21}} \simeq \frac{J_{12}}{\rho_{12}}, \quad \dots\dots\dots\dots\dots\dots \quad (7.9)$$

while N_1/N_4 is given in Table 2.

As before, the atmosphere is taken as uniform, λ_{23} and $4\pi b_{23}$ being computed for the electron concentration at $\sqrt{3\lambda}\tau_{23}=1$. If $\sqrt{3\lambda}\tau^1_{23} \ll 1$, the corresponding quantities adopted are those applicable at the base.

Since in Hα the chromosphere is superimposed on a surface of comparable brightness, transmitted photospheric radiation has also to be taken into account.

The solution of (**3.2**) for the emergent intensity of Hα is in such a case (paper I equation (**18**))

$$J_c = \frac{4\pi b}{\lambda}\left[1 - \frac{\sqrt{\lambda}+\sqrt{\lambda'}+4\sqrt{\frac{\lambda\lambda'}{3}}\exp(-\sqrt{3\lambda}\tau^1)+(\sqrt{\lambda}-\sqrt{\lambda'})\exp(-2\sqrt{3\lambda}\tau^1)}{D}\right]$$

$$+\frac{2(1+2/\sqrt{3})\sqrt{\lambda\lambda'}\exp(-\sqrt{3\lambda}\tau^1)J_w}{D}, \quad \dots\dots\dots\dots\dots\dots \quad (7.10)$$

where

$$D=(1+2\sqrt{\lambda/3})(\sqrt{\lambda}+\sqrt{\lambda'})+(1-2\sqrt{\lambda/3})(\sqrt{\lambda}-\sqrt{\lambda'})\exp(-2\sqrt{3\lambda}\tau^1)$$

and J_w is the intensity of the neighbouring continuum.

In this equation, τ^1 is the optical thickness of the chromosphere while λ' represents the photospheric scattering parameter defined by

$$\left.\begin{aligned} 1-\lambda' &= \frac{\sigma_\nu}{\sigma_\nu+\varkappa_\nu+\varkappa_0} \\ &= \frac{(\sigma_\nu+\varkappa_\nu)\lambda_L+\varkappa_0}{\sigma_\nu+\varkappa_\nu+\varkappa_0}, \end{aligned}\right\} \quad \dots\dots \quad (7.11)$$

where σ_ν and $\sigma_\nu+\varkappa_\nu$ are respectively the scattering and absorption coefficients, $\varkappa_0$ is the continuous absorption coefficient of the negative hydrogen ion, and $\lambda_L=\varkappa_\nu/(\sigma_\nu+\varkappa_\nu)$ is the line scattering parameter of Hα.

In paper I, λ_L was found from observation to have a value $\sim 1/3$ in the wings of the line (i.e. in the deeper photospheric layers). To calculate λ_L for the photosphere the rates of collision excitation by neutral atoms as well as by electrons are required. Bates and Griffing (1953) have shown that $1S \rightarrow$excited state transitions in hydrogen caused by H or H^+ impact are of the near adiabatic type with consequent very low cross section at low energies. If similar results apply for excitation from the 2-quantum state it is justifiable to neglect excitation by neutral atom collisions compared with that by electrons under photospheric conditions. In any case such neglect will give a lower limit to λ_L. From (7.5) it readily follows by considering only electron collisions that, for an atmosphere of kinetic temperature 5000 °K,

$$\lambda_L = \frac{2\cdot 3\times 10^5 + 4\cdot 2\times 10^{-7}N_e}{4\cdot 4\times 10^7 + 4\cdot 2\times 10^{-7}N_e}. \quad \ldots\ldots\ldots\ldots \quad (7.12)$$

Adopting Munch's (1947) values for the electron pressure in a model solar atmosphere we find that, at $\tau=0\cdot 01$ (corresponding to the centre of Hα) $\lambda_L=1\cdot 7\times 10^{-2}$; and at $\tau=1\cdot 0$ (corresponding to the wings) $\lambda_L=0\cdot 25$ in fair agreement with observation, τ being the optical depth in the continuum.

On substituting for the transition rates and putting $\lambda'_{\nu_0}=\lambda_L=1\cdot 7\times 10^{-2}$, the central intensities of the emergent Hα line are obtained as fractions of the adjacent continuum. Results are given in Table 4 for a variety of temperatures and electron concentrations at the base of the atmosphere. For $T=5\times 10^4$ °K, the results have again been found by interpolation.

TABLE 4

Hα CENTRAL INTENSITY

$\beta=6\times 10^{-9}$ cm^{-1}

N_e (cm^{-3}) / T (°K)	10^{12}	5×10^{11}	2×10^{11}	10^{11}
$1\cdot 0\times 10^4$	24 (*24*, *0*)*	24 (*24*, *0*)	22 (*17*, *5*)	20 (*4*, *16*)
$1\cdot 5\times 10^4$	39 (*39*, *0*)	39 (*39*, *0*)	32 (*24*, *8*)	22 (*5*, *17*)
$2\cdot 5\times 10^4$	60 (*60*, *0*)	60 (*59*, *1*)	42 (*32*, *10*)	26 (*7*, *19*)
$5\cdot 0\times 10^4$	60 (*58*, *2*)	37 (*25*, *12*)	27 (*10*, *17*)	27 (*3*, *24*)
$1\cdot 0\times 10^5$	33 (*23*, *10*)	28 (*5*, *23*)	24 (*0*, *24*)	24 (*0*, *24*)
$2\cdot 5\times 10^5$	26 (*2*, *24*)	24 (*0*, *24*)	24 (*0*, *24*)	24 (*0*, *24*)

* Chromospheric and photospheric contributions, in this order respectively, are included in parentheses after the corresponding Hα central intensity. The intensity of the surrounding continuum equals 100 units.

Comparison of these values with the corresponding ones given in paper I shows that the present chromospheric components are in general much greater.

The origin of this difference lies in the adoption in paper I of a definite value for the Hα intensity in the earlier stages of the computations. Thus, while the transition rates due to absorption of radiation are in general much greater than collisional rates, the origin of the radiation lies in these collisions together with the radiation incident on the atmosphere. It is not in general legitimate, therefore, to neglect collisional in comparison with radiative transitions; we may approximate between collisional rates or between radiative rates but not between the two. By approximating to the Hα intensity in paper I we effectively did just this and in consequence held down the computed value of the Hα intensity.

VIII. Discussion

Comparison of the results of these computations with observations of the emission from the Sun is made rather difficult by lack of knowledge of the chromospheric structure. From the results found in paper I it appeared that, if the Hα emitting regions had a temperature in the range discussed, it would lie between $1 \cdot 5 \times 10^4$ and $3 \cdot 5 \times 10^4$ °K. From the present results, however, it would seem that this temperature is less than $1 \cdot 5 \times 10^4$ °K, although the non-uniformity of the chromosphere could modify this conclusion considerably.

Until recently, Hα intensities were the only ones available for comparison with theory. However, during recent rocket flights, the solar Lα line has been photographed and this will provide very valuable data, the more so as there seems less doubt attached to computed Lα intensities than to Hα and Lβ.

At the present stage it appears that the most useful application of the results would be in interpreting observations of prominences and flares—where conditions are more likely to conform to our model of an isothermal atmosphere—and possibly, from limb observations of the chromosphere, in ascertaining the temperature gradient.

IX. References

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