A GENERAL EQUATION FOR THE CHOICE OF GLASS FOR CEMENTED DOUBLETS

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Summary

A general equation is derived for the choice of glass for cemented doublet objectives for any object distance or residual aberration. The case where the aberrations are zero is studied in detail.

I. INTRODUCTION

In the design of a cemented doublet with prescribed amounts of longitudinal chromatic aberration, spherical aberration, and coma, two of the three variables required are provided by the radii of the surfaces, and the third must be chosen from the refractive indices and dispersions of the two glasses used. As the properties of available glasses cannot be represented by continuous variables, it is most economical of time if a suitable pair of glasses is chosen from those available as the first step of the design.

Brown and Smith (1946) have published tables which make possible the choice of glass when the object is at infinity and when the spherical aberration and coma are completely corrected. In this paper, a general equation is derived to cover the cases of finite object distances and uncorrected aberrations. The present analysis, like the work of Brown and Smith, is based on third order aberration theory and the results are strictly valid only for thin, low aperture objectives, but it is probable that the equation has wider application in practice, for the author has shown (Steel 1954) that glasses chosen from the Brown and Smith tables are suitable at all practical apertures.

The final equation obtained is a quintic in the relative dispersions of the two glasses. To avoid the need for solving such an equation, the special case where the aberrations are corrected is treated here in sufficient detail to cover the object distances used in practice. However, in the general case, this quintic would have to be solved, or an alternative approximate method given by Corrias (1953) could be used to find suitable glasses.

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II. Derivation of Equation

For a doublet of power K, the total curvatures c_a and c_b of the two components must satisfy the condition

$$(n_a-1)c_a+(n_b-1)c_b=K, \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

where n_a and n_b are the refractive indices of the two glasses used ; in this paper they are taken for the e ray (5461 Å).

(a) *Chromatic Aberration*

If the chromatic aberration has a prescribed value, the two curvatures must also satisfy the condition

$$(n_a-1)c_a/\nu_a+(n_b-1)c_b/\nu_b=KA, \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

where ν_a and ν_b are the Abbe values for the two glasses, again taken here for the e ray, $\nu_{\mathrm{e}}=(n_{\mathrm{e}}-1)/(n_{\mathrm{F}}-n_{\mathrm{C}})$. The chromatic aberration is then given by

$$Lch'/l'^2-Lch/l^2=KA=2\delta E/y^2, \quad \ldots\ldots\ldots\ldots \quad (3)$$

l and l' being the object and image distance, y the semi-aperture, Lch and Lch' the longitudinal chromatic aberrations before and after the doublet, and δE the corresponding increase in the wave aberration ΣE.
Hence it follows that

$$c_a=\frac{\rho K}{(\rho-1)(n_a-1)}, \qquad c_b=-\frac{K}{(\rho-1)(n_b-1)}, \quad \ldots\ldots\ldots \quad (4)$$

where

$$\rho=\frac{\nu_a(1-A\nu_b)}{\nu_b(1-A\nu_a)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$=\frac{N+1}{N-1} \text{ in Brown and Smith's notation.}$$

When the chromatic aberration is corrected, $\rho=\nu_a/\nu_b$.

(b) *Coma*

The third order coma may be represented by C where

$$C=2OSC'/y^2K^2=2\delta W_{\mathrm{coma}}/y^3e'K^2, \quad \ldots\ldots\ldots\ldots \quad (6)$$

OSC' being the offence against the sine condition and δW_{coma} the increase in the coma wave aberration at an angular semi-field e'.

The standard equation for the third order coma of a cemented doublet (Conrady 1929) may now be solved for c_2, the contact curvature, giving

$$c_2(\rho-1)\{\rho(1+a)-(1+b)\}$$
$$=K[\{\rho+v(\rho-1)\}\{\rho(2+a)-(2+b)\}-\frac{\rho^2}{1-a}+\frac{1}{1-b}+C(\rho-1)^2], \quad \ldots \quad (7)$$

where $a=1/n_a$, $b=1/n_b$, and $v=1/Kl$, the reciprocal object distance at unit power.

The other curvatures are given by

$$c_1=c_2+c_a, \qquad c_3=c_2-c_b. \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

(c) *Spherical Aberration*

The third order spherical aberration may be represented by S where

$$LA_p'/l'^2-LA_p/l^2=K^3y^2S/2=4\delta W_p/y^2, \quad \ldots\ldots\ldots\ldots \quad (9)$$

LA_p and LA_p' being the third order longitudinal spherical aberrations before and after the doublet and δW_p the corresponding increase in the wave aberration.

If this value S and the values of c_a, c_b, and c_2 are substituted in the standard equation for the primary spherical aberration of a doublet (Conrady 1929) the following equation is found connecting ρ and x, x being $\rho+v(\rho-1)$,

$$\begin{aligned}
&\{\rho(1+a)-(1+b)\}^2\left[\frac{\rho^3}{(1-a)^2}-\frac{1}{(1-b)^2}-S(\rho-1)^3+x\{\rho^2-1-4C(\rho-1)^2\}\right.\\
&\qquad\qquad\left.-x^2\{\rho(5+2a)-(5+2b)\}\right]\\
&-\{\rho(1+a)-(1+b)\}\left[\frac{\rho^2}{1-a}-\frac{1}{1-b}-C(\rho-1)^2-x\{\rho(2+a)-(2+b)\}\right]\left\{\rho^2\frac{2+a}{1-a}-\frac{2+b}{1-b}\right\}\\
&+\left[\frac{\rho^2}{1-a}-\frac{1}{1-b}-C(\rho-1)^2-x\{\rho(2+a)-(2+b)\}\right]^2\{\rho(1+2a)-(1+2b)\}=0.
\end{aligned}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{10})$$

For given values of the reciprocal object distance v, the aberrations C and S, and the refractive indices, this equation may be expanded as a quintic in ρ, giving a generalized form of Mossotti's equation (Argentieri 1954). Only positive roots are of practical interest; in the cases studied there is only one which is greater than unity for $n_a<n_b$ (the crown leading doublet) and less than unity for $n_a>n_b$ (the flint leading doublet). It should also be noted that the root for a given value of v is the reciprocal of the root of the equation when v is replaced by $-(v+1)$ and n_a and n_b interchanged, for this represents a reversal of the light path.

The Brown and Smith tables give solutions for the case $C=S=0$ when $v=0$ and when $v=-1$.

III. Application

(a) *General Case*

The general method of choosing a suitable pair of glasses would follow closely that used by the author (Steel 1954) for telescope objectives. One glass is chosen tentatively and the curve representing the glasses which can be used with it is plotted on a glass chart of n_e and ν_e using points obtained by solving equation (**10**) for different indices of the unknown glass. If this curve does not pass sufficiently close to an available glass type, another initial choice would have to be made.

(b) *Corrected Doublets*

The numerical solution as a quintic is tedious but the reverse problem of finding the value of v for which a given glass pair is suitable is much simpler, involving only the solution of a quadratic. This method has been applied to the case of corrected doublets by solving for v for a typical selection of glass pairs.

When $C=S=0$, equation (**10**) may be expanded in powers of x to give

$$x^2(\rho-1)^3-x\Bigg[\{\rho(1-a)-(1-b)\}\{\rho(2+a)-(2+b)\}\left\{\frac{\rho^2}{1-a}-\frac{1}{1-b}\right\}$$
$$-(\rho-1)(\rho^2-1)\{\rho(1+a)-(1+b)\}\Bigg]$$
$$+\{\rho(2+a)-(2+b)\}\left\{\frac{\rho^2}{1-a}-\frac{1}{1-b}\right\}^2-\{\rho(1+a)-(1+b)\}^2\left\{\frac{\rho^3}{(1-a)^2}-\frac{1}{(1-b)^2}\right\}$$
$$-(\rho^2-1)\{\rho(1+a)-(1+b)\}\left\{\frac{\rho^2}{1-a}-\frac{1}{1-b}\right\}=0. \quad \ldots\ldots\ldots\ldots\ldots (11)$$

The two roots of this equation coincide when

$$\left\{(\rho-1)(\rho^2-1)+(a\rho-b)\left(\frac{\rho^2}{1-a}-\frac{1}{1-b}\right)\right\}^2=4\rho(\rho-1)^2\left(\frac{\rho}{1-a}-\frac{1}{1-b}\right)^2. \quad \ldots (12)$$

The value $\rho_{\min.}$ given by equation (**12**) is the minimum value of ρ corresponding to a given pair of indices n_a and n_b for which real solutions of v exist. The corresponding value of v is given by

$$v_{\min.}=-\frac{\rho}{\rho-1}+\frac{\dfrac{\rho^2}{1-a}-\dfrac{1}{1-b}}{(\rho-1)^2}-\sqrt{\rho}\,\frac{\{\rho(1+a)-(1+b)\}\left\{\dfrac{\rho}{1-a}-\dfrac{1}{1-b}\right\}}{(\rho-1)^3}. \quad \ldots . (13)$$

Several common crown and flint glasses were selected from the catalogues of Messrs. Chance Bros. and of Messrs. Schott & Gen, and the results for doublets formed of these glasses is given in Table 1 in which are listed $\rho_{\min.}$ and $v_{\min.}$ and the two solutions of v, if real. Only the crown leading form is considered, as the flint leading form can always be considered in reverse.

TABLE 1

RECIPROCAL OBJECT DISTANCES FOR TYPICAL GLASS PAIRS

Glasses Crown	Glasses Flint	Indices (e ray) n_a	Indices (e ray) n_b	ν Value (e ray) a	ν Value (e ray) b	ρ ν_a/ν_b	Minimum Values $\rho_{\min.}$	Minimum Values $v_{\min.}$	Solutions v_1	Solutions v_2
BSC_1	EDF	1·51160	1·65568	64·6	33·8	1·910	1·899	−1·13	−0·65	−1·58
	DF	1·51160	1·62667	64·6	36·2	1·784	1·849	−1·05	—	
BSC_2	EDF	1·51951	1·65568	64·3	33·8	1·901	1·878	−1·11	−0·44	−1·72
	DF	1·51951	1·62667	64·3	36·2	1·775	1·825	−1·02	—	
HC	EDF	1·52104	1·65568	60·6	33·8	1·792	1·874	−1·10	—	
PSK3	EDF	1·55483	1·65568	63·5	33·8	1·876	1·778	−0·99	−0·44	−2·12
	F4	1·55483	1·62111	63·5	36·8	1·726	1·684	−0·89	+0·03	−1·74
MBC	EDF	1·57782	1·65568	57·9	33·8	1·712	1·712	−0·93	−0·93	
	DF	1·57782	1·62667	57·9	36·2	1·599	1·603	−0·83	—	
DBC	EDF	1·59113	1·65568	62·0	33·8	1·833	1·650	−0·88	+0·76	−2·27
	DF	1·59113	1·62667	62·0	36·2	1·712	1·532	−0·78	+0·89	−2·27

The results are shown graphically in Figure 1 where $\rho/\rho_{min.}$ is plotted against $v-v_{min.}$ using extra results from the glass pairs BSC_1–EDF, PSK3–F4, and MBC–DF with hypothetical values of ρ. Although the points for the different glasses do not lie exactly on a common curve, they are sufficiently close to it within the range of v studied, for the author has shown (Steel 1954) that, for telescope objectives, the smallest tolerance on ρ is $\pm 0 \cdot 01$.

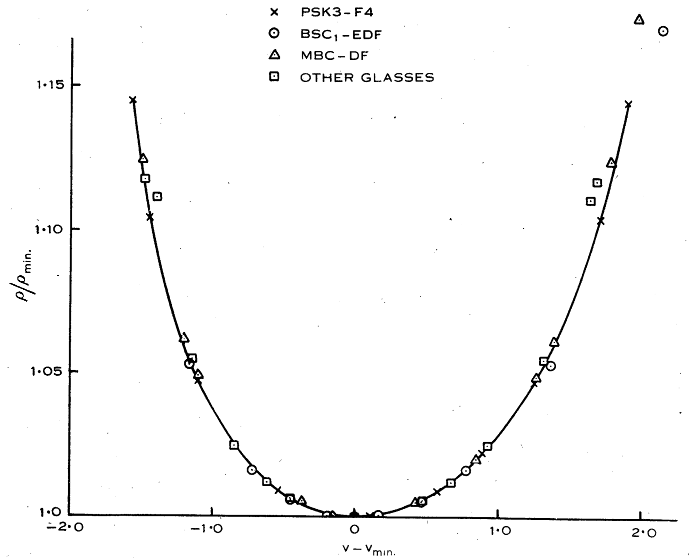

Fig. 1.—Variation of relative dispersion $\rho=\nu_a/\nu_b$ with reciprocal object distance v for unit power objectives.

This curve may be used to find the range of v that corresponds to a given tolerance on ρ. These latter tolerances are given in the article cited; for $\delta\rho=\pm 0 \cdot 01$, Table 2 has been drawn up to show the appropriate glass pair for

Table 2

GLASSES SUITABLE FOR A GIVEN VALUE OF v WHEN $\delta\rho=\pm 0 \cdot 01$

Range of v Glasses ..	$-1 \cdot 92, -1 \cdot 85$ DF–DBC	$-1 \cdot 82, -1 \cdot 64$ PSK3–F4	$-1 \cdot 82, -1 \cdot 58$ BSC_2–EDF	$-1 \cdot 79, -1 \cdot 72$ EDF–DBC	$-1 \cdot 72, -1 \cdot 34$ BSC_1–EDF
Range of v Glasses ..	$-1 \cdot 35, -0 \cdot 47$ MBC–EDF	$-1 \cdot 19, -0 \cdot 47$ MBC–DF*	$-1 \cdot 12, -0 \cdot 92$ F4–PSK3	$-0 \cdot 93, -0 \cdot 48$ BSC_1–EDF	$-0 \cdot 72, -0 \cdot 33$ EDF–PSK3
Range of v Glasses ..	$-0 \cdot 68, -0 \cdot 41$ EDF–BSC_2	$-0 \cdot 67, -0 \cdot 28$ PSK3–EDF	$-0 \cdot 59, -0 \cdot 32$ BSC_2–EDF	$-0 \cdot 52, -0 \cdot 07$ EDF–BSC_1	$-0 \cdot 53, +0 \cdot 19$ DF–MBC*
Range of v Glasses ..	$-0 \cdot 53, +0 \cdot 35$ EDF–MBC	$-0 \cdot 08, +0 \cdot 12$ PSK3–F4	$+0 \cdot 34, +0 \cdot 72$ EDF–BSC_1	$+0 \cdot 58, +0 \cdot 82$ EDF–BSC_2	$+0 \cdot 64, +0 \cdot 82$ F4–PSK3
Range of v Glasses ..	$+0 \cdot 72, +0 \cdot 79$ DBC–EDF	$+0 \cdot 85, +0 \cdot 92$ DBC–DF	$+1 \cdot 07, +1 \cdot 17$ EDF–PSK3		

* Although the pair MBC–DF has no real roots for v, the difference $\rho_{min.}-\nu_a/\nu_b$ is $0 \cdot 004$ and so lies within the tolerance $\delta\rho=\pm 0 \cdot 01$.

any given value of v in the range $-2<v<1$. Flint leading objectives are also included, the type of objective being shown by the order in which the glasses are listed.

It is seen that the range of real images, $-1<v<0$, is well covered by the glasses studied, and that the same glasses cover large regions of the total range of v. In particular, the four glasses BSC_1, BSC_2, MBC, and EDF can be used for all values of v from $-1 \cdot 82$ to $+0 \cdot 82$.

As the presence of chromatic aberration does not change the form of equations (**11**), (**12**), and (**13**), a similar method may be used for choosing the glass when this aberration is present, provided that ρ is derived from equation (**5**). But an extension of this method to the case of the other aberrations does not seem warranted and these cases would best be treated along the lines indicated in Section III (*a*).

IV. References

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