MULTIPLE IMAGES IN REFLECTING MICROSCOPES*

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Introduction

The normal image of a reflecting microscope is formed by a single reflection at each mirror surface. However, other images are possible; higher order images formed by two or more reflections at each surface, and a zero order " image " which is the object itself, if there is an unobstructed light path through the system. In a mirror system possessing central obscuration of the pupil, this unwanted light usually does not reach the axial image point, but it may give rise to a spread of light across the outer parts of the field, so reducing the contrast. The effects of these extra images are most noticeable when the objective is used in reverse as a condenser.

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These false images have been indicated by Norris, Seeds, and Wilkins (1951) and they have shown stops placed to eliminate them. In the present paper, methods of choosing the position of these stops are discussed in more detail, reference being made to the consequent vignetting of the primary image; this follows a demonstration that it is impossible to secure a corrected secondary image having any advantages over the primary image.

Correction of the Secondary Image

The secondary image, formed by two reflections at each mirror, is studied by the approximate method used by the author (Steel 1953) for investigating the primary image of a monocentric mirror pair. The objective is considered in reverse so that its magnification $-g$, where $u_1=-gu_4'$, is the reciprocal of the magnification when used as a microscope objective.

The third order spherical aberration ΣW_p is corrected when

$$\sigma=\frac{11-10g+11g^2\pm(1+g)\sqrt{(21-22g+21g^2)}}{2(5-6g+5g^2)}, \qquad \text{(1)}$$

where σ is the ratio of the radii of the two mirrors,

$$\sigma=r_1/r_2.$$

The two "geometrical" obscurations, as defined by the above article, are

$$q_1=(\sigma+5g\sigma-4g)/(2-\sigma-2g+3g\sigma), \qquad \text{(2)}$$

$$q_2'=(5-4\sigma+g)/(3-2\sigma-g+2g\sigma). \qquad \text{(3)}$$

Object at Infinity.—The case of a microscope objective of high magnification can be studied approximately by considering $1/g$ to be infinite. Equation (**1**) then gives $\sigma=(11\pm\sqrt{21})/10$.

As in the case of the primary image, the larger root, corresponding to a Cassegrain type objective, is of little practical interest as the image point is virtual. The smaller root, $\sigma=0\cdot6417$, corresponds to a Schwarzchild type objective for which $q_1=0\cdot284$, $q_2'=1\cdot417$. The physical obscuration p is given by $1/q_2'$ and is $0\cdot706$; for the primary image it is $0\cdot447$.

The fifth order spherical aberration is also found to be less favourable being $1\cdot27$ times that for the primary image at the same numerical aperture.

Finite Object.—The closest that the object can conveniently be placed to the objective is at the pole of the concave mirror. Then $q_1=0$ and it is found that $1/g=3$ and $p=0\cdot6$. For the object at the same position, the primary image gives $1/g=5$, $p=0\cdot333$.

Hence, for a given magnification, the secondary image has a higher obscuration than the primary image and its quality is therefore inferior.

Elimination of the Secondary Image

We now consider the elimination of the secondary image in a monocentric mirror pair, corrected for an object at infinity. The methods used are applicable to other types of reflecting systems.

Axial Rays.—The paths of the axial rays from infinity that form each image are shown in Figure 1, this figure corresponding to that given by Norris, Seeds, and Wilkins (1951). The rays forming the primary image are shown above the optical axis and those forming the secondary image below. When the system is used in the reverse sense as an objective, no axial rays from the object can form a secondary image.

In a monocentric objective, the obscuration for the secondary ray is $0 \cdot 644$ and it contains $0 \cdot 146$ of the light in the primary image. The secondary focal length is one-half the primary focal length and the secondary rays cross the primary field at angles above $0 \cdot 288u$, where u is the numerical aperture of the objective. A field stop should therefore be used to limit the object to half this size.

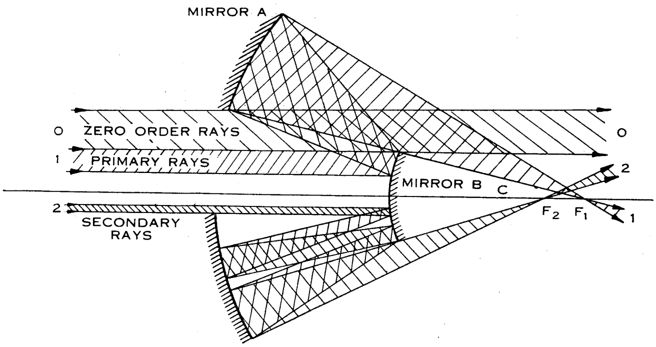

Fig. 1.—Axial rays forming primary and secondary images.

If no field stop can be used, or if a field greater than $0 \cdot 144u$ is required, the secondary rays should be blocked completely. At their first reflection at both mirrors and also in the final image space, the axial rays forming the secondary and higher order images all lie inside the primary rays and so a central stop placed at any of these positions will eliminate them. Oblique rays, however, require further consideration.

Oblique Rays.—Zero order image. This effect is caused by direct light from the object reaching the image plane. It can occur in both directions, either when the system is used as an objective or as a condenser. However, in both cases, this light can be kept outside the primary image provided a field stop is used at the object or the image, in combination with a suitable choice of the size of the hole in the concave mirror.

For a monocentric system, it is found that the angular semi-field should be limited by a stop to a value less than $0 \cdot 224u$. For non-concentric systems with obscuration p, the field would be $up/2$.

Secondary image. The effect on oblique rays of stops at the various positions suggested for axial rays can be seen in Figure 2. This shows the various apertures and obstructions presented by the system to an object at infinity, the scale being for a numerical aperture of $0 \cdot 5$. The stops for the primary rays are shown above the optical axis and those for the secondary rays below. The lines at A and B represent the positions of the concave and convex mirrors respectively, at A' and B' their first images and at A'' and B'' their second images in the whole system.

The full line at A and at its images represents the reflecting part of the concave mirror and the broken line the minimum size of the hole, both calculated for axial rays. To prevent vignetting for a field of semi-angle e, the radii of the hole and of the outer edge of the mirror should both be increased by $3 \cdot 24ef'$. The radius of the inside limit of the reflecting surface should be decreased by the same amount except when this edge is used as a stop to cut out secondary rays. An upper limit can be placed on the size of the hole if it is to block the zero order image, so in general the hole should be smaller than the inside limit of the reflecting surface, the intervening portion being blackened.

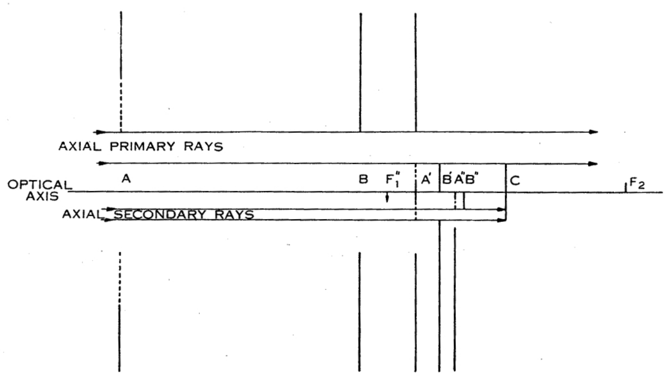

Fig. 2.—Obstructions and apertures for object at infinity.

The convex mirror cannot be increased in diameter beyond the value chosen for axial rays without increasing the obscuration. It acts as an aperture for primary rays at B and as an obscuration at B'. For secondary rays, B' acts as an aperture and rays that pass through it continue on to form the secondary image.

The primary image plane as seen by rays forming the secondary image is at F_1'', the size shown being the maximum as indicated above. If a larger field is desired, no rays should be allowed to pass through the aperture at B' and this may be done by a central stop either at A' or B and another at C, the centre of curvature.

A single stop at the focus of the secondary rays is also effective. Its image for primary rays is at F_2' and the stop should be small enough not to vignette these rays.

Vignetting of the primary image. Provided that the hole and the outside edge of the concave mirror are made larger than the size demanded for axial rays by the amount indicated above, there is no vignetting at the edge of the pupil. But, if a central stop is used at the centre of curvature in combination with one at either of the mirrors to eliminate secondary rays, there are obstructions at C and B' and either at A' or B and some vignetting at the edge of the obstruction ; it is smaller if the second stop is at the concave mirror.

If a stop at the secondary focus is used, there is no vignetting nor any light due to a secondary image for angular semi-fields less than $0 \cdot 175u$. A stop in this position, however, would reduce the working distance of the microscope, and the gain in field size would be small.

Conclusions

The detailed discussion above has been based on paraxial optics and an object at infinity, but the author has obtained similar results for particular cases treated by trigonometrical ray traces. Similar considerations would also apply to non-concentric objectives and to objectives made of more than two mirrors, although in this latter case the treatment becomes more complicated, for every pair of mirrors can give rise to multiple reflections.

Thus it is most important to use a field stop at the object or image of all mirror systems. Auxiliary central stops are also advantageous in eliminating the light due to false images.

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References

Norris, K. P., Seeds, W. E., and Wilkins, M. H. F. (1951).—*J. Opt. Soc. Amer.* **41** : 111.
Steel, W. H. (1953).—*Rev. Opt.* (*Théor. Instrum.*) **32** : 269 (Thèse, Paris (1952)).