DIFFUSION COEFFICIENTS FROM THE RATE OF DECAY OF METEOR TRAILS

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[*Manuscript received December* 29, 1954]

Summary

The effective diffusion coefficient for a meteor trail is calculated from the theory of ambipolar diffusion and the physical constants of the upper atmosphere. The absolute value of the diffusion coefficient so calculated, and also its gradient with height, are confirmed by measurement of the rates of decay of a large number of meteor echoes of known heights. The individual values show considerable scatter, most of which is attributed to a regular diurnal variation in the value of the diffusion coefficient. Amplitude fluctuations in persistent echoes are also briefly discussed.

I. Introduction

The problem of the behaviour of ionized meteor trails after formation has been considered in detail by Greenhow (1950, 1952). In the second paper the distinction between echoes of short and long duration was drawn, and evidence was presented that the dissipation of the ionization is primarily due to diffusion processes. Fluctuations in the amplitudes of long-enduring echoes were ascribed to distortion of the initially uniform ionized column into two or more reflecting centres. The observations discussed by Greenhow were all obtained by pulse techniques on wavelengths of 4·2 and 8·4 m. The more recent observational data are contained in the comprehensive report by Kaiser (1953).

Huxley (1952) has shown that the durations of meteor echoes are of the order of magnitude to be expected from ambipolar diffusion of the trails and the known behaviour of ions and electrons in gases. In this paper a revised estimate of the diffusion coefficient in the meteor zone is obtained by an extension of Huxley's theory of ambipolar diffusion, and these theoretical values are compared with data obtained at Adelaide using the c.w. technique at a wavelength of 11·2 m, described by Robertson, Liddy, and Elford (1953).†

II. The Effective Diffusion Coefficient

According to Kaiser (1953) the loss of electrons by recombination or by attachment is negligible except perhaps in the final decay of very persistent echoes, and the heat generated during the formation of the trail has little effect

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† *Note added in Proof.*—Theoretical calculations of the effective diffusion coefficient, essentially similar to those of Section II but without consideration of the height gradient, have been published by Massey and Sida (1955) after preparation of this paper. Some measurements by Greenhow and Neufeld (1955) on a much smaller sample of echoes have also appeared.

upon its subsequent behaviour. It will therefore be assumed that the dissipation of meteor ionization is due to ambipolar diffusion alone.

The radial distribution of electrons in an initially linear concentration, found by solution of the diffusion equation, is

$$n_e = (\alpha/4\pi Dt)\exp(-r^2/4Dt),$$

where D is the effective diffusion coefficient and α the line density of electrons (Huxley 1952). The distribution is therefore Gaussian. If the line density α is sufficiently small, the incident wave penetrates throughout the column and the echo amplitude decays exponentially from an initial value A_0 according to the law

$$A = A_0 \exp(-16\pi^2 Dt/\lambda^2). \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

For $\alpha < 2\cdot 4\times 10^{12}$/cm, A_0 is the maximum amplitude according to the Lovell-Clegg scattering formula, although, if predicted resonance effects (Herlofsen 1951; Kaiser and Closs 1952) are present, this is true only for parallel polarization. For transverse polarization the amplitude near the beginning of the echo is enhanced by the resonance and in this event the exponential decay law (**1**) will cease to apply. When $\alpha > 2\cdot 4\times 10^{12}$/cm (persistent echoes) the law (**1**) applies only to the final stage of decay of the echo, when the electron density has become sufficiently dilute.

The effective diffusion coefficient for electrons in a meteor trail may be written

$$D \sim D_i(1 + T_e/T_i),$$

where D_i is the diffusion coefficient for positive ions and T_e and T_i are the electron and ion temperatures respectively. Since $T_e \sim T_i$ (Huxley 1952), then $D \sim 2D_i$. The coefficient D_i may be evaluated, without recourse to the theoretical formulae of kinetic theory, following an unpublished method due to Huxley and Robertson.

The diffusion coefficient D_i and the mobility a_i of a group of singly charged ions are connected by the relation

$$D_i = a_i kT/e, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where k is Boltzmann's constant and e the electronic charge. The mobilities in nitrogen of the positive ions of a large number of metals have been measured (Tyndall 1938) and it is found that at a pressure of 760 mm Hg and a temperature of 18 °C the mobilities of these ions in most instances have values lying between 2 and 3 cm^2 sec^{-1} V^{-1}, although the masses of the ions differ greatly. Further, Pearce (1936) found that, for nitrogen and caesium ions moving in nitrogen at constant temperature, a change in temperature of 400 °C changed a_i by a factor of 2 only. If this behaviour is accepted as typical, then for temperatures between 200 and 250 °K and at a pressure of 760 mm Hg, the values of a_i for meteor ions are about $2\cdot 5$ cm^2 sec^{-1} V^{-1} $= 7\cdot 5\times 10^2$ cm^2 sec^{-1} (e.s.u.)$^{-1}$. Since a_i is inversely proportional to the molecular concentration, the value of D is

$$\begin{aligned} D &= 2D_i = 1\cdot 50\times 10^3 (760/p)(T^2/291)k/e \\ &= 1\cdot 13\times 10^{-3} T^2/p \quad \text{cm}^2/\text{sec}. \quad \ldots\ldots\ldots\ldots\ldots \quad (3) \end{aligned}$$

Using values of T and p found by means of rockets (Rocket Panel 1952) the variation of D with height may be found from (**3**). This relation, sketched in Figure 3, is to be compared with measured values of D.

The effect of the Earth's magnetic field, not so far considered, is to modify the diffusion coefficient for electrons, D_e, without affecting the more massive positive ions. For electrons moving parallel to the magnetic field in the absence of positive ions, the diffusion coefficient is $D_{||}=kT/m\nu \gg D_i$. Here ν is the collisional frequency and m the electronic mass. The rate of diffusion of the meteor trail in this direction is clearly controlled by the positive ions at all heights.

On the other hand, for electrons alone moving transverse to the magnetic field,

$$D_{\perp}=D_{||}\nu^2/(\nu^2+\omega^2)$$
$$\sim D_{||}\nu^2/\omega^2, \quad \text{if } \omega^2/\nu^2 \gg 1.$$

With the gyro-frequency $\omega \sim 10^7$ radians/sec, and using the expression for the collisional frequency $\nu=9\cdot36\times10^7 p$ (Crompton, Huxley, and Sutton 1953),

$$D_{\perp}=1\cdot44\times10^5 Tp, \quad p \text{ in mm Hg.} \quad \ldots\ldots\ldots\ (4)$$

Comparing (**4**) with (**3**),

$$D_{\perp}/D=1\cdot28\times10^8 p^2/T. \quad \ldots\ldots\ldots\ (5)$$

Again using the rocket data, it is found that $D_{\perp}=D_i=D$ at a height of approximately 92 km, and at this height electrons and positive ions diffuse at the same rate in directions transverse to the Earth's magnetic field. The asymmetry of the effective diffusion coefficient will result in elliptical cross sections for diffusing meteor trails above about 90 km. At heights above 92 km the ability of the electrons to retard the transverse motion of the more rapidly diffusing positive ions will presumably be limited, and it appears that D does not fall very much below D_i at any time. Marked departures from cylindrical symmetry are therefore not expected, and values of D derived from (**3**) should at most require reduction by a factor of 2 at high levels to take account of the presence of the Earth's magnetic field.

III. Amplitude Fluctuations in Persistent Echoes

According to the theory of radio reflections from meteor trails (Kaiser and Closs 1952) the short decay type of echo is characterized by a duration, defined as the time required for the echo amplitude to fall to 1/e of its initial value, which is independent of the line density α. The decay follows the exponential law (**1**) after an initial rapid rise to maximum amplitude. The long-enduring type of echo, on the other hand, shows a slower rise in amplitude to a flat maximum, and a final rapid exponential decay. Examples of these two types of echo are given in Figure 1, in which the amplitudes are plotted to logarithmic scale.

Echoes whose duration, for the Adelaide wavelength of 11·2 m, exceeds 2 sec rarely show the regular rise and decay in amplitude exemplified by the echo shown in Figure 1. The amplitude of such a persistent echo usually fluctuates irregularly. Often it cannot be measured, as with the c.w. technique it is necessary to pick out the times when the sky (reflected) wave is in phase or anti-phase with the ground wave, and if the echo is too confused these times cannot be identified. Such complex echoes are often associated with an irregular " Doppler " period, i.e. the beat period between sky wave and ground wave; and, less commonly, with the appearance of multiple range traces whose structure

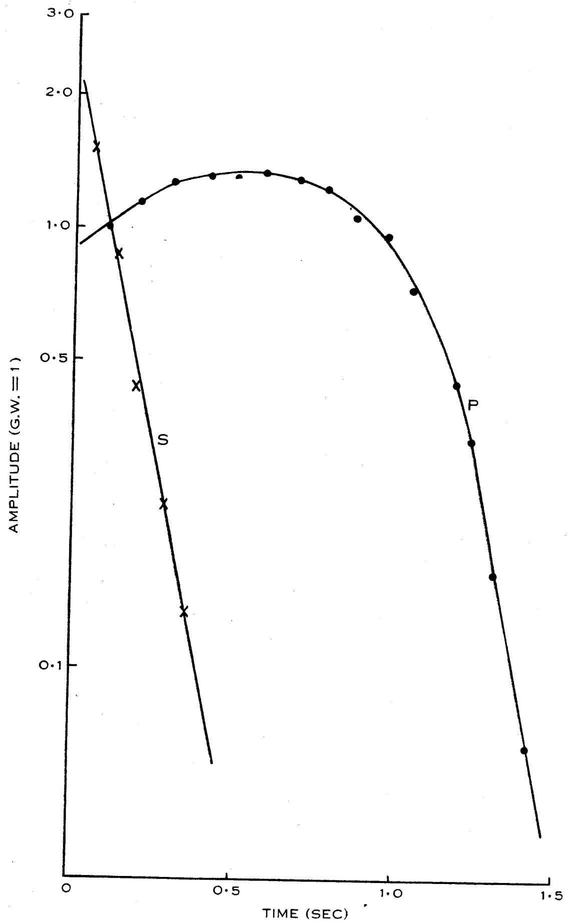

Fig. 1.—Amplitudes of typical echoes of short (curve S) and persistent (curve P) types.

may vary with time. Also, many echoes have been noted in which the amplitude fluctuations set in some time after the commencement of the echo. These facts confirm the interpretation of the irregular amplitude fluctuations in terms of the distortion of an initially uniform trail into several reflecting centres (Greenhow 1952).

Whilst it is believed that the establishment of multiple scattering centres is due to atmospheric turbulence, echoes of the type sketched in Figure 2 suggest that the scale of the turbulence is not necessarily large. Figure 2 (*a*) is an example of a trail of constant slant range, reflecting primarily from two centres. The relative velocity v of these two centres is found from the formula

$$v=\lambda/2T,$$

where T is the period of the amplitude fluctuation. Since $T=0\cdot77$ sec for this echo, $v=7$ m/sec. The Doppler period is remarkably constant over the whole duration of the echo, the mean half-period over 40 Doppler cycles being $0\cdot065\pm0\cdot006$ sec, corresponding to a line-of-sight velocity of the trail drifting in the local wind of 43 m/sec. Figure 2 (*b*) shows a more confused echo, but again the slant range remains constant and the mean Doppler half-period of

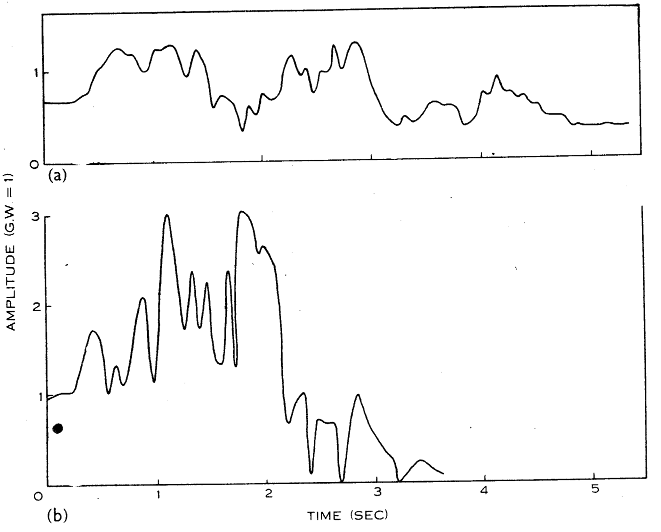

Fig. 2.—Examples of amplitude fluctuations in persistent echoes.

$0\cdot071\pm0\cdot009$ sec does not alter over the duration of the echo. Echoes such as these, which are by no means uncommon, are presumably produced by trails broken up into two or more reflecting centres which, however, all drift in the local wind with essentially the same velocity.

Finally, a small number of persistent echoes, regular in amplitude, show very rapid slant range drifts of the order of 20 km/sec or more. According to Ellyett (1950) such rapid range drifts are caused by bending of the trail in a small wind gradient.

IV. The Measured Diffusion Coefficients

(*a*) *Method of Measurement*

The rates of decay of over 1000 echoes of known heights have been determined by measurement of the amplitudes at successive maxima and minima of the Doppler beat pattern. Only those echoes which show a regular exponential decay have been used, and the rate of decay of an individual echo is found by fitting a straight line to the plot of the logarithm of the amplitude *v.* time.

The slope of this line fixes the time in seconds, τ, for the echo amplitude to decay to $1/e$ of its initial value, and from (**1**) we find for the diffusion coefficient

$$D=0\cdot 80\times 10^{4}/\tau$$

for $\lambda=11\cdot 2$ m. The method of height determination has been described by Robertson, Liddy, and Elford (1953).

TABLE 1

MEASURED DIFFUSION COEFFICIENTS

Date	Type of Meteor	Number in Group	Mean Height (km)	Mean D $(\times 10^{-4})$	Slope of $\ln D$ *v.* h $(\times 10^{-3})$
December 10–15, 1952	Geminid	70	89·3	3·21	2·28
16–20, 1952	Sporadic	112	89·7	3·61	0·89
June 5–12, 1953	ζ-Perseid	43	90·4	4·42	1·37
	Arietid	70	89·5	4·00	0·95
	Sporadic	170	89·5	3·27	1·21
September 7–30, 1953	Sporadic	539	91·2	3·06	1·23

The echoes measured comprise both shower and sporadic meteors detected during December 1952 and June and September 1953.

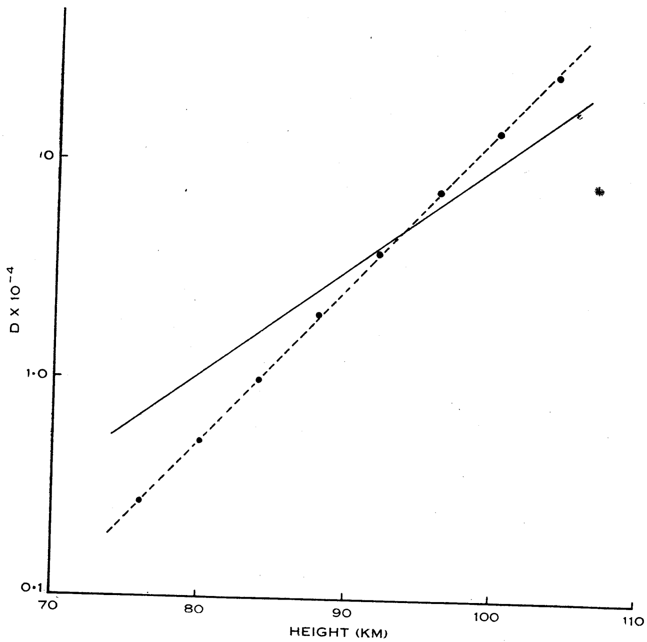

Fig. 3.—The dependence of the diffusion coefficient D upon height h.
- - - - - - - - Theoretical. ———— Measured.

(*b*) *The Height Gradient*

The mean height and mean diffusion coefficient for each group of echoes are given in Table 1, along with the slope of the linear relation between $\ln D$ and the height h, found by the method of least squares. The relation between D and h, based on all echoes measured, is sketched in Figure 3, together with the theoretical relation derived in Section II. In making the least squares analysis of the measurements it has been assumed that the diffusion coefficient increases exponentially with height, which would be true only for an isothermal atmosphere. Since, however, the temperature changes in the region of interest are much smaller than the pressure changes, the expression (3) for D is dominated by the pressure p, and this assumption is not unreasonable.

It will be seen from Figure 3 that the theory of Section II predicts correct values of D. However, the measured increase of D with height is somewhat less than that predicted theoretically. A possible reason for this may lie in the selection of echoes for measurement. At heights above 95 km the decay of many echoes is so rapid that the echo amplitude falls below receiver noise level in less than one complete Doppler cycle; the rates of decay of such short echoes cannot be measured with sufficient accuracy and they are rejected. At heights below 80 km many of the more slowly decaying echoes show an irregular decay and are likewise rejected. It is therefore to be anticipated that the agreement between the theoretical and the measured height gradients is even better than is suggested by Figure 3.

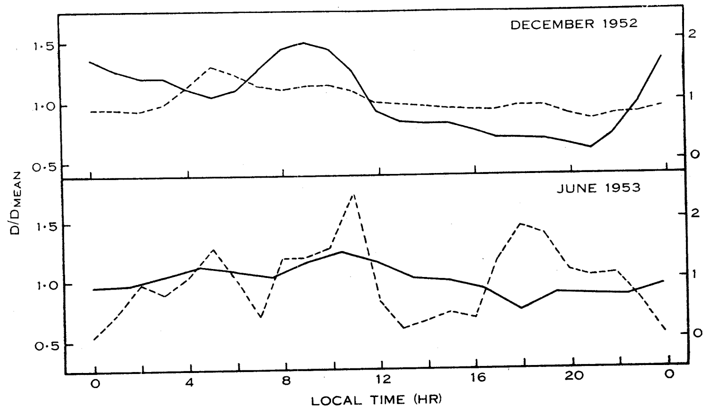

Fig. 4.—Comparison between the diurnal variations in the diffusion coefficient D and the wind speed V.
——— D/D_{mean}. - - - - - - V/V_{mean}.

(c) *The Diurnal Variation*

If the dependence of D upon height is removed by dividing the data into suitable height groups and then forming D/D_{mean}, where D_{mean} is the mean value of D for a given height group, the diurnal variation of D may be studied. The diurnal variations so found for the June and December sporadic groups are shown in Figure 4. For the more numerous September sporadic group it was possible to obtain the diurnal variation in D for different height groups separately, without resort to averaging. The September variation is similar to that during June and December, with morning maxima and evening minima. The data for September also confirm that the diurnal variation is present over the whole height range of the meteor zone, with a tendency for the relative variation in D to increase with height.

Since it is possible that a meteor trail may be deformed, and its rate of decay altered, by magneto-dynamic forces which operate as the trail moves in the Earth's magnetic field, the data have been examined for correlation between the diffusion coefficient and the local wind speed. Some evidence that a high value of D is associated with a high wind speed is contained in Figure 5, which refers only to December 1952. Each point represents the average of a selected group of echoes, occurring within a period of 4 hr or less, the groups themselves being distributed from 06 to 20 hr.

But the attempt to relate the diurnal variation in D to the pattern of the local wind speed is only partially successful. During December, when the eastwards prevailing wind is particularly strong, the diurnal variation of wind

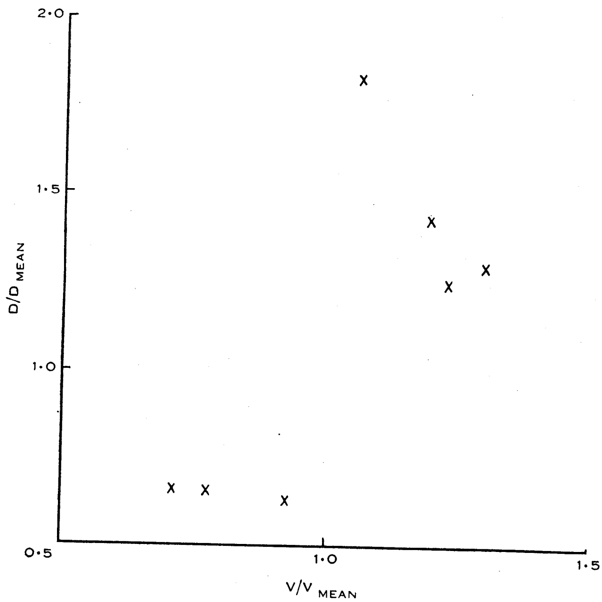

Fig. 5.—Correlation between wind speed and diffusion coefficient for selected groups of echoes, December 1952.

speed V follows closely the diurnal variation in D (see Fig. 4). For June the agreement, although not so marked, is reasonable. During September, when the wind pattern is very confused, the diurnal variation in the wind speed is almost in phase quadrature with the variation in the diffusion coefficient.

Undoubtedly, the diurnal variation in D is not wholly produced by variations in upper atmosphere winds. Although the phase of this diurnal variation appears to be incompatible with a thermal origin under solar influence, such an explanation cannot be ruled out in the present state of knowledge.

V. Scatter Amongst Individual Measurements

Diffusion coefficients for the individual echoes show considerable scatter about the mean D v. h relation. Such scatter is evident in the mean values of D, and in the mean slopes, for the different groups listed in Table 1. It is more clearly brought out in Figure 6, in which the individual values for the December 1952 sporadic group are plotted. Although the shower echoes give larger mean values of D and larger slopes than the sporadic echoes, the scatter between the groups is so large that it is doubtful whether this apparent difference between shower and sporadic echoes is significant.

The greater part of this unexpectedly large scatter amongst the values of D derived from individual echoes is to be ascribed to the diurnal variation already discussed. Two other processes which may contribute to the scatter are resonance effects and the influence of the Earth's magnetic field.

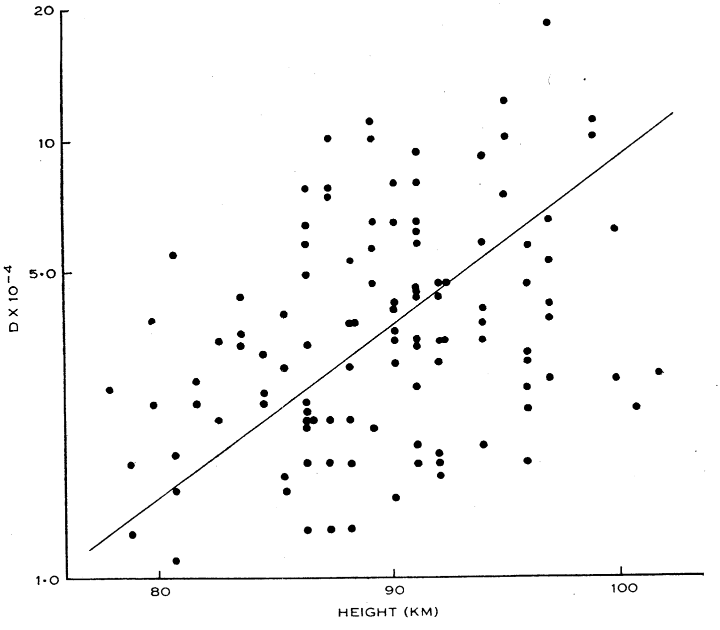

Fig. 6.—Scatter diagram of individual diffusion coefficients for 112 sporadic meteors, December 1952.

(a) *Resonance Effects*

The enhancement of echo amplitude in transverse, relative to parallel, scattering is confined to the initial stages of the echo, and is not expected to be large for the echoes detected by the equipment, for which the line density of electrons exceeds 10^{11}/cm. However, the existence of such polarization effects has been demonstrated by Robertson (1953) in echoes detected at Adelaide. Whilst they may contribute to the scatter in measured diffusion coefficients, they are not considered to be of major importance. Some abnormal echoes which gave diffusion coefficients whose values decreased with time may have been influenced by plasma resonance effects.

(b) *Magnetic Effects*

It was suggested in Section II that the rate of diffusion of a meteor trail may be influenced by its orientation relative to the Earth's magnetic field.

The reduction in D should be greatest for trails lying parallel to the magnetic field, and is expected to become effective at heights exceeding 95 km. There is no evidence in the measured data for any general falling off of the rate of increase of D with height up to 105 km, but it is apparent that magnetic effects could produce considerable scatter in D, especially at the greater heights. Unfortunately, the number of shower echoes, for which the direction of the trail is at least roughly known, is too few to test this hypothesis; and for the more numerous sporadic echoes only the directions of the reflection points, and not the orientations of the trails, are known.

VI. Conclusions

The theory of ambipolar diffusion of meteor trails, even in its present simplified form, appears adequate to account for the broad features of the decay of meteor echoes, and in particular for the variation of the diffusion coefficient with height. It does, however, appear desirable to confirm the absolute values of the diffusion coefficient by laboratory measurements of the mobilities of the meteor ions themselves.

It is clear from the scatter amongst the individual diffusion coefficients that it is impossible to determine the height of an individual meteor trail from the rate of decay of the echo produced by it. The relation between diffusion coefficient and height can only be applied, with confidence, statistically and to very large samples.

VII. Acknowledgments

The author wishes to thank Professor L. G. H. Huxley for his interest in this work and for permission to reproduce in Section II his unpublished calculation of the effective diffusion coefficient. He is also grateful to Dr. W. G. Elford for valuable discussion and for the measurement of diffusion coefficients for the Geminid echoes.

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